UNIT- I
FLUID PROPERTIES AND FLOW CHARACTERISTICS
PART – A

1. Define fluids.

Fluid may be defined as a substance which is capable of flowing. It has no definite shape of its own, but confirms to the shape of the containing vessel.

2. What are the properties of ideal fluid?

Ideal fluids have following properties

i)It is incompressible

ii) It has zero viscosity

iii) Shear force is zero

3. What are the properties of real fluid?

Real fluids have following properties

i)It is compressible

ii) They are viscous in nature

iii) Shear force exists always in such fluids.

4. Explain the Density

Density or mass density is defined as the ratio of the mass of the fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol (ρ).

$$\text{Density} = \frac{\text{Mass of the fluid (kg)}}{\text{Volume of the fluid (m3)}}$$

5. Explain the Specific weight or weight density

Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per uint volume of a fluid is called weight density and is denoted by the symbol (W).

$$(W) = \frac{\text{Weight of the fluid}}{\text{Volume of fluid}} = \frac{\text{Mass x Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$W = pg$$

6. Explain the Specific volume

Specific volume of a fluid is defined as the volume of the fluid occupied by a unit Mass or volume per unit mass of a fluid is called specific volume.

$$\text{Specific volume} = \frac{\text{Volume}}{\text{Mass}} = \frac{\text{m3}}{\text{kg}} = \frac{1}{p}$$

7. Explain the Specific gravity

Specific gravity is defined as the ratio of weight density of a fluid to the weight density of a standard fluid. For liquid, standard fluid is water and for gases, it is air.

$$\text{Specific gravity} = \frac{\text{Weight density of any liquid or gas}}{\text{Weight density of standard liquid or gas}}$$

8.Define Viscosity.

It is defined as the property of a liquid due to which it offers resistance to the movement of one layer of liquid over another adjacent layer.

9. Define kinematic viscosity.

It is defined as the ratio of dynamic viscosity to mass density. (m^2/sec)

10. Define Relative or Specific viscosity.

It is the ratio of dynamic viscosity of fluid to dynamic viscosity of water at 20°C.

11. State Newton's law of viscosity and give examples.

Newton's law states that the shear stress ( ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called co-efficient of viscosity.

$$r = \mu \frac{du}{dy}$$

12. Give the importance of viscosity on fluid motion and its effect on temperature.

Viscosity is the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. The viscosity is an important property which offers the fluid motion.

The viscosity of liquid decreases with increase in temperature and for gas it Increases with increase in temperature.

13. Explain the Newtonian fluid

The fluid which obeys the Newton's law of viscosity i.e., the shear stress is directly proportional to the rate of shear strain, is called Newtonian fluid.

$$r = \mu \frac{du}{dy}$$

14. Explain the Non-Newtonian fluid

The fluids which does not obey the Newton's law of viscosity i.e., the shear stress is not directly proportional to the ratio of shear strain, is called non-Newtonian fluid.

15. Define compressibility.

Compressibility is the reciprocal of bulk modulus of elasticity, k which is defined as the ratio of compressive stress to volume strain.

$$k = \frac{\text{Increase of pressure}}{\text{Volume strain}}$$

$$\text{Compressibility } \frac{1}{k} = \frac{\text{Volume of strain}}{\text{Increase of pressure}}$$

16. Define surface tension.

Surface tension is defined as the tensile force acting on the surface of a liquid in Contact with a gas or on the surface between two immiscible liquids such that contact surface behaves like a membrane under tension.

17. Define Capillarity.

Capillary is a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid.

18. What is cohesion and adhesion in fluids?

Cohesion is due to the force of attraction between the molecules of the same liquid. Adhesion is due to the force of attraction between the molecules of two different Liquids or between the molecules of the liquid and molecules of the solid boundary surface.

19. State momentum of momentum equation?

It states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

20. What is momentum equation

It is based on the law of conservation of momentum or on the momentum principle It states that,the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

21. What is Euler's equation of motion

This is the equation of motion in which forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream line.

22. What is venturi meter?

Venturi meter is a device for measuring the rate of fluid flow of a flowing fluid through a pipe. It consisits of three parts.

a. A short converging part b. Throat c.Diverging part.

It is based on the principle of Bernoalli's equation.

23. What is an orifice meter?

Orifice meter is the device used for measuring the rate of flow of a fluid through a pipe. it is a cheaper device as compared to venturi meter. it also works on the priniciple as that of venturi meter. It consists of a flat circular plate which has a circular sharp edged hole called orifice.

24. What is a pitot tube?

Pitot tube is a device for measuring the velocity of a flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of kinetic energy into pressure energy.

. What are the types of fluid flow?

Steady & unsteady fluid flow
Uniform & Non-uniform flow
One dimensional, two-dimensional & three-dimensional
flows Rotational & Irrotational flow

25. State the application of Bernouillie's equation ?

It has the application on the following measuring devices.

1.Orifice meter.
2.Venturimeter.
3.Pitot tube.

UNIT – I
FLUID PROPERTIES AND FLOW CHARACTERISTICS
PART - B

1. A Liquid has a specific gravity of 0.72. Find its density, specific weight and also the weight per liter of the liquid. If the above liquid is used for lubrication between a shaft and a sleeve, find the power lost in liquid for a sleeve length of 100 mm. The diameter of the shaft is 0.5 m and the thickness of the liquid film is 1 mm. Take the viscosity of fluid as 0.5 N-s/$m^2$and the speed of the shaft as 200rpm.[N/D-14]

Sol: Volume V = 1 litre = 0.001 m^3

Specific gravity S = 0.72

(i) Density ρ = S * 1000 = 0.72 * 1000 = 720 kg/m^3

(ii) Specific weight W = ρ*g = 720 * 9.81 = 7063.2 N/m^3

(iii) Weight w = W*V = 7063.2 * 0.001 = 7.063 N

(iv) μ = 0.5 Ns/m^2

Diameter D = 0.5 m

Speed of shaft N = 200 rpm0

Sleeve length L = 100 mm

Thickness of oil film t = 1 mm

u = (πDN) / 60 = (π*0.5*100)/60 = 2.62 m/s

du = 2.62 m/s

dy = 1mm = 0.001 m

τ = μ (du/dy) = 0.5 * (2.62/0.001) = 1310 N/mm^2

F = τ * A = 1310 * πDL = 205.77 N

Torque T = F * D/2 = 205.77 * 0.5/2 =51.44 Nm

Power lost = 2πNT/60 = (2*π*100*51.44)/60 = 538.68 W

2. If the velocity distribution over a plate is given by u = 2/3 y – y2 in which u is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y = 0.15m. Take dynamic viscosity of fluid as 8.63 poise. [N/D-14]

$$u = \frac{2}{3}y - y^2$$

$$\therefore \frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{at\ y=0} \text{ (or) } \left(\frac{du}{dy}\right)_{y=0}$$

$$= \frac{2}{3} - 2(0)$$

$$= \frac{2}{3} = 0.667$$

$$\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times (0.15)$$

$$= 0.367$$

Value of μ = 8.63 poise

$$= \frac{8.63}{10} \text{ SI units}$$

$$= 0.863\ Ns/m^2$$

Shear stress $\tau = \mu \frac{du}{dy}$

(i) shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy} \right)_{y=0}$$

$$= 0.863 \times 0.667$$

$$\boxed{\tau_0 = 0.5756 \, N/m^2}$$

(ii) shear stress at $y = 0.15$ m is given by

$$(\tau)_{y = 0.15}$$

$$= \mu \left(\frac{du}{dy} \right)_{y=0.15}$$

$$= 0.863 \times 0.367$$

$$\boxed{(\tau)_y = 0.3167 \, N/m^2}$$

3. A 150mm diameter vertical cylinder rotates contacted to the another cylinder of diameter 151 mm. Both the cylinder are 250mm high. If the torque of 12 Nm is required to rotate the inner cylinder at 100 r.p.m. determine the viscosity of the fluid in the space between the above two cylinders.[A/M-15]

Given:

Diameter of cylinder $= \frac{15 cm}{100} = 0.15 m$

Dia of outer cylinder $= \frac{15.10 cm}{100}$

$= 0.151 m$

Length of cylinder, $L = \frac{25 cm}{10}$

$= 0.25 m$

Torque $T = 12.0$ Nm

Speed, $N = 100$ r.p.m

Let the Viscosity $= \mu$

Tangential velocity of cylinder, $\boxed{u = \frac{\pi D N}{60}}$

$$u = \frac{\pi \times 0.15 \times 100}{60}$$

$$\boxed{u = 0.7854 m/s}$$

Surface area of the cylinder $= \pi D L$

$A = \pi \times 0.15 \times 0.25$

$\boxed{A = 0.1178 m^2}$

Now this relation $\tau = \mu \frac{du}{dy}$

$du = u - 0$

$du = u = 0.7854 \, m/s$

$dy = \frac{0.151 - 0.150}{2}$

$dy = 0.0005 \, m$

$\tau = \frac{\mu \times 0.7854}{.0005}$

shear force, F = shear stress × Area

$= \frac{\mu \times 0.7854}{0.0005} \times 0.1178$

Torque, $T = F \times D/2$

$12.0 = \frac{\mu \times 0.7854 \times 0.1178 \times \frac{0.15}{2}}{0.0005}$

$\mu = \frac{12.0 \times 0.0005 \times 2}{0.7854 \times 0.1178 \times 0.15}$

$\mu = 0.864 \, NS/m^2$

$\mu = 0.864 \times 10$

$\boxed{\mu = 8.64 \text{ poise}}$ //

5. (a) (i) Differentiate

(1) Real fluids and ideal fluids

(2) Newtonian and non-Newtonian fluids

(4)

(ii) What is the difference between U – tube differential manometer and inverted U- tube differential manometer?

(4)

(1) Real Fluid:
A fluid, which possesses viscosity, is known as real fluid. All fluids, in actual practice, are real fluids.

Ideal Fluid:
A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

(2) Newtonian fluids

A Newtonian fluid's viscosity remains constant, no matter the amount of shear applied for a constant temperature.. These fluids have a linear relationship between viscosity and shear stress.

Non-Newtonian fluids

You can probably guess that non-Newtonian fluids are the opposite of Newtonian fluids. When shear is applied to non-Newtonian fluids, the viscosity of the fluid changes.

(3) U-Tube Manometer:
It consist a U – shaped bend whose one end is attached to the gauge point 'A' and other end is open to the atmosphere. It can measure both positive and negative (suction) pressures. It contains liquid of specific gravity greater than that of a liquid of which the pressure is to be measured.

Inverted U-Tube Manometer:
Inverted U-Tube manometer consists of an inverted U – Tube containing a light liquid. This is used to measure the differences of low pressures between two points where where better accuracy is required. It generally consists of an air cock at top of manometric fluid type.

6. If the velocity profile of a liquid over a plate is a parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. calculate the velocity gradients and shear stress at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.

Soln

Given :

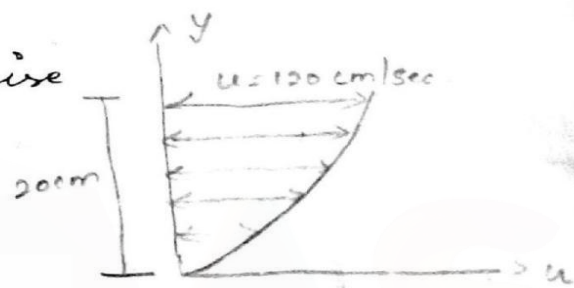
Distance of Vertex from plate = 20 cm

Velocity at Vertex, $u = 120$ cm/sec

Viscosity, $\mu = 8.5$ poise

$$\mu = \frac{8.5}{10} \frac{Ns}{m^2}$$

$$\boxed{\mu = 0.85}$$



The Velocity profile is given parabolic and equation of Velocity profile is

$$\boxed{u = ay^2 + by + c} \quad \text{—(1)}$$

where a, b and c constants. Their values are determined from boundary conditions as

(a) at $y = 0, u = 0$

(b) at $y = 20cm, u = 120$ cm/sec.

(c) at $y = 20cm, \frac{du}{dy} = 0$

substituting boundary condition (a) in equation (1), we get

$c = 0$

BC's (b) in (1)

$$120 = a(20)^2 + b(20) = 400a + 20b$$

$$120 = 400a + 20b \longrightarrow (2)$$

BC's (c) in (1)

$$\frac{du}{dy} = 2ay + b$$

$$0 = 2 \times a \times 20 + b$$

$$0 = 40a + b \longrightarrow (3)$$

Solve (2) & (3) for a and b

from (3)

$$40a + b = 0$$

$$\boxed{b = -40a}$$

sub the value in (2)

$$120 = 400a + 20(-40a)$$

$$120 = 400a - 800a$$

$$120 = -400a$$

$$a = \frac{120}{-400} = -3/10$$

$$\boxed{a = -0.3}$$

sub the values of a, b, c in (1)

$$\boxed{u = -0.3y^2 + 12y}$$

$$\frac{du}{dy} = -0.3(2y) + 12$$

Velocity gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12$$

$$\boxed{\frac{du}{dy} = -0.6y + 12}$$

(i) at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0}$

$$= -0.6 \times 0 + 12 = \boxed{12/s}$$

(ii) at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12$

$$= -6 + 12$$

$$\boxed{= 6/s}$$

(iii) at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12$

$$= -12 + 12$$

$$\boxed{= 0}$$

Shear stress

shear stress $\tau = \mu \frac{du}{dy}$

(i) shear stress at $y = 0$ $\tau = \mu \left(\frac{du}{dy}\right)_{y=0}$

$$= 0.85 \times 12.0$$

$$= \boxed{10.2\ N/m^2}$$

(ii) shear stress at $y = 10$ $\tau = \mu \left(du/dy\right)_{y=10}$

$$\tau = 0.85 \times 6.0$$

$$= \boxed{5.1\ N/m^2}$$

(iii) shear stress at $y = 20$ $\tau = \mu \left(du/dy\right)_{y=20}$

$$\tau = 0.85 \times 0$$

$$\boxed{\tau = 0}$$

In a two – two dimensional incompressible flow, the fluid velocity components are given by u = x – 4y and v= - y – 4x. Show that velocity potential exists and determine its form. Find also the stream function. [N/D-14]

Solution. Given:

$u = x - 4y$ and $v = - y - 4x$

$(\partial u / \partial x) = 1$ & $(\partial v / \partial y) = -1.$

$(\partial u / \partial x) + (\partial v / \partial y) = 0$

hence flow is continuous and velocity potential exists.

Let Φ = Velocity potential.

Let the velocity components in terms of velocity potential is given by

$\partial \Phi / \partial x = - u = - (x - 4y) = - x + 4y$ ———————— (1)

$\partial \Phi / \partial y = - v = - (-y - 4x) = y + 4x.$ ———————— (2)

Integrating equation(i), we get $\Phi = - (x^2 / 2) + 4xy + C$ —— (3)

Where C is a constant of Integration, which is independent of 'x'.

This constant can be a function of 'y'.

Differentiating the above equation, i.e., equation (3) with respect to 'y', we get

$\partial \Phi / \partial y = 0 + 4x + \partial C / \partial y$

But from equation (3), we have $\partial \Phi / \partial y = y + 4x$

Equating the two values of $\partial \Phi / \partial y$, we get

$4x + \partial C / \partial y = y + 4x$ or $\partial C / \partial y = y$

Integrating the above equation, we get

$C = (y^2 / 2) + C_1.$

Where C_1 is a constant of integration, which is independent of 'x' and 'y'.

Taking it equal to zero, we get $C = y^2/2$.

Substituting the value of C in equation (3), we get.

$$\Phi = -(x^2/2) + 4xy + y^2/2.$$

Value of stream functions

Let $\partial\psi/\partial x = v = -y - 4x$. ------------------------ (4).

Let $\partial\psi/\partial y = -u = -(x - 4y) = x + 4y$ -----(5)

Integrating equation (4) w.r.t. 'x' we get

$$\Psi = -yx - (4x^2/2) + k \text{ ----------------(6)}$$

Where k is a constant of integration which is independent of 'x' but can be a function 'y'.

Differentiating equation (6) w.r.to. 'y' we get,

$$\partial\psi/\partial x = -x - 0 + \partial k/\partial y$$

But from equation (5), we have $\partial\psi/\partial y = -x + 4y$

Equating the values of $\partial\psi/\partial y$, we get $-x + \partial k/\partial y$ or $\partial k/\partial y = 4y$.

Integrating the above equation, we get $k = 4y^2/2 = 2y^2$.

Substituting the value of k in equation (6), we get.

$$\Psi = -yx - 2x^2 + 2y^2$$

A ventrimeter of inlet diameter 300 mm and throat diameter 150 mm is inserted in vertical pipe carrying water flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 200 mm. Find the discharge if the co-efficient of discharge of meter is 0.98. [N/D-14]

Throat

Dia of Throat $d_2 = 15cm$

$$a_2 = \pi/4 \times 15^2$$

$$= 176.7 cm^2$$

$$Cd = 0.98.$$

Reading of differential Manometer

$x = 20cm$ of mercury

Difference in pressure head

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

S_h = Sp. gr. of mercury = 13.6

S_0 = Sp. gr. of water = 1

$$h = 20\left[\frac{13.6}{1} - 1\right]$$

$$\boxed{h = 252.0 \text{ cm of Water}}$$

$$Q = Cd\, \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 25}$$

$$Q = 125756 \text{ cm}^3/s$$

$$Q = \frac{125756}{1000} \text{ lit}/s$$

$$\boxed{Q = 125.756 \text{ l}/s}$$

An oil of specific gravity 0.8 is flowing through a horizontal venturimeter having a inlet diameter 200 mm and throat diameter 100 mm. The oil – mercury differential manometer shows a reading of 250 mm, calculate the discharge of oil through the venturimeter. Take C_d = 0.98.

[A/M-15]

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$$\therefore \quad a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$$

$d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$$= 70465 \text{ cm}^3/\text{s} = \textbf{70.465 litres/s. Ans.}$$

Water flows through a pipe AB 1.2m diameter at 3m/s and then passes through a pipe BC 1.5m diameter. At C, the pipe branches. Branch CD is 0.8m in diameter and carries one- third of the flow in AB. The flow velocity in branch CE is 2.5m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and diameter of CE. [M/J-16

Solution. Given:

Diameter of Pipe AB,	$D_{AB} = 1.2$ m
Velocity of flow through AB	$V_{AB} = 3.0$ m/s.
Dia. of Pipe BC,	$D_{BC} = 1.5$m.
Dia. of Branched pipe CD,	$D_{CD} = 0.8$m.
Velocity of flow in pipe CE,	$V_{CE} = 2.5$ m/s.
Let the rate of flow in pipe	$AB = Q\ m^3/s$.
Velocity of flow in pipe	$BC = V_{BC}\ m^3/s$.
Velocity of flow in pipe	$CD = V_{CD}\ m^3/s$.

Diameter of pipe	$CE = D_{CE}$
Then flow rate through	$CD = Q / 3$
And flow rate through	$CE = Q - Q/3 = 2Q/3$

(i). Now the flow rate through $AB = Q = V_{AB}$ X Area of AB

$$= 3 \times (\pi / 4) \times (D_{AB})^2 = 3 \times (\pi / 4) \times (1.2)^2$$

$$= 3.393\ m^3/s.$$

(ii). Applying the continuity equation to pipe AB and pipe BC,

V_{AB} X Area of pipe AB = V_{BC} X Area of Pipe BC

$$3 \times (\pi / 4) \times (D_{AB})^2 = V_{BC} \times (\pi / 4) \times (D_{BC})^2$$

$$3 \times (1.2)^2 = V_{BC} \times (1.5)^2$$

$$V_{BC} = (3 \times 1.2^2)/ 1.5^2 = 1.92\ m/s.$$

(iii). The flow rate through pipe

$$CD = Q_1 = Q/3 = 3.393 / 3 = 1.131\ m^3/s.$$

$Q_1 = V_{CD}$ X Area of pipe $C_D \times (\pi / 4) (C_{CD})^2$

$$1.131 = V_{CD} \times (\pi / 4) \times (0.8)^2$$

$$V_{CD} = 1.131 / 0.5026 = 2.25\ m/s.$$

(iv). Flow through CE,

$$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262\ m^3/s$$

$Q_2 = V_{CE}$ X Area of pipe CE $= V_{CE} \times (\pi / 4) (D_{CE})^2$

$$2.263 = 2.5 \times (\pi / 4) (D_{CE})^2$$

$$D_{CE} = \sqrt{(2.263 \times 4)/ (2.5 \times \pi)} = 1.0735\ m$$

Diameter of pipe CE = 1.0735m.

State Bernoulli's theorem for steady flow of an incompressible fluid. Derive an expression for Bernoulli's equation from first principle and state the assumptions made for such a derivation. [M/J-16]

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in s-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are :

1. Pressure force pdA in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction s.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta$$
$$= \rho dA ds \times a_s \quad \text{...(6.2)}$$

where a_s is the acceleration in the direction of s.

Now $$a_s = \frac{dv}{dt}, \text{ where } v \text{ is a function of } s \text{ and } t.$$
$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{dS}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g\, dA ds \cos\theta = \rho dA ds \times \frac{v\partial v}{\partial s}$$

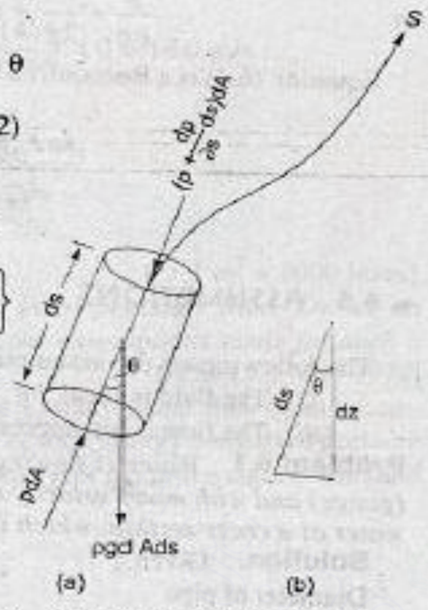


Fig. 6.1 *Forces on a fluid element.*

Dividing by $\rho ds dA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $$\frac{\partial p}{\rho\partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho}\frac{\partial p}{\partial \rho} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0 \quad \text{or} \quad \frac{\partial p}{\rho} + gdz + vdv = 0$$

or $$\frac{\partial p}{\rho} + gdz + vdv = 0 \quad \text{...(6.3)}$$

▶ 6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \quad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or
$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or
$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

▶ 6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero (*ii*) The flow is steady

(*iii*) The flow is incompressible (*iv*) The flow is irrotational.

Find the displacement thickness and the momentum thickness for the velocity distribution in the boundary layer given by $u/U = 2(y/\delta)-(y/\delta)^2$. [A/M-15]

Soln

Velocity distribution

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

(i) Displacement thickness δ^*

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^{\delta} \left(1 - \left[2\left(\frac{y}{\delta}\right) - \left(y/\delta\right)^2\right]\right) dy$$

$$= \int_0^{\delta} \left\{ 1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2 dy \right.$$

$$= \left[y - \frac{\cancel{2}y^2}{\cancel{2}\delta} + \frac{y^3}{3\delta^2} \right]_0^{\delta}$$

$$= \left[\delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} \right]$$

$$= \delta - \delta + \frac{\delta}{3}$$

$$\boxed{\delta^* = \delta/3}$$

(ii) momentum thickness

$$\theta = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy = \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right) \left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right) \right] dy$$

$$= \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right] \left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2} \right] dy$$

$$= \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4} \right] dy$$

$$= \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4} \right] dy$$

$$= \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4} \right]_0^{\delta}$$

$$= \left[\frac{\delta^2}{\delta} - \frac{5\delta^3}{3\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4} \right]$$

$$= \delta - \frac{5\delta}{3} + \delta - \delta/5$$

$$= \frac{15\delta - 25\delta + 15\delta - 3\delta}{15} = \frac{30\delta - 28\delta}{15} = \boxed{\frac{2\delta}{15}}$$

Unit – 2 FLOW THROUGH CIRCULAR CONDUITS

PART – A

1. Define viscosity (u).

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.Viscosity is also defined as the shear stress required to produce unit rate of shear strain.

2. Define kinematic viscosity.

Kniematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by μ.

3. What is minor energy loss in pipes?

The loss of head or energy due to friction in a pipe is known as major loss while loss of energy due to change of velocity of fluid in magnitude or direction is called minor loss of energy. These include,

a. Loss of head due to sudden enlargement.
b. Loss of head due to sudden contraction.
c. Loss of head at entrance to a pipe.
d. Loss of head at exit of a pipe.
e. Loss of head due to an obstruction in a pipe.
f. Loss of head due to bend in a pipe.
g. Loss of head in various pipe fittings.

4. What is total energy line?

Total energy line is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing sum of the pressure head and kinetic head from the centre of the pipe.

5. What is hydraulic gradient line?

Hydraulic gradient line gives the sum of (p/w+z) with reference to datum line. Hence hydraulic gradient line is obtained by subtracting v2 / 2g from total energy line.

6. What is meant by pipes in series?

When pipes of different lengths and different diameters are connected end to end, pipes are called in series or compound pipe. The rate of flow through each pipe connected in series is same.

7. What is meant by pipes in parallel?

When the pipes are connected in parallel, the loss of head in each pipe is same. The rate of flow in main pipe is equal to the sum of rate of flow in each pipe, connected in parallel.

8. What is boundary layer and boundary layer theory?

When a solid body immersed in the flowing fluid, the variation of velocity from zero to free stream velocity in the direction normal to boundary takes place in a narrow region in the vicinity of solid boundary. This narrow region of fluid is called boundary layer. The theory dealing with boundary layer flow is called boundary layer theory.

9. What is turbulent boundary layer?

If the length of the plate is more then the distance x, the thickness of boundary layer will go on increasing in the downstream direction. Then laminar boundary becomes unstable and motion of fluid within it, is disturbed and irregular which leads to a transition from laminar to turbulent boundary layer.

10. What is boundary layer thickness?

Boundary layer thickness (S) is defined as the distance from boundary of the solid body measured in y-direction to the point where the velocity of fluid is approximately equal to 0.99 times the free steam (v) velocity of fluid.

11. Define displacement thickness

Displacement thickness (S*) is defined as the distances, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction inflow rate on account of boundary layer formation.

12. What is momentum thickness?

Momentum thickness (0) is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of flowing fluid on account of boundary layer formation.

13.Mention the general characteristics of laminar flow.

- There is a shear stress between fluid layers
- 'No slip' at the boundary
- The flow is rotational
- There is a continuous dissipation of energy due to viscous shear

14. What is Hagen poiseuille's formula ?

$P_1\text{-}P_2 \,/\, \rho g = h_f = 32\ \mu\ UL \,/\, \_gD^2$

The expression is known as Hagen poiseuille formula .

Where $P_1\text{-}P_2 \,/\, \_g$ = Loss of pressure head
U = Average velocity
μ = Coefficient of viscosity
D = Diameter of pipe
L = Length of pipe

15.What are the factors influencing the frictional loss in pipe flow ?

Frictional resistance for the turbulent flow is

i. Proportional to v_n where v varies from 1.5 to 2.0 . ii. Proportional to the density of fluid .
iii. Proportional to the area of surface in contact .
iv. Independent of pressure .
v. Depend on the nature of the surface in contact .

16. What is the expression for head loss due to friction in Darcy formula

? $h_f = 4fLV^2 / 2gD$

Where f = Coefficient of friction in pipe
L = Length of the pipe
D = Diameter of pipe
V = velocity of the fluid

17. What do you understand by the terms

a) major energy losses , b) minor energy losses Major energy losses : -

This loss due to friction and it is calculated by Darcy weis bach formula and chezy's formula .

Minor energy losses :- This is due to

i. Sudden expansion in pipe .ii. Sudden contraction in pipe .
iii. Bend in pipe .iv. Due to obstruction in pipe .

18. Give an expression for loss of head due to sudden enlargement of the pipe :

$he = (V1-V2)^2/2g$

Wherehe = Loss of head due to sudden enlargement of pipe .

V1 = Velocity of flow at section 1-1

V2 = Velocity of flow at section 2-2

19.Give an expression for loss of head due to sudden contraction :

$hc = 0.5\ V^2/2g$

Where hc = Loss of head due to sudden contraction .

V = Velocity at outlet of pipe.

20. Give an expression for loss of head at the entrance of the pipe

hi =0.5V2/2g

where hi = Loss of head at entrance of pipe .

V = Velocity of liquid at inlet and outlet of the pipe .

21. What is sypon ? Where it is used: _

Sypon is along bend pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level . Uses of sypon : -

1. To carry water from one reservoir to another reservoir separated by a hill ridge .
2. To empty a channel not provided with any outlet sluice .

UNIT – II
FLOW THROUGH CIRCULAR CONDUITS
PART- B

A Horizontal pipe of 250 mm diameter and 60m long is connected to a water tank at one end and discharge freely to atmosphere through the other end. If height of the water in tank is 4.5 m above the centre of the pipe, calculate the rate of flow of water. Consider all losses and take $f = 0.008$. Also draw the Hydraulic grade line (H.G.L) and total energy line (T.E.L) [A/M-13]

Givendata :-

Dia of pipe $D = 250mm = 0.25m$

length of pipe $L = 60m$

Height of water $H = 4.5m$

Co. efficient of friction, $f = 0.008$

Sol

Head loss at the entrance of the pipe, $h_i = \frac{0.5V^2}{2g}$

Head loss due to friction in the pipe, $h_f = \frac{4fLV^2}{2gD}$

Head loss at the exit from a pipe $h_o = \frac{V^2}{2g}$

Applying Bernoulli's equation at the top of the water surface in the tank and at the outlet of the pipe

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{All losses}$$

$$0 + 0 + 4.5 = 0 + \frac{V_2^2}{2g} + 0 + \frac{0.5V^2}{2g} + \frac{4fLV^2}{2gD} + \frac{V^2}{2g}$$

But the velocity in the pipe $V = V_2$

$$4.5 = \frac{V^2}{2g} + 0 + \frac{0.5V^2}{2g} + \frac{4fLV^2}{2gD} + \frac{V^2}{2g}$$

But the velocity in the pipe $V = V_2$

$$4.5 = \frac{V^2}{2g}\left[1 + 0.5 + \frac{4fL}{D} + 1\right]$$

$$= \frac{V^2}{2g}\left[1 + 0.5 + \frac{4 \times 0.008 \times 60}{0.25} + 1\right]$$

$$4.5 = \frac{V^2}{2g} \times (10.18)$$

$$V = \sqrt{\frac{4.5 \times 2 \times 9.81}{10.18}} = 2.945 \text{ m/s}$$

Rate of flow $Q = A \times V$

$$= \frac{\pi}{4} \times (0.25)^2 \times 2.945 = 0.1445 \text{ m}^3/\text{s}$$

Hydraulic Gradient line (H.G.L) gives the sum of $\left(\frac{P}{w}+Z\right)$ with reference to the datum line. Hence, H.G.L is obtained by subtracting $\frac{V^2}{2g}$ from total energy available at that point.

Head loss at the entrance of the pipe $h_i = \frac{0.5\times V^2}{2g}$

$$= \frac{0.5\times(2.945)^2}{2\times 9.81} = 0.221\,m$$

Head loss due friction, $h_f = \frac{4flv^2}{2gD} = \frac{4\times 0.008\times 60\times(2.945)^2}{2\times 9.81\times 0.25}$

$$= 3.39\,m$$

Head loss at exit of the pipe, $h_o = \frac{V^2}{2g} = \frac{2.945^2}{2\times 9.81} = 0.442\,m$

Total energy available at the entrance of the pipe

$$= h - h_i = 4.5 - 0.221 = 4.279\,m$$

The piezometric head $\left(\frac{P}{w}+Z\right)$ at the entrance = Total energy at entrance $-\frac{V^2}{2g}$ at entrance of the pipe $= 4.2749 - \left(\frac{(2.945)^2}{2\times 9.81}\right) = 3.833\,m$

Similarly, total at exit of the pipe,

$$= h - (h_i + h_f + h_o)$$
$$= 4.5 - (0.221 + 3.3949 + 0.442)\ 0.44$$

The piezometer head $\left(\frac{P}{w}+Z\right)$ available at exit of the pipe

$$= 0.442 - \frac{V^2}{2g} = 0.442 - \frac{(2.945)^2}{2\times 9.81} = 0\,m$$

... energy line (T.E.L)

1. point A lies on the free surface of water since total energy at A =

$$\frac{p}{w}+\frac{v^2}{2g}+z=0+0+4.5=4.5\,m$$

2. a point B is noted at a distance $AB = h_i = 0.221$ because the total energy at entrance of the pipe B = Total energy at A − h_i = 4.5 − 0.221 = 4.279 m

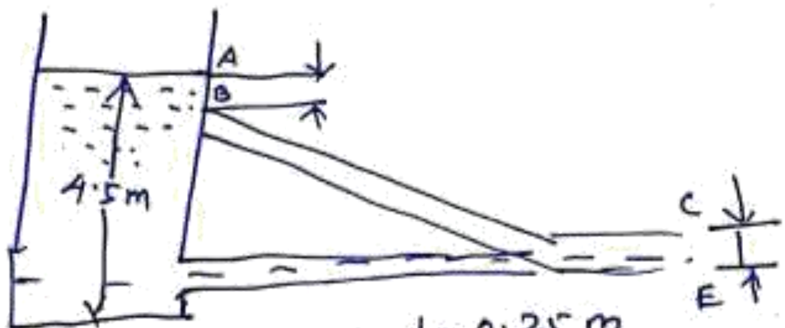
3. Total energy available at the exit of the pipe. i.e at C is already found out as 0.442 m. therefore, a point C is placed at a distance 0.442 m from the centre line as shown in fig. 2.

4. A, B and C are joined by straight lines. Then ABC represents the total energy line.

H.G.L

It gives the piezometric head i.e (sum of $\frac{p}{w}+z$) with reference to the datum line.

1. Piezometric head at the entrance of the pipe is already found. as 3.836 m. A point D is placed at a distance of 3.836 m from the datum



L = 60 m & d = 0.25 m

2. piezo head at the exit of the pipe line is 0 m. ∴ a point E is placed on the datum line.

3. D & E are joined by a straight line / H.G.L.

A pipe of 12 cm ... velocity of 4.5 m/s

and $\rho = 1250\ kg/m^3$) ... the wall surface.
Determine the shear stress at the length of the pipe, head loss.
Of the pipe, head loss, if the length of the pipe is 25 m and the
loss, of the length of the pipe
power lost.

Given data :- $D = 12cm = 0.12m$, Dynamic viscosity $\mu = 2.2\ Pa \cdot s = 2.2\ N.s/m^2$
$L = 25\ m$,
$\rho = 1250\ kg/m^3$, $V = 4.5\ m/s$

Discharge $Q = A \times V$

$$Q = \frac{\pi}{4} \times 0.12^2 \times 4.5\ m = 0.051\ m^3/s$$

From Hagen – Poiseuille's equation

$$P_1 - P_2 = \frac{128 \mu Q L}{\pi D^4} = P_1 - P_2 = \frac{128 \times 2.2 \times 0.051 \times 25}{\pi \times 0.12^4}$$

$$P_1 - P_2 = 551{,}147.67\ Pa$$

Shear stress at the pipe wall,

$$\tau_{max} = \left(-\frac{dp}{dx}\right)\frac{R}{2} - \frac{P_1 - P_2}{L} \times \frac{R}{2}$$

diff the Pre
dif. length

$$= \frac{551147}{25} \times \frac{0.12}{2 \times 2} = 661.38\ N/m^2$$

Head loss due to friction, $h_L = \frac{32 \mu V L}{w D^2} = \frac{32 \mu V L}{\rho g D^2}$

$$= \frac{32 \times 2.2 \times 4.5 \times 25}{(1250 \times 9.81) \times (0.12)^2}$$

$$= 44.85m$$

Power lost, $P = w Q h_L$

$$P = (1250 \times 9.81) \times 0.051 \times 44.85)$$

$$= 28.05\ kw.$$

3) Oil with a density of $900\ kg/m^3$ and kinematic viscosity of $6.2 \times 10^{-4}\ m^2/s$ is being discharged by a 6mm diameter 40m long horizontal pipe from a storage tank open to the atmosphere. The height of the liquid level above the center of the pipe is 3m. Neglecting the minor losses, determine the flow rate of oil through the pipe. [A/M-16]

Given data:-

Density $\rho = 900 kg/m^3$

Kinematic viscosity $= \nu = 6.2\times10^{-4} m^2/s$

Diameter, $D = 6mm$

Length $L = 40m$

Height $h = 3m$

SOL

$\mu = \rho\nu = 900\times6.2\times10^{-4} = 0.558 N\text{-}s/m^2$

Pressure at the bottom of the tank $\} p = \rho g h$

$P_1 gauge = 900\times9.81\times3 = 26487 Pa.$

$= 26.49 kPa$

Neglecting inlet and outlet losses, the pressure drop across the pipe,

$\Delta p = p_1 - p_2 = p_2 - P_{atm} = P_1 gauge$

$= 26.49 kpa$

The flow rate through a horizontal pipe in laminar flow.

$$Q = \frac{\Delta p \pi D^4}{128\mu L} = \frac{26487\times\pi\times\left(\frac{6}{1000}\right)^4}{128\times0.558\times40} = 3.78\times10^{-8}\frac{m^3}{s}.$$

(AU 2018)

(4) Two tanks of fluids ($\rho = 998 kg/m^3$ and $\mu = 0.001 kg/m^3$) at 20°C are connected by a capillary tube 4mm in diameter and 3.5m long. The surface of tank 1 is 30 cm higher than the surface of tank 2. Estimate the flow rate in m^3/h. Is the flow laminar? For what tube diameter will Reynolds number be 500?

Given data

$\rho = 998 kg/m$

$\mu = 0.001 kg/ms = 0.001 N\text{-}s/m^2$

$D = 4mm = 0.004m$

$L = 3.5m$

$z_1 - z_2 = H = 30cm = 0.3m$

Sol

Exit velocity of flow in the capillary tube $V = \sqrt{2gH}$

$= \sqrt{2 \times 9.81 \times 0.3} = 2.426 \frac{m}{s}$

From continuity equation, Flow rate $Q = AV$

$= \frac{\pi}{4} D^2 V$

$= \frac{\pi}{4} \times (0.004)^2 \times 2.426$

$= 3.05 \ m^3/s$

Reynolds number, $Re = \frac{\rho V D}{\mu}$

$= \frac{998 \times 2.426 \times 0.004}{0.001} = 9684.59$

75000

Therefore, the flow is turbulent since the flow through tube is considered as flow through pipes and Reynolds number of the flow is more than 5000.

For the Reynolds number 500, the diameter of the tube is given by

$D = \frac{Re \mu}{\rho V} = \frac{500 \times 0.001}{998 \times 2.426}$

$= 0.206 \ mm$

5) NOV 2012

A plate of 600mm length and 400 mm wide is immersed in a fluid of specific gravity 0.9 and kinematic viscosity $\nu = 10^{-4} m^2/s$. The fluid is moving with the velocity of 6 m/s Determine.

(1) Boundary layer thickness

(2) shear stress at the end of the plate &

(3) Drag force on one side of the plate

Given data :-

(i) Boundary layer thickness :-

$Re = \frac{UL}{\nu} = \frac{6 \times 0.6}{10^{-4}} = 36000$

Since $Re < 5\times10^5$, The flow is laminar. Therefore the thickness of boundary layer and shear stress for laminar flow are obtained as follows:-

The empirical relation for thickness of boundary layer for laminar flow is given by Prandtl-Blasius as

$$\delta_{lam} = \frac{5x}{\sqrt{Re}} = \frac{5\times0.6}{\sqrt{3600}} = 0.0158$$

ii) Shear stress at the end of the plate

$$\text{shear stress } \tau_0 = \frac{\mu U_\infty}{2\delta} = \frac{0.9\times(1\times10^{-4})\times6\times\pi}{2\times0.0158}$$

$$= 0.0536\ N/m^2$$

iii) Drag force on one side of the plate

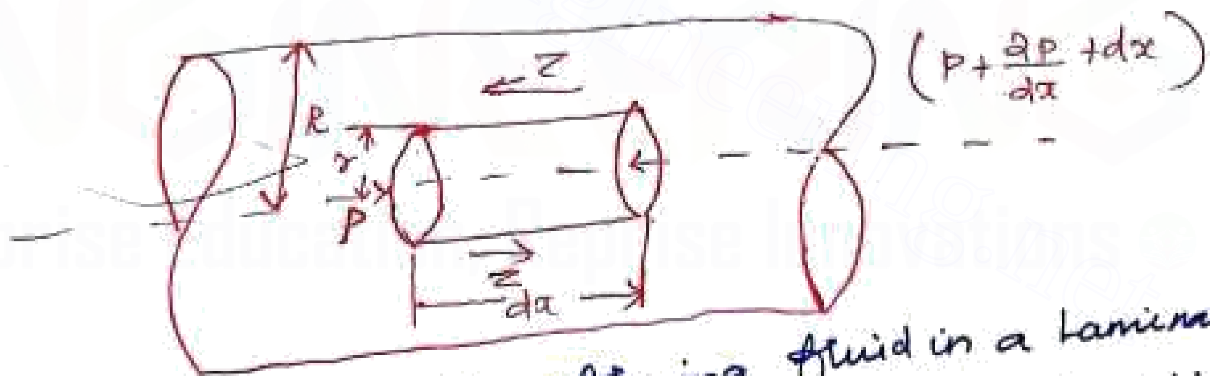
force = stress x Area

$$F_D = \tau_0 \times b \times L$$

$$= \frac{\mu U_\infty}{2\delta} \times b \times L = 0.0536 \times 0.4 \times 0.6 = 0.01288\ N$$

HAGEN POISEUILLE'S EQUATION [N/D - 2016]

5)



Due to viscosity of the flowing fluid in a laminar flow, some losses of head take place, the equation which gives us the value of loss of head due to viscosity in laminar flow is known as Hagan-poiseuille's Law.

$Re < 2000 \rightarrow$ Laminar flow

$2000 < Re < 4000 \rightarrow$ Transition flow

$Re > 4000 \rightarrow$ Turbulent flow