$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \text{Mass} \times \text{acceleration}$$

$$\text{Inertia force} = \text{Mass} \times \text{acceleration}$$
$$= \rho \times \text{volume} \times \text{acceleration}$$
$$= \rho \times L^3 \times L/T^2 = \rho^2 L^2 v^2$$

$$\text{Viscous force} = \text{Shear stress} \times \text{Area}$$
$$= \mu \left[\frac{dv}{dy}\right] \times L^2 = \mu v L$$

$$Re = \frac{\rho^2 L^2 v^2}{\mu v L} = \frac{\rho v L}{\mu} = \frac{vD}{\nu} \quad \left(\frac{\mu}{\rho} = \nu\right)$$

$v \Rightarrow$ velocity, $\rho \Rightarrow$ density, L = Characteristic of Linear diamension $\}$ = Dia of pipe

μ = viscosity of the fluid. ν = kinematic viscosity of the fluid.

Flow through pipes in series and parallel

7) [Apr.10] For a town water supply, a main pipe line of diameter 0.4m is required. As pipes more than 0.35m diameter are not readily available, two parallel pipes of same diameter are used for water supply. If the total discharge in the parallel pipes is same as in the single main pipe, find the diameter of parallel pipe. Assume coefficient of discharge to be the same for all the pipes.

Sol

For a single pipe system,

The head loss in 0.4m diameter pipe,

$$h_f = \frac{4flv^2}{2gD} = \frac{4flv^2}{2g \times 0.4}$$

When pipes are connected in parallel,

$$Q = Q_1 + Q_2$$

The head loss in parallel pipes,

$$(h_f)_p = h_{f1} = h_{f2}$$

$$(h_f)_p = \frac{4fl_1v_1^2}{2gD_1} = \frac{4fL_2v_2^2}{2gD_2} = \frac{4fLv_p^2}{2gD_p}$$

It is given that the total discharge in the parallel pipe is same as in the single main pipe and therefore, equating both the head losses in single pipe an pipes in parallel

$$\frac{v_p^2}{v^2} = \frac{D_p}{0.4}$$

Using continuity principle

$Q = A \times V$

$$v \times \frac{\pi}{4} \times (0.4)^2 = 2v_p \times \frac{\pi D_p^2}{4}$$

$$\frac{v_p}{v} = \frac{0.16}{2D_p^2}$$

Using ① & ②

$$\frac{v_p^2}{v^2} = \frac{D_p}{0.4} = \frac{0.16^2}{(2D_p^2)^2} = \frac{0.0256}{4D_p^4}$$

$$\frac{D_p}{0.4} = \frac{0.0256}{4D_p^4}$$

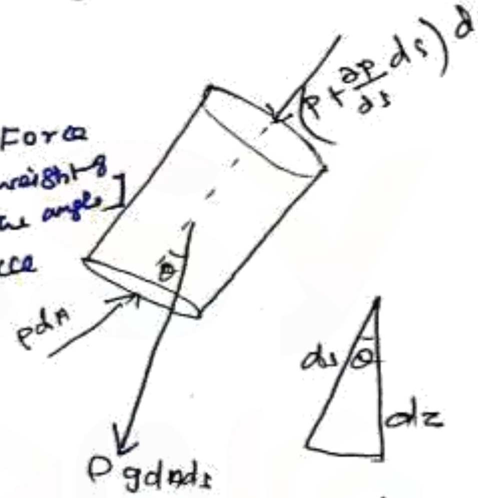
$$\boxed{D_p = 0.30314\ m}$$

8) Eular Equation of motion [N/D-15]

this is equation of motion in which the forces due to gravity and pressure are taken into consideration. this is derived by considering the motion of fluid element along a stream line as

Consider a stream line in which flow is taking place in s-direction. consider a cylindrical element of c/s dA and length ds. the forces acting on the cylindrical element are

1. Pressure force pdA in the direction of flow.
2. Pressure force $\left[p + \frac{\partial p}{\partial s} ds\right] dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$



Let θ is the angle b/w the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction s.

$$pdA - \left[p + \frac{dP}{ds} ds\right] dA - \rho g dA ds \cos\theta$$

$$= \rho dA ds \times a_s$$

where a_s is the acceleration in the direction s

Now

$$a_s = \frac{dv}{dt} \text{ where } v \text{ is the function of s\&t}$$

$$= \frac{dv}{ds}\frac{ds}{dt} + \frac{dv}{dt} = \frac{vdv}{ds} + \frac{dv}{dt}$$

$$\therefore \frac{ds}{dt} = v$$

[Force + op. Force + op. weight & the angle]

= wight × acce

if the flow is steady $\frac{dv}{dt} = 0$

$$\boxed{a_s = \frac{vdv}{\partial s}}$$

substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g dA ds \cos\theta = \rho dA ds \times \frac{dv}{ds}$$

Dividing by $\rho ds dA$, $-\frac{\partial p}{\rho ds} - g\cos\theta = \frac{vdv}{ds}$

(or)

$$\frac{\partial P}{\rho ds} + g\cos\theta + v\frac{dv}{ds} = 0$$

But from, we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0$$

(or)

$$\frac{dp}{\rho} + g\,dz + v\,dv = 0 \qquad g = \rho$$

(or) $$\frac{dp}{\rho} + \rho\,dz + v\,dv = 0$$

Equation is know as Euler's equation of motion

Bernouli's equation is obtained by integrating the Euler's equation of motion

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant} \qquad \text{Pg } 264$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid (or) pressure head.

$\frac{v^2}{2g}$ = kinetic energy per unit weight (or) kinetic head

z = potential energy per unit weight

Assumptions

The following.

1* The fluid is ideal (viscosity is zero) ② The flow is steady

3* The flow is incompressible ④ The flow is irrotational

9) Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. Find the total head loss total energy per unit weight of the water at c/s, which is 5 m above the datum line

Sol

Given

Ø of pipe = 5cm = 0.5m

Pressure = p = 29.43 N/cm^2 = $29.43 \times 10^4 \, N/m^2$

velocity v = 2.0 m/s

Datum head z = 5m

Total head = Pressure head + kinetic head + datum head

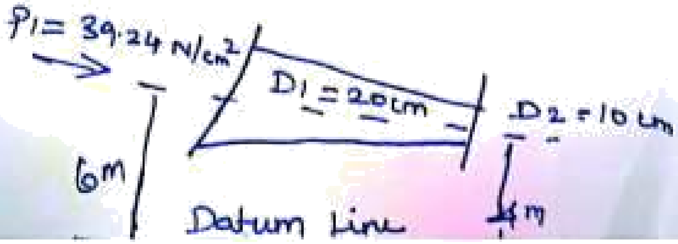
$$\text{Pressure head} = P/\rho g = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30\,m \quad \left[\rho = 1000 \, kg/m^3\right]$$

$$\text{Kinetic head} = \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204\,m$$

$$\text{Total head} = \frac{P}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5$$

$$= 35.204\,m$$

The water is flowing through a pipe having diameter 20cm and 10cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 l/s the section 1 is 6 m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm^2, find the intensity of pressure at section 2



Sol

At section 1 $\quad D_1 = 20cm = 0.2m$

$$A_1 = \frac{\pi}{4}(.2)^2 = 0.0314\, m^2$$

$$P_1 = 39.24\, N/cm^2$$

$$= 39.24 \times 10^4\, N/m^2$$

$$Z_1 = 6.0m$$

At section 2, $\quad D_2 = 0.10m$

$$A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\, m^2$$

$$Z_2 = 4m$$

$$P_2 = ?$$

Rate of flow $\quad Q = 35\, lit/s = \frac{35}{1000} = .035\, \frac{m^3}{s}$

Now

$$Q = A_1V_1 = A_2V_2$$

$$V_1 = Q/A_1 = \frac{.035}{.0314} = 1.114\, m/s$$

and $\quad V_2 = Q/A_2 = \frac{.035}{.00785} = 4.456\, m/s$

Applying Bernoulli's equation at sec 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4$$

$$40 + 0.063 + 6 = \frac{P_2}{9810} + 1.012 + 4$$

$$46.063 = \frac{P_2}{9810} + 1.012 + 4$$

$$46.063 = \frac{P_2}{9810} + 5.012$$

$$P_2 = 41.051 \times 9810\, N/m^2$$

$$= 40.27\, N/cm^2 \times 10^4$$

Reynolds number, $Re = \frac{\rho v D}{\mu}$

Density of water at 25°C, $\rho = 997.05\ kg/m^3$

Viscosity of water at 25°C, $\mu = 0.00089\ N\text{-}s/m^2$

$$Re = \frac{997.05 \times 3.06 \times 0.025}{0.00089} = 85701.49$$

Which is greater than 4000 so, the flow is turbulent.

From moody chart $f = 0.04$

Total head loss $h_L = h_{L,bend} + h_{L,entrance} + h_{L,bend}$

$= k_L\ bend \frac{v^2}{2g} + k_{L,entrance} \frac{v^2}{2g} + k_{L,bend} \frac{v^2}{2g}$

We know that

k_L entrance $= 0.12$

k_L bend $= 0.3$

$$= 0.3 \times \frac{3.06^2}{2 \times 9.81} + 0.12 \times \frac{3.06^2}{2 \times 9.81} + 0.3 \left[\frac{3.06}{2 \times 9.81}\right]^2$$

Total head loss $h_L = 0.344\ m$

pressuredrop, $\Delta p = \rho g h_L = 997.05 \times 9.81 \times 0.344 = 3364.69\ Pa$

pressure drop, $\Delta p = p_1 - p_2$

$= p_1 - P_{atm}$

$3364.69 = p_1 - 101300$

Absolute pressure at the top

$$\boxed{p_1 = 104664.69\ Pa}$$

11) A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head b/w A & B. [A/M-16]

Sol

Dia of pipe $D = 400mm = 0.4m$

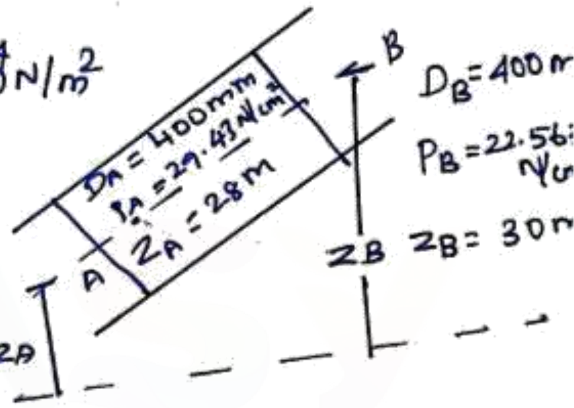
$V = 25\ m/s$

At point A,

$P_A = 29.43\ N/cm^2 = 29.43 \times 10^4\ N/m^2$

$Z_A = 28m$

$V_A = V = 25\ m/s$



∴ Total energy at 'A'

$$E_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$$

$$= \frac{29.48 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28 = 89.85m$$

At point B,

$P_B = 22.563\ N/cm^2 = 22.563 \times 10^4\ N/m^2$

$Z_B = 30m$

$V_B = V = V_A = 25\ m/s$

Total energy at 'B' $E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 84.85m$$

loss of energy $= E_A - E_B = 89.85 - 84.85 = \underline{5.0\ m}$

12 Explain & details '8

Practical Applications of Bernoulli's Equation

* Incompressible fluid flow

(269) derivation

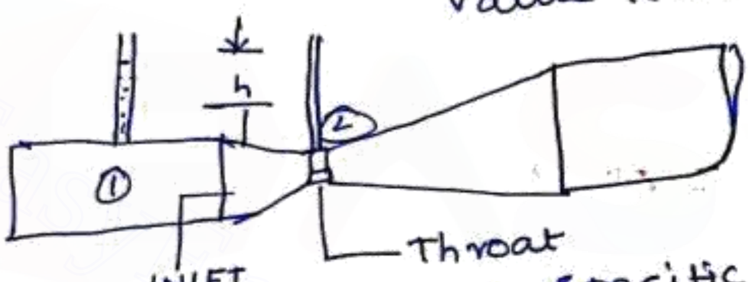
1* Venturimeter

2* Orifice meter

3* Pitot-tube

$$Q_{act} = C_d \times \frac{a_1 \times a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

C_d = Co-efficient of venturimeter and it's value is less than 1



In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16cm and 8cm respectively. A is 2m above B. The pressure gauge readings have shown that the pressure at B is greater than A by 0.981 N/cm². Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U-tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.

In b/w the vena-contracta and the wall of the pipe, a lot of eddies are formed as shown in fig 2.23. These eddies cause a considerable dissipation of energy. Most of the energy is lost due to these eddies and dissipation of energy.

Let $A_c = c\text{-}c$, V_c = Velocity $c\text{-}c$ $A_2 = 2\text{-}2$ h_c = loss of head due to sudden contraction

h_c = Loss up to Vena-contraction + loss due to sudden enlargement beyond Vena-contracta

Actually, losses upto Vena contracta are very small and may be neglected

$$\boxed{h_c = \frac{(V_c - V_2)^2}{2g}}$$

$\therefore$ loss due to sudden enlargement $\Big\}\; h_e = \frac{(V_1 - V_2)^2}{2g}$

From continuity equation

$$A_c V_c = A_2 A_2$$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{A_c/A_2} = \frac{1}{C_c}$$

$$\therefore C_c = \frac{A_c}{A_2}$$

$$V_c = \frac{V_2}{C_c}$$

Sub the of value of V_c in eqn ①

$$h_c = \left[\frac{V_c}{C_c} - V_2\right]^2 \Big/ 2g$$

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right]^2$$

$$h_c = \frac{k V_2^2}{2g}$$

where $k = \left(\frac{1}{C_c} - 1\right)^2$

The value of C_c (or) k is not constant; it depends on the ratio (A_2/A_1).

Generally, the k value varies from 0·375 to 0·5

if C_c value (or) k ~~value~~ is not given, Then the head loss due to friction is taken as

$$\boxed{h_c = \frac{0.5\, V_2^2}{2g}}$$

13 The rate of flow of water pumped into a pipe ABC, which is 200 m long, is 20 l/s. The pipe is laid on an upward slope of 1 in 40. The length of the portion AB is 100 m and its diameter is 100 mm. While the length of the portion BC is also 100 m but its diameter is 200 mm. The change of diameter at B is sudden. The flow is taking place from A to C where the pressure at A is 19.62 N/cm² and end C is connected to a tank. Find the pressure at C and draw the hydraulic gradient and total energy line. Take $f = 0.008$ [A/M-16]

Given data:-

Length of pipe, $ABC = 200\,m$

Discharge, $Q = 20\,l/s = 0.02\,m^3/s$

Slope of pipe $i = 1$ in $40 = 1/40$

Length of pipe, $AB = 100\,m$, Dia of pipe $AB = 100\,mm = 0.1\,m$

Length of pipe $BC = 100\,m$, Dia of pipe $BC = 200\,mm = 0.2\,m$

pressure at A $P_A = 19.62\,N/cm^2 = 19.62 \times 10^4\,N/m^2$

co-efficient of friction, $f = .008$

velocity of water in pipe AB, $V_1 = \dfrac{Q}{\text{Area of AA}} = \dfrac{0.02}{\frac{\pi}{4}(.1)^2} = 2.54\,m/s$

velocity of water in pipe BC, $V_2 = \dfrac{Q}{\text{Area of BC}} = \dfrac{0.02}{\frac{\pi}{4} \times (.2)^2} = 0.63\,m/s$

Applying Bernoulli's equation to point 'A' and 'C'

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C + \text{total loss from A to C.}$$

Total loss from A to C = loss due to friction in pipe AB + loss of head due to enlargement at B + loss of head due to friction in pipe BC

Now loss of head due to friction in pipe AB

$$h_{f_1} = \frac{4flv^2}{d \times 2g} = \frac{4 \times .008 \times 100 \times 2.54^2}{0.1 \times 2 \times 9.81} = 10.52\,m$$

loss of head due to friction in pipe BC,

$$h_{f_2} = \frac{4 \times .008 \times 100 \times (0.63)^2}{0.2 \times 2 \times 9.81} = 0.323\,m$$

loss of head due to enlargement at 'B'

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(2.54 - .63)^2}{2 \times 9.81} = 0.186\,m$$

$\therefore$ Total loss from A to C $= h_{f_1} + h_e + h_{f_2}$

$$= 10.52 + 0.186 + .323 = 11.029$$
$$\simeq 11.03\,m$$

Substituting this value in (i) we get

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C + 11.03$$

Taking datum line passing through A, we have

$$Z_A = 0$$
$$Z_C = \frac{1}{40} \times \text{total length of pipe}$$
$$= \frac{1}{40} \times 200 = 5\,m$$
$$P_A = 19.62 \times 10^4\ N/m^2$$
$$V_A = V_1 = 2.52\,m/s,\ V_C = V_2 = 0.63\,m/s$$

Substituting these values in (ii) we get

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{(2.54)^2}{2 \times 9.81} + 0 = \frac{P_C}{\rho g} + \frac{(0.63)^2}{2 \times 9.81} + 5 + 11.03$$

(or) $$20 + 0.328 = \frac{P_C}{\rho g} + 0.02 + 5.0 + 11.03$$

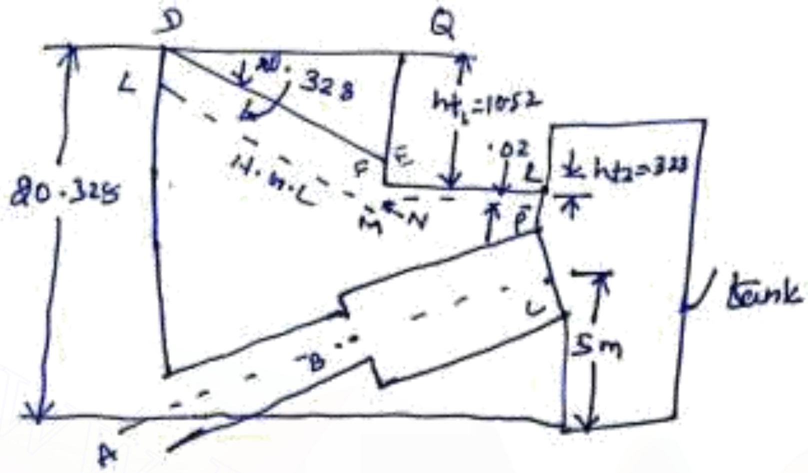
$$20.328 = \frac{P_C}{\rho g} + 16.05$$

$$\frac{p_c}{\rho g} = 20.328 - 16.05 = 4.278 \text{ m}$$

$$p_c = 4.278 \times 1000 \times 9.81 \text{ N/m}^2$$

$$= \frac{4.278 \times 1000 \times 9.81}{10^4} \text{ N/cm}^2 = 4.196 \text{ N/cm}^2$$

Hydraulic gradient and total Energy line



Pipe AB. Assuming the datum line passing through A, Then total energy at 'A'

$$= \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{(2.54^2)}{2 \times 9.81} + 0$$

$$= 20 + 0.328 = 20.328 \text{ m}$$

Total energy at B

$$= \text{Total energy at } A - h_{f_1}$$

$$= 20.328 - 10.52 = 9.808 \text{ m}$$

Also $V_c^2/2g = \frac{(0.63)^2}{2 \times 9.81} = 0.02$

Total Energy line

Hydraulic Gradient line

LM parallel to the line DE at a distance in the downward direction equal to 0.328 m. Also draw the line PN parrel to the line GF at a distance of $V_c^2/2g = 0.02$. Joint point M to N. Line LMNP ∴ hydraulic gradient line.

UNIT III - DIMENSIONAL ANALYSIS
PART – A

1. What are the methods of dimensional analysis

There are two methods of dimensional analysis.
They are, a. Rayleigh - Retz method
b. Buckingham's theotem method.
Nowadays Buckingham's theorem method is only used.

2. Describe the Rayleigh's method for dimensional analysis.

Rayleigh's method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables becomes more than four, then it is very difficult to find the expression for dependent variable.

3. What do you mean by dimensionless number

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension or elastic force. As this is a ratio of one force to other force, it will be a dimensionless number.

4. Name the different forces present in fluid flow

Inertia force
Viscous force
Surface tension
force Gravity
force

5. State Buckingham's Π theorem

It states that if there are 'n' variables in a dimensionally homogeneous equation and if these variables contain 'm' fundamental dimensions (M,L,T), then they are grouped into (n-m), dimensionless independent Π-terms.

6. State the limitations of dimensional analysis.

1. Dimensional analysis does not give any due regarding the selection of variables.
2. The complete information is not provided by dimensional analysis.
3.The values of coefficient and the nature of function can be obtained only by experiments or from mathematical analysis.

7. Define Similitude

Similitude is defined as the complete similarity between the model and prototype.

8. State Froude's model law

Only Gravitational force is more predominant force. The law states 'The Froude's number is same for both model and prototype'

9.What are the similarities between model and prototype?

(i) Geometric Similarity
(ii) Kinematicc Similarity
(iii) Dynamic Similarity

UNIT –III
DIMENSIONAL ANALYSIS
PART-B

What are the needs of dimensional Analysi
1) Need for dimensional analysis [A/M-13]

* Fluid flow problems are difficult to solve for obtaining analytical solutions.

* Fluid flow problems like a comb Physical analysis and experimental studies used.

* Experimental studies are use determine the effect of variables associat fluid flow Phenomenon. It is used to fu the dependency of the variable with the oth

* Dimensional analysis is a mathe tool (or) technique to study dimensions of problems. In this, each phenomenon is expr an equation having number of variables.

DIMENSIONS

Engineers and scientists use various Parameters to describe a given phenomenon these physical parameters are indepen each other called fundamental (or) pri quantities/ parameters.

SL.NO	Physical quantity	symbol	Dimensions
1	Fundamental quantities		
	a) Mass	M	M
	b) Length	L	L
	c) Time	T	T
2.	Geometric quantities		
	a) Area	A	L^2
	b) Volume	V	L^3
	c) Moment of inertia	I	L^4
3)	Kinetic quantities		
	a) velocity	v	LT^{-1} $(\frac{L}{T})$
	b) Angular velocity	ω	T^{-1}
	c) Acceleration	a	LT^{-2}
	d) Angular acceleration	α	T^{-2}
	e) Gravity	g	LT^{-2}
	f) Discharge	Q	L^3T^{-1}
	g) kinematic viscosity	ν	L^2T^{-1}
4)	Dynamic quantities		
	a) Force	F	MLT^{-2}
	b) weight	W	MLT^{-2}
	c) Specific weight	w	$ML^{-2}T^{-2}$

d) Density ρ ML^{-3}

e) Dynamic viscosity μ $ML^{-1}T^{-1}$

f) work W $ML^{2}T^{-2}$

g) Energy E $ML^{2}T^{-2}$

h) Power P $ML^{2}T^{-2}$

i) Torque T $ML^{2}T^{-2}$

j) Momentum M MLT^{-1}

2) What is Meant by Dimensional Homogenity and Explain in detail ?

Dimensional Homogeneity

The Law of Fourier principle of dimensional homogeneity states "an equation which expresses a physical phenomenon of fluid flow should be algebraically correct and dimensionally homogeneous."

Dimensionally homogeneous means, the dimensions of the terms of left hand side should be same as the dimensions of the terms on right hand side.

Example: Consider a flow over rectangular weir having discharge

$$Q = 2/3\ cd \sqrt{2g}\ L H^{3/2}$$

cd = coefficient of discharge

g = Gravitional force

L = Length of the weir

H = Head of water

Dimensions of each given parameter

$Q = L^3T^{-1}$

cd = No dimension

$g = LT^{-2}$

$L = L$

$H = L$

For numerical numbers, there are no dimensions.
So, the non - dimension equation

$$L^3 T^{-1} = \sqrt{L T^{-2}} \times L \times L^{3/2}$$

$$L^3 T^{-1} = L^{1/2 + 1 + 3/2} T^{-1}$$

$$= L^3 T^{-1}$$

As per Fourier principle of dimensional homogeneity, the left hand side dimensions are equal to the right hand side dimensions. So, the given equation is dimensionally homogeneous.

Uses of Dimensional Homogeneity

1* To check the dimensional homogeneity of the given equation

2* To determine the dimension of a physical variable.

3* To convert units from one system to another through dimensional homogeneity.

4* It is a step towards dimensional analysis

Methods of Dimensional Analysis

i) Rayleigh's method

ii) Buckingham π- Theorem

3) a) State Backingham π - Theorm b) The efficiency η of a fan depends on dencity ρ, dynamic viscosity μ of the fluid, angular velocity w, diameter D of the rotar and the discharge Q. Express η in terms of dimensionless parameters

$$\eta = f(\rho, \mu, w, D, Q)$$

(or)

$$f_1(\eta, \rho, \mu, w, D, Q) = 0$$

Hence total number of variables, $n = 6$. The value of m, number of fundamental dimensions for the problem is obtained by writing dimensions of each variable. Dimensions of each variables are

η = Dimensionless (η, Δp)

$$\left.\begin{array}{l} \rho = ML^{-3}, \quad w = T^{-1} \quad Q = L^3T^{-1} \\ \mu = ML^{-1}T^{-1} \quad D = L \end{array}\right\} m = 3$$

Number of π-terms $= n - m = 6 - 3 = 3$

Equation (i) is written as $f_1(\pi_1, \pi_2, \pi_3) = 0$

Each π-term contains $m+1$ variables, where m is equal to three and is also repeating variable choosing D, w, and ρ as repeating variables, we have

$$\pi_1 = D^{a_1}, w^{b_1}, \rho^{c_1} \cdot \eta$$

$$\pi_2 = D^{a_2} \cdot w^{b_2} \rho^{c_2} \cdot \mu$$

$$\pi_3 = D^{a_3} \cdot w^{b_3} \cdot \rho^{c_3} \cdot Q$$

Dimensionaless parameters

since the inertia force is always present in a fluid flow, ~~its~~ its ratio with variables.

Method of selecting Repeating variables.

$(n-m) \rightarrow$ constant - 3

Method of selecting Repeating variables.

number of repeating variables = number of fundamental diamensions of fluid properties.

1. Geometric, ② flow property ③ fluid properties.

1. (i) Length (l) ii) d iii) H → Fundamental.

Verables with flow property are dependent

(i) Velocity (V), acceleration

variables with ~~fluid~~ property (i) μ ii) ρ iii) w..

3) The repeating variables selected should not form a dimensionless group

4) The repeating variables together must have the same number of fundamental dimensions

5) No two repeating variables should have the same dimensions.

In most of fluid mechanics problems, the choice of repeating variables may be

(i) d v ρ ii) l, v, ρ (or) iii) l, v, μ (or) d v μ)

First π-term

substituting dimensions on both sides of π_1,

$$M^0 L^0 T^0 = L^{a_1} \cdot (T^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot M^0 L^0 T^0$$

Equating the powers of MLT on both sides

Power of M, $0 = c_1 + 0 \quad \therefore \quad c_1 = 0$

Power of L $0 = a_1 + 0 \quad \therefore \quad a_1 = 0$

Power of T $0 = -b_1 + 0, \quad \therefore \quad b_1 = 0$

substituting the values of a_1, b_1 and c_1 in π_1 we get

$$\pi_1 = D^0 \omega^0 \rho^0 \cdot \eta = \eta$$

[if a variable is dimensionless, it itself is a π-term. Here the variable η is a dimensionless and hence η is a π-Term.

$$\pi_1 = \eta.$$

$$\boxed{\pi_2 = D^{-2} \cdot \omega^{-1} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{D^2 \omega \rho}}$$

Second π-term

$$\pi_2 = D^{a_2} \cdot \omega^{b_2} \cdot \rho^{c_2} \cdot \mu$$

Substituting the dimensions on both sides.

$$M^0 L^0 T^0 = L^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1}T^{-1}$$

Equating the power of M, L, T on both sides

Power of M, $0 = c_2 + 1$ $\therefore$ $c_2 = -1$

Power of L, $0 = a_2 - 3c_2 - 1$ $\therefore$, $a_2 = 3c_2 + 1 = -3+1 = -2$

Power of T, $0 = -b_2 - 1$, $\therefore$ $b_2 = -1$

Substituting the value of a_2, b_2 and c_2 in π_2

$$\boxed{\pi_2 = D^{-2} \cdot \omega^{-1} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{D^2 \omega \rho}}$$

Third π-term

Substituting the dimensions on both sides.

$$M^0 L^0 T^0 = L^{a_3} \cdot (T^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot L^3 T^{-1}$$

Equating the powers of M, L and T on both side

Power of M, $0 = c_3$ $c_3 = 0$

L, $0 = a_3 - 3c_3 + 3$ $a_3 = 3c_3 - 3 = -3$

$0 = -b_3 - 1$, $b_3 = -1$

Substituting the value of a_3, b_3 and c_3 in π_3

$$\pi_3 = D^{-3} \cdot \omega^{-1} \cdot \rho^0 \cdot Q = \frac{Q}{D^3 \omega}$$

Substituting the values of π_1, π_2, π_3 in equ (ii)

$$f_1\left(\eta, \frac{\mu}{D^2 \omega \rho}, \frac{Q}{D^2 \omega}\right) = 0 \text{ (or)}$$

$$\eta = \phi\left(\frac{\mu}{D^2 \omega \rho}, \frac{Q}{D^2 \omega}\right).$$

27) Derive on the basis of dimensional analysis suitable Parameters to present the ~~thrust~~ force developed by a propeller. Assume that the thrust P depends upon the angular velocity w. speed of advance v, diameter D, dynamic viscosity μ, mass density ρ, elasticity of the fluid medium which can be denoted by the speed of sound in the medium c. [N/D-12]

SOL

Thrust F is a function of w, V, D, μ, ρ, c

$$F = f(w, V, D, \mu, \rho, c)$$

$$f_1 = (P, w, V, D, \mu, \rho, c) = 0$$

Total number of variables, $n = 7$

writing dimensions of each variable, we have

$P = MLT^{-2}$, $w = T^{-1}$, $V = LT^{-1}$, $D = L$, $\mu = ML^{-1}T^{-1}$

$\rho = ML^{-3}$, $C = LT^{-1}$

Number of fundamental dimensions, $m = 3$

Number of π-terms $= n - m = 7 - 3 = 4$

Hence, equation (i) can be written as

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

Each π-term contains $m+1 = 3+1 = 4$ variables

Out of four, three are repeating variables.

Choosing D, v, ρ as repeating variables, we get π-terms as

$$\pi_1 = D^{a_1}, v^{b_1}, \rho^{c_1}, P$$

$$\pi_2 = D^{a_2}, v^{b_2}, \rho^{c_2}, w$$

$$\pi_3 = D^{a_3}, v^{b_3}, \rho^{c_3}, M$$

$$\pi_4 = D^{a_4}, v^{b_4}, \rho^{c_4}, C$$

First π-term

writing dimensions on both sides

$$M^0 L^0 T^0 = L^{a_1} \cdot (LT^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot MLT^{-2}$$

Equating powers of M, L, T on both sides

power of M, $0 = c_1 + 1 \quad \therefore c_1 = -1$

power of L $0 = a_1 + b_1 - 3c_1 + 1$

$a_1 = -b_1 + 3c_1 - 1 = 2 - 3 - 1 = -2$

power of T $0 = -b_1 - 2$ $b_1 = -2$

Sub the values of a_1, b_1 and c_1 in π_1

$$\pi_1 = D^{-2}, v^{-2}, \rho^{-1} \; P = \boxed{\frac{P}{D^2 v^2 \rho}}$$

Second π-term

$$M^0 L^0 T^0 = L^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} \cdot T^{-1}$$

M = 0 = c2, c2 = 0 $c_2 = 0$

L = 0 = $a_2 + b_2 - 3c_2$ $a_2 = -b_2 + 3c_2 + 1 = 1$

T = 0 = $-b_2 - 1$ $b_2 = -1$

$$\pi_2 = D^1 \cdot v^{-1} \cdot \rho^0 \cdot w = \boxed{Dw/V}$$

Third - Π _term

$$c_3 = -1,\ a_3 = -1,\ b_3 = -1$$

$$\Pi_3 = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu = \boxed{\mu / DV\rho}$$

Fourth Π_term

$$M^0 L^0 T^0 = L^{a_4} \cdot (LT^{-1})^{b_4} \cdot (ML^{-3})^{c_4} \cdot LT^{-1}$$

$$c_4 = 0,\ a_4 = 0,\ b_4 = -1$$

$$\Pi_4 = D^0 \cdot V^{-1} \cdot \rho^0 \cdot C = \boxed{C/V}$$

$\Pi_1, \Pi_2, \Pi_3, \Pi_4$ in equation (i)

$$P = D^2 V^2 \rho\, \phi \left(\frac{D\omega}{V}, \frac{\mu}{DV\rho}, C/V \right)$$

5) Using Buckingham's Π_theorem, show that the discharge Q consumed by an oil ring is given by

$$Q = Nd^3 \phi \left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{w}{\rho N^2 d} \right]$$

where d is the internal diameter of the ring, N is rotational speed, ρ is density, μ is viscosity, σ is surface tension and w is the specific weight of oil.

Sol

Given $\quad Q = f(d, N, \rho, \mu, \sigma, w)$ (or)

$$f_1(Q, d, N, \rho, \mu, \sigma, w) = 0$$

Total number of variables, $n = 7$

Dimensions of each variables are

$Q = L^3T^{-1}$, $d = L$, $N = T^{-1}$, $\rho = ML^{-3}$, $\mu = ML^{-1}T^{-1}$, $\sigma = MT^{-2}$

$w = ML^{-2}T^{-2}$

Total number of fundamental dimensions $m = 3$

$\therefore$ Total number of Π_term $= n - m = 7 - 3 = 4$

Equation (i) becomes as $f_1(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0$

Choosing d, N, ρ as repeating variables, the π-terms are,

$$\pi_1 = d^{a_1}, N^{b_1}, \rho^{c_1} \cdot Q$$

$$\pi_2 = d^{a_2}, N^{b_2}, \rho^{c_2} \cdot \mu$$

$$\pi_3 = d^{a_3}, N^{b_3}, \rho^{c_3} \cdot \sigma$$

$$\pi_4 = d^{a_4}, N^{b_4} \cdot \rho^{c_4} \cdot w$$

First π-term

substituting dimensions on both sides

$$M^0 L^0 T^0 = L^{a_1} \cdot (T^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot L^3 T^{-1}$$

M $\quad 0 = c_1 \qquad \therefore \; c_1 = 0$

L $\quad 0 = a_1 - 3c_1 + 3 \qquad \therefore \; a_1 = 3c_1 - 3 = 0 - 3 = -3$

T $\quad 0 = -b_1 - 1 \qquad \therefore \; b_1 = -1$

Substituting a_1, b_1, c_1 in π_1, $\pi_1 = d^{-3} \cdot N^{-1} \cdot \rho^0 \cdot Q = \boxed{\frac{Q}{d^3 N}}$

Second π-term $[\pi_2 = d^{a_2} \cdot N^{b_2} \cdot \rho^{c_2} \cdot \mu]$

$$M^0 L^0 T^0 = L^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1} T^{-1}$$

$c_2 = -1$, $a_2 = -2$, $b_2 = -1$

$$\pi_2 = d^{-2}, N^{-1} \cdot \rho^{-1} \cdot \mu = \boxed{\frac{\mu}{d^2 N \rho}}$$

Third π-term

$$M^0 L^0 T^0 = L^{a_3} \cdot (T^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot MT^{-2}$$

$c_3 = -1$, $a_3 = -3$, $b_3 = -2$,

$$\pi_3 = d^{-3} \cdot N^{-2} \cdot \rho^{-1} \cdot \sigma = \frac{\sigma}{d^3 N^2 \rho}$$

Fourth π-term

$$M^0 L^0 T^0 = L^{a_4} \cdot (T^{-1})^{b_4} \cdot (ML^{-3})^{c_4} \cdot ML^2 T^{-2}$$

$c_4 = -1$, $a_4 = -1$, $b_4 = -2$

$$\pi_4 = d^{-1} \cdot N^{-2} \cdot \rho^{-1} \cdot w = w / d N^2 \rho$$

$$f\left(\frac{Q}{d^3 N}, \frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3 N^2 \rho}, \frac{w}{d N^2 \rho}\right) = 0; \quad \boxed{Q = d^3 N \phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3 N^2 \rho}, \frac{w}{d N^2 \rho}\right]}.$$

6) What is meant by Model & Analysis & Model in detail?

Model :-

It is smaller scale size of phototype.

Phrotype :

It is original size of the structure.

Model Analysis :-

Studied about the model

Advantages of Model testing :-

1) The model test are economical & convient.

2) problems may predicated in advance.

3) Model testing can be used to detect and rectify the defects of an existing structure.

~~Applying~~ Application of the model testing :-

1* Civil Engineering structures such dams, weirs, canals

2* Design of harbour, ships and submarines

3* Aeroplanes, rockets and machines missiles.

* similitude

similarity b/w model and prototype.

types :-

1. Geometric similarity
2. Kinematic similarity
3. Dynamic similarity

Geometric

$L_r \rightarrow$ length scale ratio

$L_r^2 \rightarrow$ Area

$L_r^3 \rightarrow$ Volume.

Kinematic similarity

Time scale ratio, $T_r = \frac{T_P}{T_m}$

Velocity scale ratio, $C_r = \frac{L_r}{T_r} \left(\frac{L_P T_P}{L_m T_m} \right)$

Acceleration scale ratio, $a_r = \frac{L_P T_P^2}{L_m T_m^2} = \frac{L_r}{T_r^2}$

Discharge scale ratio, $Q_r = \frac{L^3 P / T_P}{L^3_m / T_m} = \frac{L_r^3}{T_r}$

Dynamic Similarity $= \frac{(F_i)_P}{(F_i)_m} = \frac{(F_v)_P}{(F_v)_m} = \frac{(F_g)_P}{(F_g)_m} = F_r$

What is meant by similarity laws and Explain in detail?

7) MODEL (or) SIMILARITY laws [AU - M/J - 14]

Dynamic similarity is known as model (or) similarity laws. It means, the models are designed on the basis of force which influences flow. The following are various similarity laws along with its applications :-

1. Reynolds model law
2. Froude model law
3. Euler model law
4. Model Law
5. Mach model Law

1. Reynolds Model law

$$(Re)_{model} = (Re)_{prototype}$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_P V_P L_P}{\mu_P} \quad \text{--- (1)} \qquad Re = \frac{\rho V L}{\mu}$$

$\rho_m \rightarrow$ Density of fluid in model

$V_m \rightarrow$ velocity of fluid in model

$L_m \rightarrow$ length of model

$\mu_m \rightarrow$ viscosity of fluid

Reynolds model significance

1* Motion of air planes

2* Flow of incompressible fluid in closed pipes.

3* Motion of submarines and

4* Flow around structures and other bodies immersed fully in moving fluids.

8) The ratio of length of a submarine and its model is 25:1 The speed of sub-marine (prototype) is 15m/s. The model is to be tested in wind tunnel. Find the speed of air in wind tunnel. Also determine the ratio of the drag (resistance) b/w the model and its prototype. Assume the value of kinematic viscosity for water and air as 0.012 strokes and 0.016 stokes respectively. The density for sea water and air is given as 1030 kg/m3 and 1.24 kg/m3 respectively. [A/M-15]

Given data

$L_r = 25$

For prototype $V_p = 15 m/s$

Fluid sea water, $\nu_p = 0.012$ stokes $= 0.012 cm^2/s$ ∴ (1 stroke $= 1 cm^2/s$)

$= 0.012 \times 10^{-4} m^2/s$

$\rho_p = 1030 kg/m^3$

For model, fluid = air

$\nu_m = 0.016$ stokes $= 0.016 cm^2/s = 0.016 \times 10^{-4} m^2/s$

$\rho_m = 1.24 kg/m^3$

Sol According to Reynolds model law

$Re_{model} = (Re)_{prototype}$

$$\frac{\rho_p v_p D_p}{\mu_p} = \frac{\rho_m V_m D_m}{\mu_m} \qquad \left[\because\ Re = \frac{\rho v L}{\mu}\right.$$

$$\frac{V_p D_p}{\mu_p \rho_p} = \frac{V_m D_m}{\mu_m \rho_m}, \quad V_m = \frac{V_p D_p \nu_m}{\nu_p D_m} = V_p \frac{D_p}{D_m} \times \frac{\nu_m}{\nu_p}$$

$$= V_p \times L_r \times \frac{\nu_m}{\nu_p} = 15 \times 25 \times \frac{0.016 \times 10^{-4}}{0.012 \times 10^{-4}} = 50$$

Drag force, $F = ma$ $\qquad \left(\because\ m = \rho L^3 \text{ \& } a\right.$

$$= \rho L^3 \frac{v}{T}$$

$$= \rho L^2 \times L/T \times v$$

$$= \rho L^2 v^2 \qquad \because \left[\frac{L}{T} = v\right]$$

For model

$$F_m = \rho_m L_m^2 V_m^2$$

For prototype $F_p = \rho_p L_p^2 v_p^2$

Dividing equation (1) by (2) $\qquad \because \left(\frac{L_m}{L_p} = L\right.$

$$\frac{F_m}{F_p} = \frac{\rho_m}{\rho_p} \times \left(\frac{L_m}{L_p}\right)^2 \times \left(\frac{V_m}{V_p}\right)^2$$

$$= \frac{1.24}{1030} \times \left(\frac{1}{25}\right)^2 \times \left(\frac{500}{15}\right)^2$$

$$= 3.082 \times 10^{-3} = 0.0031 \text{ // Ans}$$

9. Obtain an expression in non-dimensional form for the pressure gradient in a horizontal pipe of circular cross-section. Show how this relates to the familiar expression for frictional head loss.

[N/D-14]

Step 1. Identify the relevant variables.

$dp/dx, \rho, V, D, k_s, \mu$

Step 2. Write down dimensions.

$$\frac{dp}{dx} \quad \frac{[force/area]}{length} = \frac{MLT^{-2} \times L^{-2}}{L} = ML^{-2}T^{-2}$$

ρ ML^{-3}

V LT^{-1}

D L

k_s L

μ $ML^{-1}T^{-1}$

Step 3. Establish the number of independent dimensions and non-dimensional groups.

Number of relevant variables: $n = 6$

Number of independent dimensions: $m = 3$ (M, L and T)

Number of non-dimensional groups (Πs): $n - m = 3$

Step 4. Choose m (= 3) dimensionally-independent scaling variables.

e.g. geometric (D), kinematic/time-dependent (V), dynamic/mass-dependent (ρ).

Step 5. Create the Πs by non-dimensionalising the remaining variables: dp/dx, k_s and μ.

$$\Pi_1 = \frac{dp}{dx} D^a V^b \rho^c$$

Considering the dimensions of both sides:

$$M^0L^0T^0 = (ML^{-2}T^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c$$
$$= M^{1+c}L^{-2+a+b-3c}T^{-2-b}$$

Equate powers of primary dimensions. Since M only appears in [ρ] and T only appears in [V] it is sensible to deal with these first.

M: $0 = 1 + c \Rightarrow c = -1$

T: $0 = -2 - b \Rightarrow b = -2$

L: $0 = -2 + a + b - 3c \Rightarrow a = 2 - b + 3c = 1$

Hence,

$$\Pi_1 = \frac{dp}{dx} D V^{-2} \rho^{-1} = \frac{D\frac{dp}{dx}}{\rho V^2}$$ (Check: OK – ratio of two pressures)

$$\Pi_2 = \frac{k_s}{D}$$ (by inspection, since k_s is a length)

$$\Pi_3 = \mu D^a V^b \rho^c$$

In terms of dimensions:

$$M^0L^0T^0 = (ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c$$
$$= M^{1+c}L^{-1+a+b-3c}T^{-1-b}$$

Equating exponents:

M: $0 = 1 + c \quad \Rightarrow \quad c = -1$

T: $0 = -1 - b \quad \Rightarrow \quad b = -1$

L: $0 = -1 + a + b - 3c \quad \Rightarrow \quad a = 1 - b + 3c = -1$

Hence,

$$\Pi_3 = \frac{\mu}{\rho V D}$$ (Check: OK – this is the reciprocal of the Reynolds number)

<u>Step 6</u>. Set out the non-dimensional relationship.

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

or

$$\frac{D\dfrac{dp}{dx}}{\rho V^2} = f(\frac{k_s}{D}, \frac{\mu}{\rho V D}) \qquad (*)$$

<u>Step 7</u>. Rearrange (if required) for convenience.

We are free to replace any of the Πs by a power of that Π, or by a product with the other Πs, provided we retain the same number of independent dimensionless groups. In this case we recognise that Π_3 is the reciprocal of the Reynolds number, so it looks better to use $\Pi_3' = (\Pi_3)^{-1} = \mathrm{Re}$ as the third non-dimensional group. We can also write the pressure gradient in terms of head loss: $\frac{dp}{dx} = \rho g \frac{h_f}{L}$. With these two modifications the non-dimensional relationship (*) then becomes

$$\frac{g h_f D}{L V^2} = f(\frac{k_s}{D}, \mathrm{Re})$$

or

$$h_f = \frac{L}{D} \times \frac{V^2}{g} \times f(\frac{k_s}{D}, \mathrm{Re})$$

Since numerical factors can be absorbed into the non-specified function, this can easily be identified with the Darcy-Weisbach equation

$$h_f = \lambda \frac{L}{D} \frac{V^2}{2g}$$

where λ is a function of relative roughness k_s/D and Reynolds number Re, a function given (Topic 2) by the Colebrook-White equation.

10. Describe briefly the types of forces in moving fluid and the importance of three types of similarity. [N/D-14]

Forces encountered in flowing fluids include those due to inertia, viscosity, pressure, gravity, surface tension and compressibility. These forces can be written as follows; Inertia force: $m.a = \rho V (dV/dt) \propto \rho V^2 L^2$

Viscous force: $\tau A = \mu A \, du/dy \propto \mu V L$

Pressure force: $(\Delta p) A \propto (\Delta p) L^2$

Gravity force: $mg \propto g \rho L^3$

Surface tension force: σL

Compressibility force: $E_v A \propto E_v L$

Parameter	Mathematical expression	Qualitative definition	Importance
Prandtl number	$P_r = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$E_c = \frac{V^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific heat ratio	$\gamma = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow
Roughness ratio	$\frac{\varepsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent rough walls
Grashof number	$G_r = \frac{\beta(\Delta T) g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural onvection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{(1/2)\rho V^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Hydrodynamics, Aerodynamics
Lift coefficient	$C_L = \frac{L}{(1/2) A \rho V^2}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Hydrodynamics,Aero dynamics
Drag coefficient	$C_D = \frac{D}{(1/2) A \rho V^2}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Hydrodynamics, Aero dynamics

11. **The tip deflection δ of a cantilever beam is a function of tip load *W*, beam length *l*, second moment of area *I* and Young's modulus *E*. Perform a dimensional analysis of this problem.[A/M-15]**

Step 1. Identify the relevant variables.

$\delta, W, l, I, E.$

Step 2. Write down dimensions.

δ	L
W	MLT^{-2}
l	L
I	L^4
E	$ML^{-1}T^{-2}$

Step 3. Establish the number of independent dimensions and non-dimensional groups.

Number of relevant variables: $n = 5$

Number of independent dimensions: $m = 2$ (L and MT^{-2} - *note*)

Number of non-dimensional groups (Πs): $n - m = 3$

Step 4. Choose m (= 2) dimensionally-independent scaling variables.

e.g. geometric (l), mass- or time-dependent (E).

Step 5. Create the Πs by non-dimensionalising the remaining variables: δ, I and W.

These give (after some algebra, not reproduced here):

$$\Pi_1 = \frac{\delta}{l}$$

$$\Pi_2 = \frac{I}{l^4}$$

$$\Pi_3 = \frac{W}{El^2}$$

Step 6. Set out the non-dimensional relationship.

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

or

$$\frac{\delta}{l} = f\left(\frac{I}{l^4}, \frac{W}{El^2}\right)$$

This is as far as dimensional analysis will get us. Detailed theory shows that, for small elastic deflections,

$$\delta = \frac{1}{3}\frac{Wl^3}{EI}$$

or

$$\frac{\delta}{l} = \frac{1}{3}\left(\frac{W}{El^2}\right) \times \left(\frac{I}{l^4}\right)^{-1}$$