UNIT - IV

SHEAR STRENGTH

Shear strength of cohesive and cohesion less soils - Mohr- Coulomb failure theory - Measurement of shear strength - Direct shear - Triaxial Compression, UCC & Vane shear tests - Pore pressure parameters Cyclic mobility - Liquefaction.

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SHEAR STRENGTH:

When soil is loaded, shear stresses are induced in it. When shear stress reaches its limiting value, shear deformation takes place, leading to failure of the soil mass.

The failure may be in the form of

- Sinking of a footing
- Movement of a wedge of soil behind retaining wall forcing it to move out.
- Slide in Earth Embankment.

Definition: The shear strength of soil is the resistance to deformation by continuous shear displacement of soil particles upon the action of shear stress. The failure conditions for a soil may be expressed in terms of limiting shear stress, called shear strength.

The shearing resistance of soil is constituted basically of following components.

* Structural resistance to displacement of soil because of interlocking of particles.
* Frictional resistance to translocation b/w individual soil particles at their contact points.
* Cohesion or adhesion b/w surface of soil particles.

Shear strength of cohesionless soils:

The shear strength in cohesionless soil results from intergranular friction alone, while in all other soils it results from both internal friction as well as cohesion. However, plastic undrained clay doesnot possess internal friction.

MOHR'S STRESS CIRCLE:

Through a point in loaded soil mass, innumerous planes pass & stress components on each plane depends upon direction of plane.

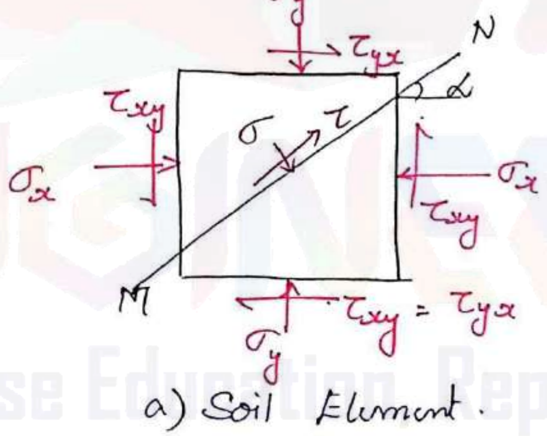
On every plane, there will be three typical planes, mutually orthogonal to each other, on which stress is wholly normal & no shear stress acts. These planes are called principal planes & normal stress acting on these planes are called principal stresses.

The plane which is devoid of shear is principal plane.

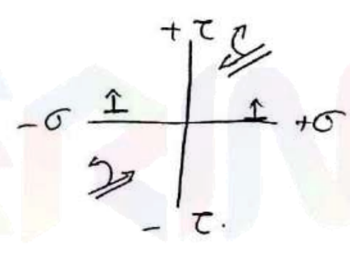
In the order of decreasing magnitude ②
of normal stress, these plane are called
* Major principal planes
* Intermediate Principal planes
* Minor principal planes.

The corresponding normal stress on them are
* Major principal stress (σ_1)
* Intermediate principal stress (σ_2)
* Minor principal stress (σ_3).

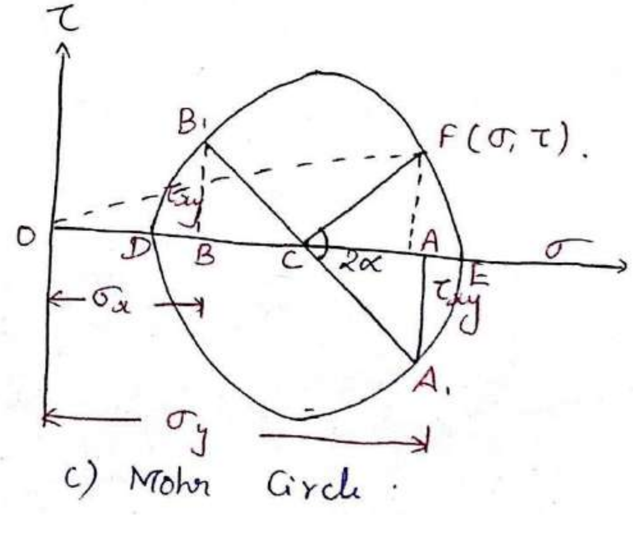
Many problems can be approximated by considering 2D stress conditions.



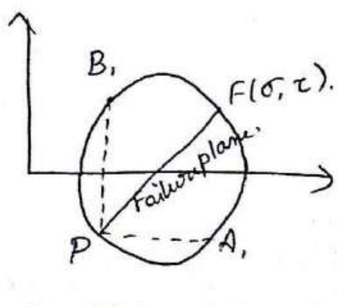
a) Soil Element.



b) Sign Convention



c) Mohr Circle.



P – Pole (or) Origin of plane.

Figure (a) shows a soil element subjected to 2D stress system.

From equilibrium of element, the following expressions were found for normal stress σ & shearing stress τ on any plane MN inclined at α with x-direction.

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \left[\frac{\sigma_y - \sigma_x}{2}\right]\cos 2\alpha + \tau_{xy}\sin 2\alpha \quad (1)$$

$$\tau = \frac{\sigma_y - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha \quad (2) \text{ when } (\sigma_y > \sigma_x)$$

σ_x - Normal stress on plane $\perp$ to x axis.

σ_y - Normal stress on plane $\perp$ to y-axis.

$\tau_{xy} = \tau_{yx}$ - Shear stress on these two plane

From (1),

$$\sigma - \frac{\sigma_y + \sigma_x}{2} = \left[\frac{\sigma_y - \sigma_x}{2}\right]\cos 2\alpha + \tau_{xy}\sin 2\alpha \quad (3)$$

Squaring & adding (3) & (2).

$$\left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 = \left[\frac{\sigma_y - \sigma_x}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha\right]^2$$

$$= \left[\frac{\sigma_y - \sigma_x}{2}\cos 2\alpha\right]^2 + \left[\tau_{xy}\sin 2\alpha\right]^2 + \cancel{2}\left[\frac{\sigma_y - \sigma_x}{\cancel{2}}\cos 2\alpha \cdot \tau_{xy}\sin 2\alpha\right]^1$$

$$\left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2\cos^2 2\alpha + \tau_{xy}^2\sin^2 2\alpha + (\sigma_y - \sigma_x)\tau_{xy}\sin 2\alpha\cos 2\alpha.$$

$$\tau^2 = \left[\frac{\sigma_y - \sigma_x}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha\right]^2 \quad (3)$$

$$= \left(\frac{\sigma_y - \sigma_x}{2}\right)^2 \sin^2 2\alpha + \tau_{xy}^2 \cos^2 2\alpha -$$

$$\tau_{xy}(\sigma_y - \sigma_x) \sin 2\alpha \cos 2\alpha .$$

Adding .

$$\left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 + \tau^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2 \left[\underbrace{\sin^2 2\alpha + \cos^2 2\alpha}_{1}\right] +$$

$$\tau_{xy}^2 \left[\underbrace{\sin^2 2\alpha + \cos^2 2\alpha}_{1}\right] +$$

$$(\sigma_y - \sigma_x)\, \cancel{\tau_{xy} \sin 2\alpha \cos 2\alpha} -$$

$$(\sigma_y - \sigma_x)\, \cancel{\tau_{xy} \sin 2\alpha \cos 2\alpha}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 + \tau^2 = \left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2 \quad (4)$$

Equation 4 is the equation of circle like

$x^2 + y^2 = R^2$ $\quad (x-a)^2 + (y-b)^2 = R^2$.

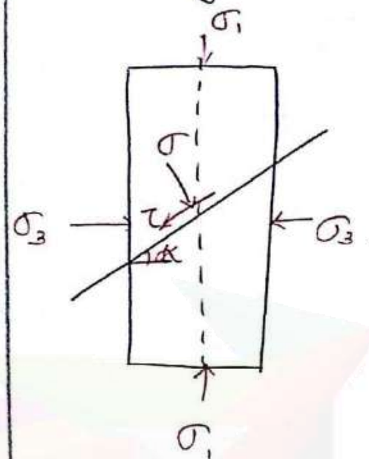
Coordinates (a, b) $= \left[\frac{\sigma_y + \sigma_x}{2}, 0\right]$

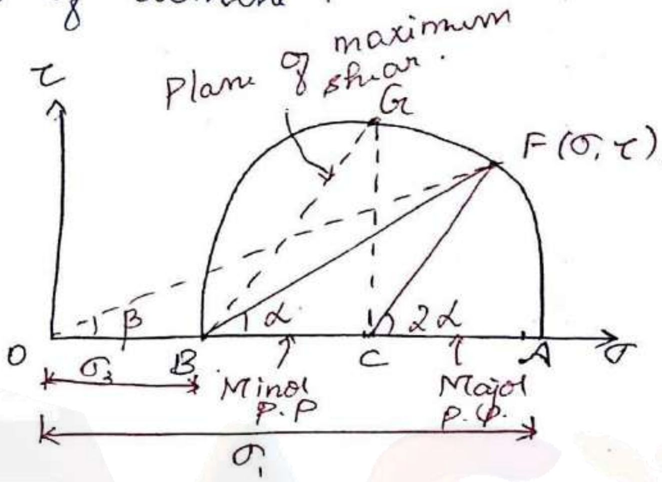
Radius $R^2 = \left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2$

$$R = \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

Coordinates of points on circle represent normal & shearing stress on inclined planes at a given point. This circle is known as Mohr's circle of stress.

Let us take the case of soil element whose sides are principal planes - consider the state of stress where only normal stresses are acting on faces of element.





Expression for σ, τ are.

$$\boxed{\begin{aligned}\sigma &= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\alpha. \\ \tau &= \frac{\sigma_1 - \sigma_3}{2}\sin 2\alpha.\end{aligned}}$$

Resultant stress $= \sqrt{\sigma^2 + \tau^2}$

Angle of obliquity $\beta = \tan^{-1}\left(\frac{\tau}{\sigma}\right)$

Max. Shear stress

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}}$$

It occurs on plane $\alpha = 45°$.

At max. shear stress,

Normal stress $= \dfrac{\sigma_1 + \sigma_3}{2}$.

MOHR- COULOMB FAILURE THEORY: (4)

Essential points of Mohr's strength theory are:

* Material fails essentially by shear. The critical shear stress causing failure depends upon the properties of material as well as on normal stress on failure plane.

* The ultimate strength of material is determined by stresses on potential failure plane.

* When the material is subjected to 3D principal stress ($\sigma_1, \sigma_2, \sigma_3$) the intermediate principal stress does not have any influence on strength of material. i.e: the failure criterion is independent of intermediate principal stress.

The theory can be expressed algebraically by equation.

$$\boxed{\tau_f = S = F(\sigma)} \longleftarrow \text{Mohr.}$$

τ_f = s = Shear stress on failure plane @ failure
= Shear resistance of material.

$F(\sigma)$ = Function of normal stress.

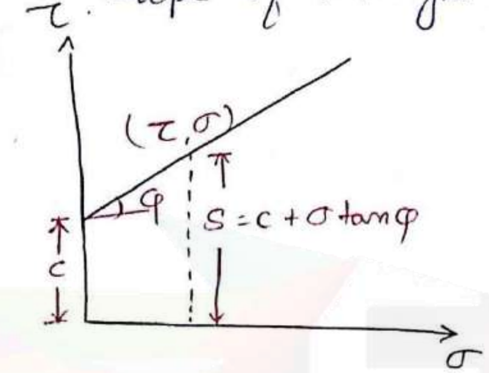
If normal stress & shear stress corresponding to failure are plotted, then a curve is obtained. That curve is called strength envelope.

Coulomb defined the function $F(\sigma)$ as a linear function of σ & gave strength equation : $\boxed{S = c + \sigma \tan\varphi}$ ← Coulomb.

c & φ - Empirical constants.

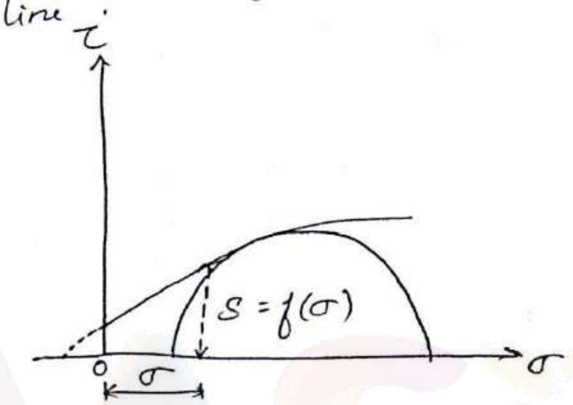
c - cohesion - Intercept on shear axis.

φ - Angle of internal friction/shearing resistance - Slope of straight line.



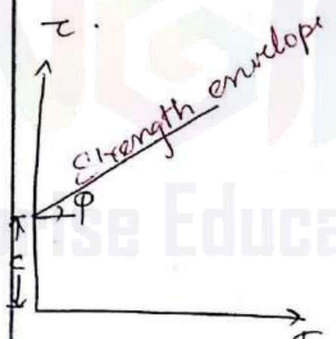
a) Coulomb Envelope (Straight line)

Shear strength ∝ Normal stress

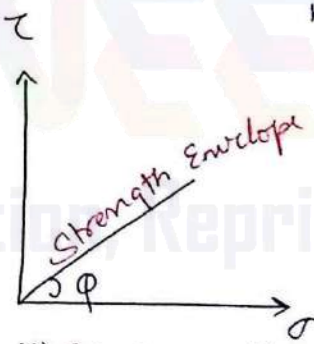


b) Mohr's Envelope. (Curve).

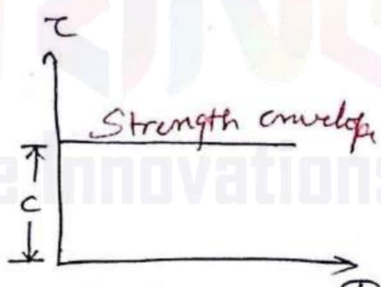
Shear stress & normal stress relation is not linear.



i) Cohesionless soil



ii) Cohesive soil (Pure friction).



iii) Purely cohesive soil.

EFFECTIVE STRESS PRINCIPLE :

In equation $S = c + \sigma \tan\varphi$, it is assumed that total normal stress governs shear strength of soil. This assumption is not always correct.

Extensive tests on remoulded clays have C sustained beyond doubt Terzaghi's early concept that effective normal stresses control shearing resistance of soils.

$$\tau_f = c' + \sigma' \tan \varphi'$$

$$\boxed{\tau_f = c' + (\sigma - u) \tan \varphi'}$$

c' - Effective cohesion intercept.

φ' - Effective angle of shearing resistance.

In terms of total stresses,

$$\boxed{\tau_f = c_u + \sigma \tan \varphi_u}.$$

c_u - Apparent cohesion

φ_u - Apparent angle of shearing resistance.

Normal stress σ' & shear stress τ on any plane inclined @ an angle α to major principal plane can be expressed by

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha.$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\alpha.$$

σ_1' - Effective Major principal stress.

σ_3' - Effective minor principal stress.

Substituting values of σ'

$$\tau_f = c' + \tan \varphi' \left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \right].$$

Most dangerous plane — Failure occurs - $(\tau_f - \tau)$, b/w shear strength & shear stress is minimum.

$$\tau_f - \tau = c' + \frac{\sigma_1' + \sigma_3'}{2}\tan\varphi' + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\alpha \tan\varphi' - \frac{\sigma_1' - \sigma_3'}{2}\sin 2\alpha .$$

Differentiate w.r. to α,

$$\frac{d}{d\alpha}(\tau_f - \tau) = \frac{\sigma_1' - \sigma_3'}{\cancel{2}}\tan\varphi' \left[-\cancel{2}\sin 2\alpha\right] - \frac{\sigma_1' - \sigma_3'}{\cancel{2}}\left[\cancel{2}\cos 2\alpha\right].$$

$$= -(\sigma_1' - \sigma_3')\tan\varphi' \sin 2\alpha - (\sigma_1' - \sigma_3')\cos 2\alpha .$$

For minimum, $(\tau_f - \tau)$,

$$\frac{d}{d\alpha}(\tau_f - \tau) = 0 .$$

$$\cancel{(\sigma_1' - \sigma_3')}\cos 2\alpha = -\cancel{(\sigma_1' - \sigma_3')}\tan\varphi' \sin 2\alpha .$$

$$\tan\varphi = -\frac{\cos 2\alpha}{\sin 2\alpha} .$$

$$-\tan\varphi = \cot 2\alpha .$$

$$\cot(90^\circ + \varphi) = \cot 2\alpha .$$

$$2\alpha = 90^\circ + \varphi$$

$$\alpha = \frac{90^\circ + \varphi}{2}$$

$$\boxed{\alpha_f = \alpha = 45^\circ + \frac{\varphi}{2}}$$

It can also be derived from Mohr's circle.

MEASUREMENT OF SHEAR STRENGTH:

The measurement of shear strength of soil involves certain test observations at failure, with the help of which failure envelope or strength envelope can be plotted.

Laboratory tests:

1. Direct shear test.
2. Triaxial shear test
3. Unconfined Compression test
4. Vane shear test.

Depending on drainage conditions, 3 types of shear tests.

* Undrained test (or) quick test
* Consolidated drained test.
* Drained test.

Undrained test:

No drainage of water is permitted. There is no dissipation of pore pressure during entire test

Direct Shear test - Drainage not permitted during application of both normal stress & shear stress.

Triaxial compression test: Drainage not permitted during period of both pore pressure & deviated pressure.

Drained test:

Drainage is permitted throughout the test during application of both normal & shear stresses so that full consolidation occurs & no excess pore pressure is set up at any stage of test.

Consolidated undrained test

Drainage is permitted under initially applied normal stress only & full primary consolidation or softening is allowed to take place. No drainage is allowed afterwards.

C & φ - vary with drainage conditions.

Direct shear test - Allowed to consolidate fully under applied normal stress & shear for high rate of strain to prevent dissipation of pore pressure during shearing.

Triaxial compression test: Allowed to consolidate fully under applied self pressure & then pore water outlet is closed & specimen subjected to increasing deviated stress at high rate of strain.

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1. DIRECT SHEAR TEST:

- Simple & commonly used test.

Apparatus: Shear box Apparatus.

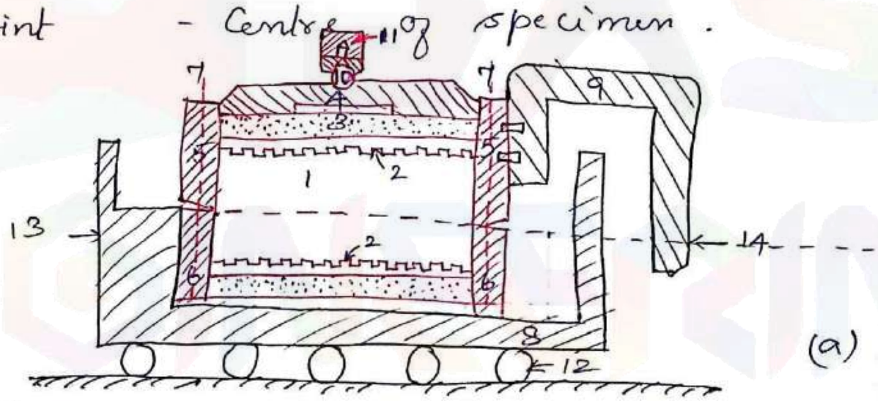
The apparatus consists of two piece shear box of square or circular cross-section.

The lower half of the box is rigidly held in position in a container which rests over slides or rollers and can be pushed forward at a constant rate by geared jack, driven either by electric motor or by hand.

The upper half of the box butts against a proving ring.

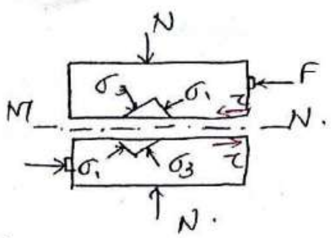
The soil sample is compacted in the shear box & is held b/w metal grids and porous stones.

Upper box - Upper half of specimen
Lower box - Lower half of specimen
Joint - Centre of specimen.

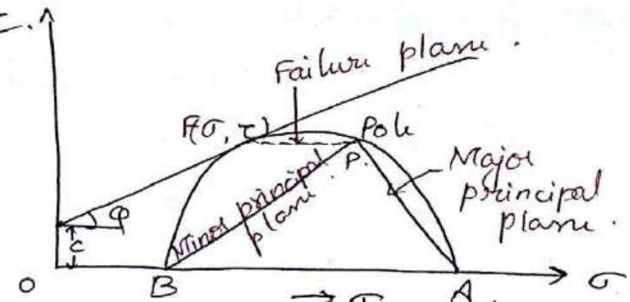


(a)

1. Soil specimen
2. Metal grids
3. Porous stones
4. Loading pad
5. Upper part
6. Lower part
7. Screws to fix two halves of shear box
8. Container for shear box
9. U-Arm
10. Steel ball
11. Loading yoke
12. Rollers
13. Shear force applied by jack
14. Shear resistance Measured by proving ring.



b) Principle of direct shear box.



c) Mohr's envelope.

Procedure:

Normal pressure is applied on the specimen from loading yoke bearing upon steel ball of pressure pad.

When shearing force is applied to lower box through geared jack, the movement of lower part of box is transmitted through specimen to upper box ~~of~~ and hence on proving ring.

Deformation of proving ring indicates shear force.

The volume change during consolidation & during shearing process is measured by mounting a dial guage at top of box.

The soil specimen can be compacted in shear box by clamping both the parts together with the help of two screws.

These screws are removed before shearing force is applied.

Metal grids. placed above top & below ~~bottom~~ bottom of specimen may be perforated if drained test is required. or plain if undrained test is required.

Serrations in grids are $\perp$ to direction of shear ~~force~~:

* Strain controlled test
* Stress controlled test.

Strain controlled Test:

Shear strain is made to increase at constant rate. Fig(b) shows strain controlled shear test. Fig(c) shows failure envelope plotted as a function of shear stress & normal stress.

Stress controlled test:

There is an arrangement to increase the shear stress at a desired rate and measure the shearing strain.

Test can be performed under all three conditions of drainage.

* Undrained test - Plain grids are used.
* Drained test (Slow test) - Perforated grids are used. Consolidated under normal load & then sheared slowly so that complete dissipation of pore pressure takes place. (2-5 days).

Consolidated undrained test - Perforated grids are used. Consolidated under normal load & sheared quickly in about 5-10 minutes.

Advantages:

* Simple test.
* The relatively thin thickness of sample permits quick drainage & quick dissipation of pore pressure developed during test.

Disadvantages:

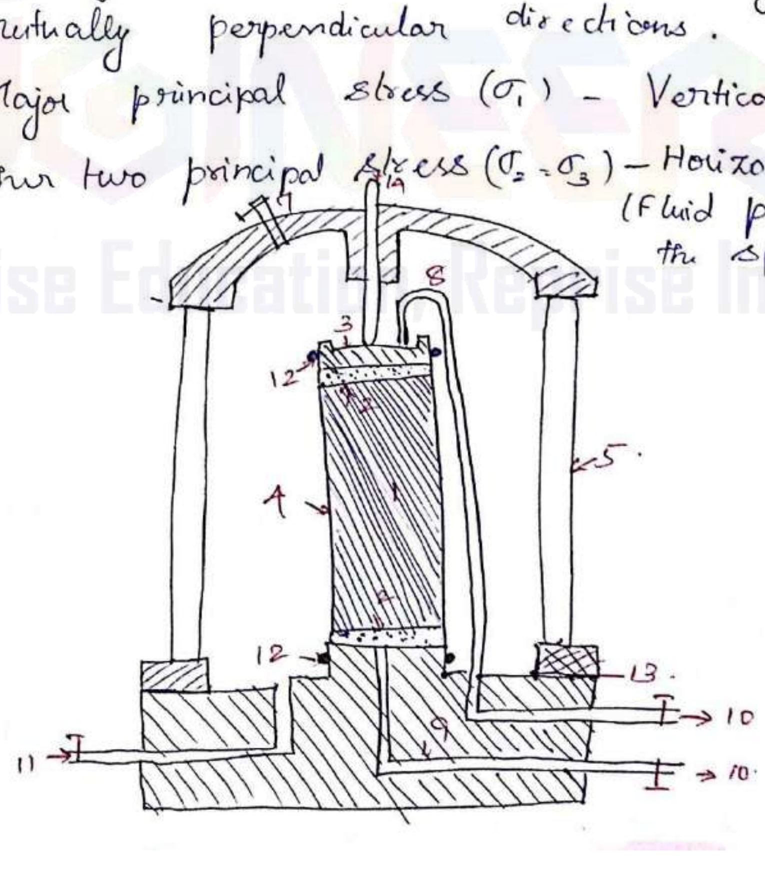
* Stress conditions across soil sample are very complex.
* As test progresses, the area under shear gradually decreases.
* As compared to triaxial test, there is little control on drainage of soil.
* The plane of shear failure is predetermined, which may not be weakest one.
* There is effect of lateral restraint by side walls of shear box.

TRIAXIAL COMPRESSION TEST:

The solid specimen, cylindrical in shape, is subjected to direct stresses acting in three mutually perpendicular directions.

Major principal stress (σ_1) - Vertical direction.

Other two principal stress ($\sigma_2 = \sigma_3$) - Horizontal direction. (Fluid pressure round the specimen)



1. Soil specimen
2. Porous disc
3. Top cap
4. Rubber membrane
5. Perspex cylinder
6. Loading ram
7. Air Release Valve
8. Top Drainage Tube
9. Bottom Drainage Tube
10. Connections for drainage (or) Pore pressure measurement.
11. Cell fluid inlet
12. Rubber rings
13. Sealing ring
14. Axial load through proving ring.

Apparatus :

- High pressure cylindrical cell - Perspex or other transparent material - fitted b/w base & top cap.

- 3 outlet connections are provided through base :
  * Cell fluid inlet
  * Pore water outlet from bottom of specimen
  * Drainage outlet from top of specimen.

- A seperate compressor is used to apply fluid pressure in the cell.

- Pore pressure developed in specimen during test can be measured with help of seperate pore pressure measuring equipment, such as Bishop's apparatus.

- A stainless steel piston running through centre of top cap applies the vertical compressive load (deviator stress) on specimen under test.

- The load is applied through a proving ring, with the help of mechanically operated load frame.

- Depending on drainage conditions of test, solid nonporous or porous discs are placed on top & bottom & rubber membrane is sealed on to the end caps

Length of specimen = 2 to 2½ times its diameter

Cell pressure σ_3 $(=\sigma_2)$ acts all round specimen it also acts on top of specimen as well as vertical piston meant for among deviator stress

Vertical stress $= (\sigma_1 - \sigma_3)$

Total stress on top $= (\sigma_1 - \sigma_3) + \sigma_3$

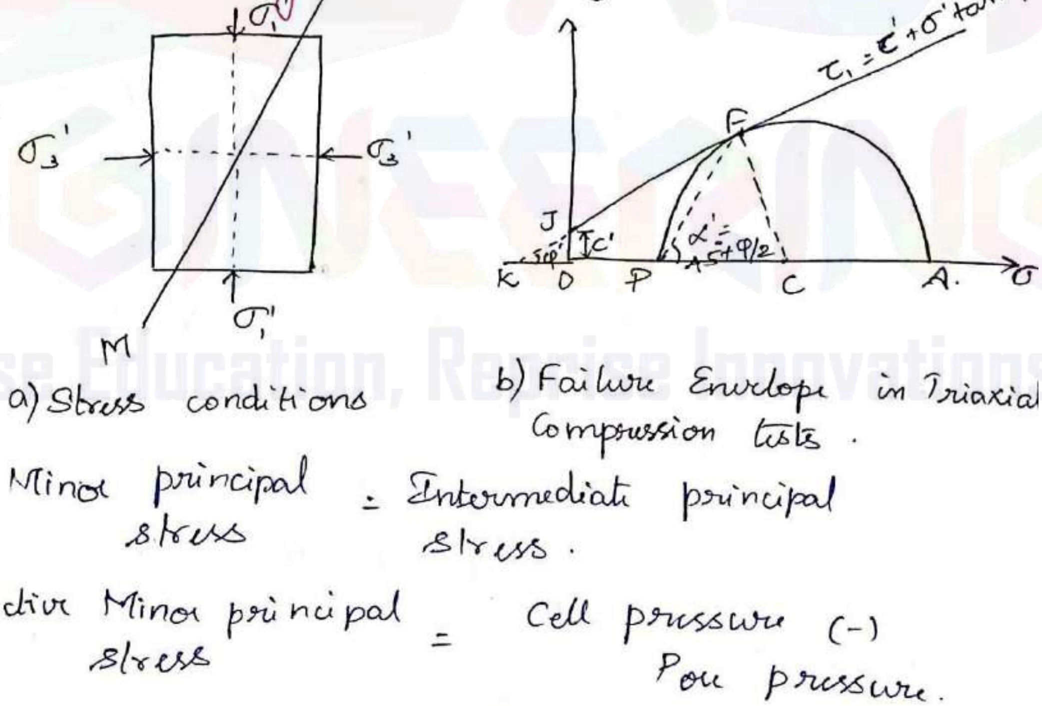
$= \sigma_1$ - Major principal stress

Stress difference $(\sigma_1 - \sigma_3)$ - Deviator stress.

Record on proving ring dial

Another dial - Vertical deformation.

Stress condition in soil specimen during triaxial testing:



a) Stress conditions

b) Failure Envelope in Triaxial Compression tests.

Minor principal stress = Intermediate principal stress.

Effective Minor principal stress = Cell pressure (-) Pore pressure.

Major Principal stress = Deviator stress + Cell pressure

Failure plane inclined at an angle α' to major principal plane

$$\alpha' = 45^\circ + \varphi'/2 .$$

FC - Radius of Mohr's circle $= \frac{1}{2}(\sigma_1' - \sigma_3')$.

$$OC = \frac{1}{2}(\sigma_1' + \sigma_3')$$

$$OK = c' \cot \varphi'$$

$$\sin\varphi' = \frac{FC}{KC} = \frac{FC}{KO + OC}$$

$$= \frac{\frac{\sigma_1' - \sigma_3'}{2}}{c'\cot\varphi' + \frac{\sigma_1' + \sigma_3'}{2}}$$

$$\sin\varphi' = \frac{\sigma_1' - \sigma_3'}{2c'\cot\varphi' + (\sigma_1' + \sigma_3')}$$

$$(\sigma_1' - \sigma_3') = 2c'\cot\varphi' \sin\varphi' + (\sigma_1' + \sigma_3')\sin\varphi'$$

$$= 2c' \frac{\cos\varphi'}{\sin\varphi'} \sin\varphi' + (\sigma_1' + \sigma_3')\sin\varphi'$$

$$\sigma_1' - \sigma_3' = 2c'\cos\varphi' + (\sigma_1' + \sigma_3')\sin\varphi'$$

$$\sigma_1' - \sigma_1'\sin\varphi' = 2c'\cos\varphi' + \sigma_3'\sin\varphi' + \sigma_3'$$

$$\sigma_1'(1 - \sin\varphi') = 2c'\cos\varphi' + \sigma_3'(1 + \sin\varphi')$$

$$\sigma_1' = 2c' \frac{\cos\varphi'}{(1 - \sin\varphi')} + \sigma_3' \frac{1 + \sin\varphi'}{1 - \sin\varphi'}$$

$$\sigma_1' = 2c' \tan\left(45^\circ + \frac{\varphi'}{2}\right) + \sigma_3' \tan^2(45^\circ + \varphi'/2)$$

$$\boxed{\sigma_1' = \sigma_3' \tan^2\alpha' + 2c' \tan\alpha'.}$$

$$\sigma_1' = \sigma_3' N_\varphi' + 2c'\sqrt{N_\varphi'}$$

$$\boxed{N_\varphi' = \tan^2\alpha' = \tan^2[45^\circ + \varphi'/2]}$$

In terms of total stress,

$$\boxed{\sigma_1 = \sigma_3 \tan^2\alpha + 2c_u \tan\alpha.}$$

$$\boxed{\alpha = 45^\circ + \frac{\varphi_u}{2}}$$

The calculation of deviator stress must be done on basis of changed area of c/s at failure, or during any stage of test. The area A_2 at failure, or during any stage of test can be found by relation -

$$\boxed{A_2 = \frac{V_1 \pm \Delta V}{L_1 - \Delta L}}$$

V_1 - Initial volume of specimen

L_1 - Initial length of specimen

ΔV - change in volume of specimen

ΔL - change in length of specimen

Deviator stress σ_d

$$\sigma_d = \frac{\text{Additional axial load.}}{A_2}$$

σ_3 = Fluid pressure

$\sigma_1 = \sigma_3 + \sigma_d$.

Advantages:

* Shear tests under all three drainage conditions can be performed with complete control.
* Precise measurement of pore pressure & volume change during test are possible
* Stress distribution on failure plane is uniform.
* State of stress within specimen during any stage of test, as well as @ failure is completely determinate.

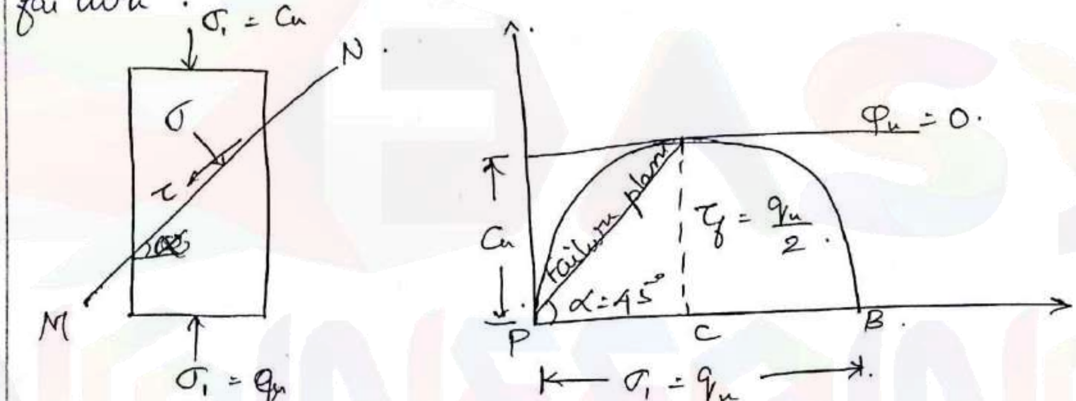
UNCONFINED COMPRESSION TEST

It is a special case of biaxial compression test in which $\sigma_2 = \sigma_3 = 0$.

Cell pressure = Confining pressure.

Absence of confining pressure - Uniaxial test - Unconfined compression test.

Cylindrical specimen of soil is subjected to major principal stress σ_1 till specimen fails due to shearing along a critical plane or failure.



Apparatus:

* Small load frame fitted with proving ring to measure vertical stress applied to soil specimen.
* Deformation is measured with dial gauge.

$\sigma_3 = 0$, Mohr's circle passes through origin which is also a pole.

$$\boxed{\sigma_1 = 2C_u \tan\alpha = 2C_u \tan\left(45^\circ + \frac{\varphi_u}{2}\right)}$$

Unknowns - C_u & φ_u - can't be found by unconfined test.

It gives the value of σ_1.

This test is generally applicable to saturated clays for which the apparent angle of shearing resistance φ_u is zero.

$$\sigma_1 = 2C_u$$

$$\boxed{\sigma_1 = 2C_u}$$

When Mohr's circle is drawn, its radius is equal to $\sigma_1/2 = C_u$.

$$\boxed{\sigma = \frac{\sigma_1}{2} = \frac{q_u}{2}} \qquad \tau_f = \frac{\sigma_1}{2} = \frac{q_u}{2} = C_u.$$

$$\boxed{\tau_f = C_u = \frac{q_u}{2}}$$

q_u = Unconfined compressive strength @ failure.
Compressive stress is calculated on the basis of changed C/S area A_2 at failure.

$$A_2 = \frac{V}{L_1 - \Delta L} = \boxed{\frac{A_1}{1 - \frac{\Delta L}{L_1}} = A_2}$$

V - Initial volume of specimen
L_1 - Initial length of specimen
ΔL - Change in length at failure.

VANE SHEAR TEST (Quick test).

- Used in laboratory & in field.
- To determine undrained shear strength of cohesive soil.
- The tester consists of 4 thin steel plates, called vanes, welded orthogonally to steel rod.
- A torque measuring arrangement, such as

a calibrated torsion spring, is attached to the rod which is rotated by worm gear and worm wheel arrangement.

After pushing the vanes gently into soil, the torque rod is rotated at uniform speed.

The rotation of spring in degrees is indicated by a pointer moving on graduated dial attached to worm wheel shaft.

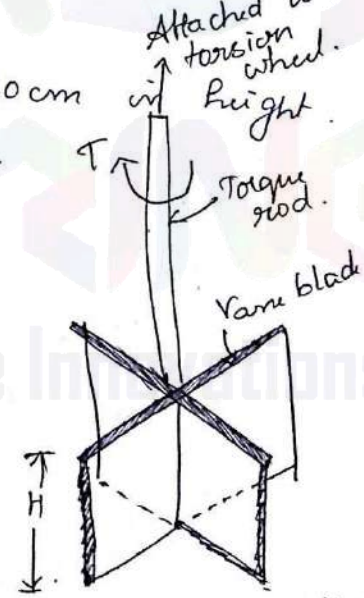
The torque T is then calculated by multiplying dial reading with spring constant.

A typical laboratory vane is 20mm high & 12mm in diameter with blade thickness from 0.5 to 1mm.

Blades - High tensile steel.

Field shear vane - 10 to 20cm in height.

Blade thickness - 2.5m.



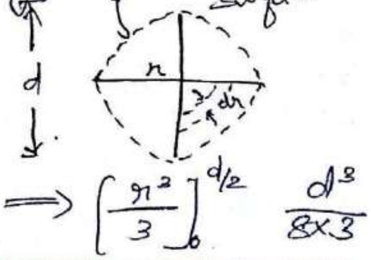
τ_f - Unit strength of soil

H - Height of vane

d - diameter of vane.

Case I:

Vane is pushed in soil with its top end below surface of soil so that both top & bottom ends partake in shearing of soil.



$$T_f = \pi d H \tau_f \frac{d}{2} + 2\int_0^{d/2} (2\pi r\,dr)\,\tau_f\, r.$$

$$= \pi d H \tau_f \frac{d}{2} + 4\pi \tau_f \int_0^{d/2} r^2\,dr \implies \left[\frac{r^3}{3}\right]_0^{d/2} \quad \frac{d^3}{8\times 3}$$

$$T = \pi d H \tau_f \frac{d}{2} + 4\pi \tau_f \frac{d^3}{\cancel{8}_{2} \times 3}$$

$$= \pi \tau_f \left[\frac{d^2 H}{2} + \frac{d^3}{6}\right]$$

$$\boxed{T_f = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6}\right]}$$

Case II :

The vane is pushed inside the soil with its top end flushed with surface of soil so that only bottom end partakes in shearing the soil.

$$T_f = (\pi d H \tau_f)\, d/2 + \int_0^{d/2} 2\pi r\, dr\, \tau_f\, r.$$

$$= \frac{\pi d^2}{2} H \tau_f + \cancel{2}\pi \tau_f \frac{d^3}{\cancel{8}_{4} \times 3}$$

$$= \frac{\pi d^2}{2} H \tau_f + \pi \tau_f \frac{d^3}{12}$$

$$\boxed{T_f = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{12}\right]}$$

Knowing T_f, H & d, Shear strength τ_f can be determined.

x

PROBLEMS:

1. A direct shear test was carried out on cohesive soil sample under the following results are obtained.

Normal Stress (kN/m^2)	Shear stress (kN/m^2)
150	110
250	120

What would be deviated stress at failure (13) if the triaxial shear test was carried on the same soil with a cell pressure of 150 kN/m^2.

Coulomb's shear strength equation is given by

$$\tau_f = c + \sigma \tan\varphi .$$

$$110 = c + 150 \tan\varphi$$

$$120 = c + 250 \tan\varphi .$$

Solving, $\boxed{c = 95 \ kN/m^2}$

$$\tan\varphi = 0.1$$

$$\boxed{\varphi = 5.71°}$$

Deviated stress at failure,

$$\sigma_d = \sigma_1 - \sigma_3 .$$

$$\sigma_1 = \sigma_3 \tan^2 (45 + \varphi/2) + 2c \tan(45 + \varphi/2)$$

$$= 150 \tan^2 (45 + 5.71/2) + 2 \times 95 \tan\left(45 + \frac{5.71}{2}\right).$$

$$\sigma_1 = 183.146 + 209.95$$

$$\boxed{\sigma_1 = 393.09 \ kN/m^2}$$

$$\sigma_d = \sigma_1 - \sigma_3$$

$$= 393.09 - 150$$

$$\boxed{\sigma_d = 243.1 \ kN/m^2}$$

---

2. A consolidated undrained test was conducted on a dry sample on following results were obtained.
Determine the shear strength parameters w.r.to i) Total stress concept ii) Effective stress concept

Cell pressure (kN/m^2)	Deviator stress at failure (kN/m^2)	Pore water pressure at failure (kN/m^2)
200	118	110
400	240	220
600	352	320.

Solution :

σ_3 (kN/m^2)	σ_d (kN/m^2)	u (kN/m^2)	$\sigma_1 = \sigma_3 + \sigma_d$ (kN/m^2)	$\sigma_1' = \sigma_1 - u$ (kN/m^2)	$\sigma_3' = \sigma_3 - u$ (kN/m^2)
200	118	110	318	208	90
400	240	220	640	420	180
600	352	320	952	632	280.

$$\sigma_1 = \sigma_3 \tan^2(45° + \varphi/2) + 2c \tan(45 + \varphi/2)$$

$$318 = 200 \tan^2\alpha + 2c \tan\alpha$$

$$640 = 400 \tan^2\alpha + 2c \tan\alpha.$$

$$322 = 200 \tan^2\alpha$$

$$\tan^2\alpha = 1.61$$

$$\boxed{\alpha = 51.76°}$$

$$45 + \varphi/2 = 51.76$$

$$\varphi/2 = 6.76$$

$$\boxed{\varphi = 13.52°}$$

$$318 = (200 \times 1.61) + 2c \tan 51.76$$

$$\boxed{c = -1.5}$$

$$\sigma_1' = \sigma_3' \tan^2(45+\varphi/2) + 2c' \tan(45+\varphi/2) \quad (1A)$$

$$208 = 90 \tan^2\alpha' + 2c' \tan\alpha'$$

$$420 = 180 \tan^2\alpha' + 2c' \tan\alpha'$$

$$212 = 90 \tan^2\alpha'$$

$$\boxed{\alpha' = 56.91^\circ}$$

$$45 + \varphi/2 = 56.91$$

$$\boxed{\varphi' = 23.83^\circ}$$

$$208 = 90 \times \tan^2(56.91) + 2c' \tan(56.91)$$

$$\boxed{c' = -1.3}$$

3. An unconfined compression test was conducted on an undisturbed sample of clay. The sample has a diameter of 38mm & length 76mm. Load at failure was 30N. & axial deformation of sample is 11mm. Determine the undrained shear strength parameter if failure plane makes an angle of 50° with horizontal.

Initial length of sample = 76mm.

Diameter of sample = 38 mm.

Initial area of C/S.

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 38^2}{4} = 1134.11\,mm^2$$

change in length $\Delta L = 11mm$.

Axial strain at failure $= \frac{\Delta L}{L} = \frac{11}{76} = 0.145$.

Area of c/s at failure.

$$A_f = \frac{A}{1-\epsilon} = \frac{1134.11}{1-0.145} = 1326.44 \text{ mm}^2.$$

$$q_u = \frac{P}{A_f} = \frac{30}{1326.44} = 0.023 \text{ N/mm}^2.$$

$$\boxed{q_u = 0.023 \text{ N/mm}^2}$$

$$\alpha = 50^\circ$$

$$45 + \varphi/2 = 50^\circ \qquad \sigma_3 = 0$$

$$\varphi = 10^\circ \rightarrow 0$$

$$\sigma_1 = \cancel{\sigma_3 \tan^2\left(45 + \frac{\varphi_u}{2}\right)} + 2c \tan\left(45 + \frac{\varphi_u}{2}\right).$$

$$\sigma_1 = q_u = 2c \tan(45^\circ + \varphi/2)$$

$$2c = \frac{0.023}{\tan 50}$$

$$2c = 0.0193$$

$$\boxed{c = 0.0096 \text{ N/mm}^2}$$

A. An unconfined compression test is conducted on a saturated clay specimen of 40mm diameter & 90mm length measured on its sides. The specimen has cored edge & its length b/w apex of cone is 80mm. The specimen fails under an axial compression load of 460N with axial deformation of 10mm. Calculate the unconfined compressive strength of clay.

Length of sample on its sides = 90mm.

Length b/w apex of cone = 80 mm.

Diameter of sample = 40mm.

$$\text{Actual Length} = 90 - \frac{2 \times 5}{3} = 86.67 \text{ mm}. \qquad \text{Area} = \frac{\pi d^2}{4} = 1256.64 \text{ mm}^2$$

$\Delta L = 10.$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{10}{86.67} = 0.1154$$

$$A_f = \frac{A}{1-\varepsilon} = \frac{1256.64}{1-0.1154}$$

$$A_f = 1420.5 \, mm^2.$$

$P = 460 \, KN.$

$$q_u = \frac{P}{A_f} = 0.324 \, N/mm^2$$

$$= \frac{0.324 \times 10^6}{10^3} \, KN/m^2$$

$$= 324 \, KN/m^2.$$

$$C_u = \frac{q_u}{2} = \frac{324}{2}$$

$$\boxed{C_u = 162 \, KN/m^2}$$

5. In a vane shear test conducted in a soft clay deposit, failure occured at a torque of 42Nm. Afterwards, the vane was allowed to rotate rapidly and the test was repeated in the rebounded soil. The torque at failure in the remoulded soil was 17Nm. Calculate the sensitivity of soil in both cases. In both cases, the vane was pushed completely inside the soil. The height & diameter of vane was 100mm & 80mm respectively.

$$\text{Sensitivity of clay} = \frac{\text{cohesion in undisturbed state}}{\text{cohesion in remoulded state}}.$$

$$S = \frac{C_{undisturbed}}{C_{remoulded}} \qquad S = \frac{q_u/2 \, (u)}{q_u/2 \, (r)}$$

Height of vane (H) = 100 mm.
Diameter (d) = 80 mm.

$$T_f = 17 Nm = 17000 Nmm.$$

In this problem, both top & bottom ends particle in shearing the soil.

$$T_f = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6}\right].$$

Case - I

For natural soil,

$$T = 42 Nm = 42000 Nmm.$$

$$42000 = \pi \times 80^2 \times \tau_f \left[\frac{100}{2} + \frac{80}{6}\right].$$

$$\boxed{\tau_f = 0.033 N/mm^2.}$$

Case - II

For remoulded state.

$$17000 = \pi \times 80^2 \times \tau_f \left[\frac{100}{2} + \frac{80}{6}\right]$$

$$\boxed{\tau_f = 0.0134 N/mm^2}$$

$$\text{Sensitivity} = \frac{0.033}{0.0134} = 2.54. \qquad \boxed{S = 2.47}$$

Hyxotrophic Effect:

If the soil is disturbed, its shear strength decreases. If we leave the soil, it regain its shear strength after sometimes. This is known as hyxotrophic effect.

SKEMPTON'S PORE PRESSURE PARAMETERS

The change in the pore pressure due to change in the applied stress, during an undrained shear, may be explained in terms of empirical coefficients called pore pressure parameters.

A pore pressure parameter may be defined as a dimensionless number that indicates the fraction of total stress increment that show up an excess pore pressure for condition of no drainage.

Let us consider a soil mass subjected to increase in 3 principal stresses, $\Delta\sigma_1$, $\Delta\sigma_2$ & $\Delta\sigma_3$, resulting in volume decrease ΔV and a consequent increase in pore pressure of Δu.

The increase in effective stresses

$$\Delta\sigma_1' = \Delta\sigma_1 - u$$
$$\Delta\sigma_2' = \Delta\sigma_2 - u$$
$$\Delta\sigma_3' = \Delta\sigma_3 - u.$$

$\varepsilon_1, \varepsilon_2, \varepsilon_3$ - Strains in 3 directions.

Youngs modulus $E = \frac{\text{Stress}}{\text{Strain}}$.

$$E\,\varepsilon_1 = \Delta\sigma_1' - \mu\,(\Delta\sigma_2' + \Delta\sigma_3')$$
$$E\,\varepsilon_2 = \Delta\sigma_2' - \mu\,(\Delta\sigma_3' + \Delta\sigma_1')$$
$$E\,\varepsilon_3 = \Delta\sigma_3' - \mu\,(\Delta\sigma_1' + \Delta\sigma_2').$$

$$E(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = E \cdot \varepsilon_v = E\,\frac{\Delta V}{V}$$

$$= \Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3' - \mu\,[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_2' + \Delta\sigma_3' + \Delta\sigma_3' + \Delta\sigma_1']$$

$= \Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3' - 2\mu\,[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3']$

$= (1-2\mu)\,[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3']$.

$$E\,\frac{\Delta V}{V} = (1-2\mu)\,[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3']$$

$$\frac{\Delta V}{V} = \frac{(1-2\mu)}{E}\,[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3']$$

Multiply & divide by 3.

$$\frac{\Delta V}{V} = \frac{3(1-2\mu)}{E} \cdot \frac{1}{3}\,[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3'].$$

$\frac{3(1-2\mu)}{E} = C_c$: Compressibility of soil skeleton

Substituting values of effective stress in terms of total stresses. $\Delta\sigma_1' = \Delta\sigma - u$.

$$\frac{\Delta V}{V} = \frac{C_c}{3}\,[\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u]$$

$$= C_c\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right].$$

n - Porosity

Volume of voids $= nV$.

If the pore fluid is assumed to show a linear relationship b/w volume change & stress, and its coefficient of volume compressibility is represented by C_v, the change in volume of pore fluid ΔV_w due to increase in pore pressure Δu under the condition of no drainage is given by

$$\Delta V_w = nVC_v\,\Delta u.$$

The decrease in volume of soil skeleton (17)
is almost entirely due to decrease in volume of voids.

$$nVC_v\Delta u = VC_c\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right].$$

$$\Delta u = \frac{VC_c}{nVC_v}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right]$$

$$nVC_v\Delta u + C_c\Delta u = \frac{VC_c}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)$$

$$\Delta u(nVC_v + VC_c) = \frac{VC_c}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)$$

$$\Delta u = \frac{VC_c}{(nVC_v + VC_c)}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\right]$$

$$= \frac{1}{1 + n\cancel{V}\frac{C_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\right]$$

$$\boxed{\Delta u = \frac{1}{1 + \frac{nC_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\right].}$$

In a conventional triaxial test,

$$\Delta\sigma_2 = \Delta\sigma_3$$

$$\Delta u = \frac{1}{1 + \frac{nC_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + 2\Delta\sigma_3)\right].$$

$$= \frac{1}{1 + \frac{nC_v}{C_c}}\left[\frac{1}{3}[\Delta\sigma_1 + 3\Delta\sigma_3 - \Delta\sigma_3]\right]$$

$$\boxed{\Delta u = \frac{1}{1 + \frac{nC_v}{C_c}}\left[\Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)\right]} \leftarrow (1)$$

The equation can be written in the form

$$\boxed{\Delta u = B\left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\right]} \leftarrow (2)$$

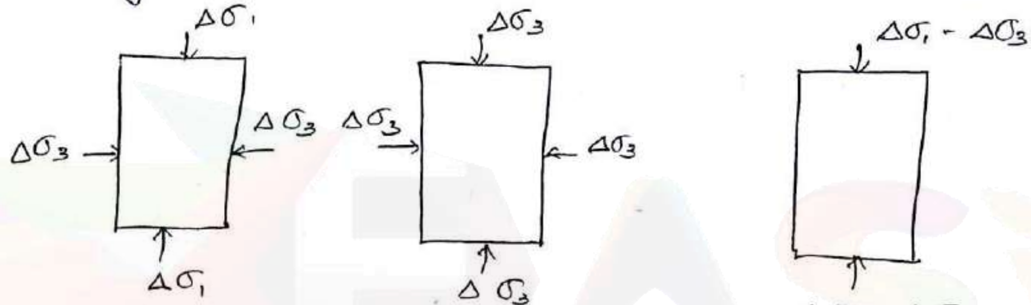
A & B - Skempton's pore pressure parameters

These paramaters are to be determined experimentally.

In an undrained triaxial test, stress changes are usually made in two stages.

i) an increase in cell pressure $\Delta\sigma_3$ resulting in an all round change in stress.

ii) an increase in axial load resulting in change in deviator stress $\Delta\sigma_d = (\Delta\sigma_1 - \sigma_3)$



$(u) \quad = \quad (\Delta u_1) \quad + \quad (\Delta u_2).$

Let

Δu_1 - change in pore pressure during first stage of test when cell pressure is applied.

Δu_2 - change in pore pressure when deviator stress is applied.

$$\Delta u = \Delta u_1 + \Delta u_2 \quad \text{--- } (3).$$

Comparing (2) & (3).

$$\Delta u_1 + \Delta u_2 = B\,\Delta\sigma_3 + AB\,(\Delta\sigma_1 - \Delta\sigma_3)$$

$\Delta u_1 = B\,\Delta\sigma_3$ $\qquad$ $\Delta u_2 = AB(\Delta\sigma_1 - \Delta\sigma_3)$

$= \bar{A}\,(\Delta\sigma_1 - \Delta\sigma_3)$

$$\boxed{B = \frac{\Delta u_1}{\Delta\sigma_3}} \qquad \boxed{\bar{A} = \frac{\Delta u_2}{(\Delta\sigma_1 - \Delta\sigma_3)}}$$

From equation (1) & (2), $\quad B = \dfrac{1}{1 + \dfrac{n\,C_v}{C_c}}$

Factors affecting A & B

Fully Saturated Soil - $C_v <<< C_c$

$\frac{C_v}{C_c} = 1 \quad \therefore B = 1.$

Perfectly dry soil - $\frac{C_v}{C_c} = \infty, \quad \therefore B = 0.$

Partially saturated soils = $0 < B < 1.$

Proctor's optimum water content & density,
$B = 0.1$ to 0.5

The coefficient A varies with stresses & strains. It depends on whether total stresses are increasing or decreasing.

Preconsolidation reduces A.

Other factors affecting A - Type of shear
Sample disturbance
Environment (Temp & nature of fluid)

Determination of parameters A & B:

B is determined in lab by measuring change in pore pressure Δu_1 due to change in cell pressure $\Delta \sigma_3$, in first part of test.

$$\boxed{B = \frac{\Delta u_1}{\Delta \sigma_3}}.$$

$\bar{A}$ is measured during second stage of test when deviator stress $(\Delta\sigma_1 - \Delta\sigma_3)$ is applied at constant cell pressure, when Δu_2 is measured.

$$\bar{A} = A \cdot B = \frac{\Delta u_2}{\Delta \sigma_1 - \Delta \sigma_3}.$$

$$\boxed{A = \frac{\Delta u - \Delta \sigma_3}{\Delta \sigma_1 - \Delta \sigma_3}}$$

For usual undrained triaxial test, $\Delta \sigma_3 = 0$, when deviator stress is applied.

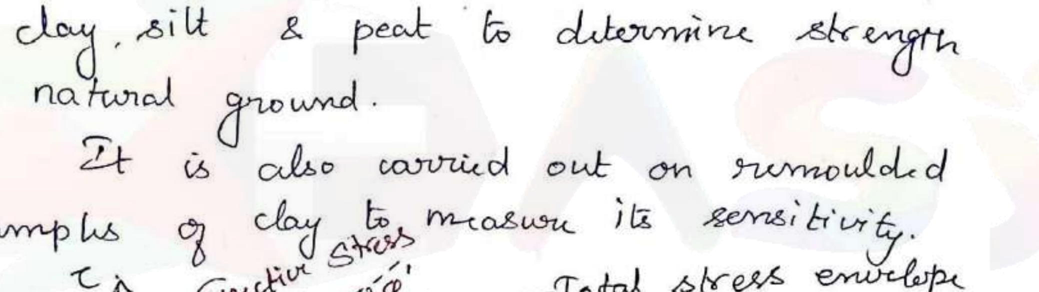
$$\boxed{A = \frac{\Delta u}{\Delta \sigma_1}}$$

SHEAR STRENGTH OF COHESIVE SOILS:

a) Undrained test on saturated cohesive soils:

It is carried on undisturbed sample of clay, silt & peat to determine strength of natural ground.

It is also carried out on remoulded samples of clay to measure its sensitivity.



For 1st circle,

Diameter $= (\sigma_1)_1 - (\sigma_3)_1$

u - Pore pressure measured @ failure.

Diameter of effective stress circle $= (\sigma_1')_1 - (\sigma_3')_1$

$$= ((\sigma_1)_1 - u) - ((\sigma_3)_1 - u)$$

$$= (\sigma_1)_1 - (\sigma_3)_1$$

Diameter of effective stress circle $\}$ = Diameter of total stress circle.

Major effective principal stress doesnot change

B = 1, both Major principal Effective stress σ_1' (19)
& minor principal Effective stress σ_3' are independent of magnitude of cell pressure applied.

$\therefore$ We get only one Mohr circle in terms of effective stress, for all identical specimens tested under increased pressure.

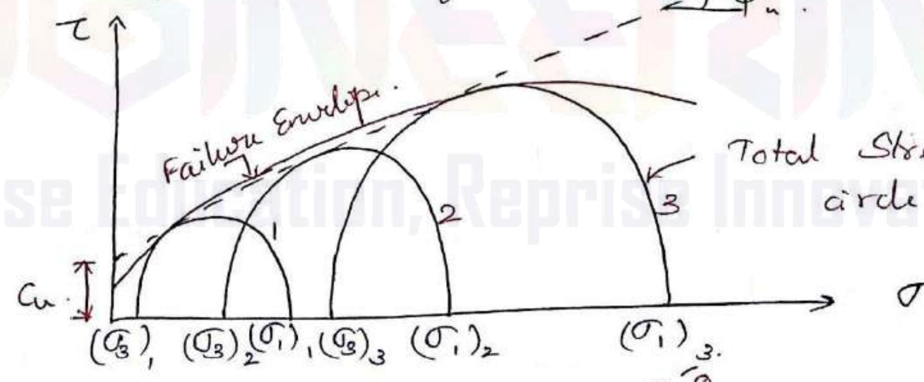
$\boxed{\varphi_u = 0}$
$\boxed{C_u = d}$

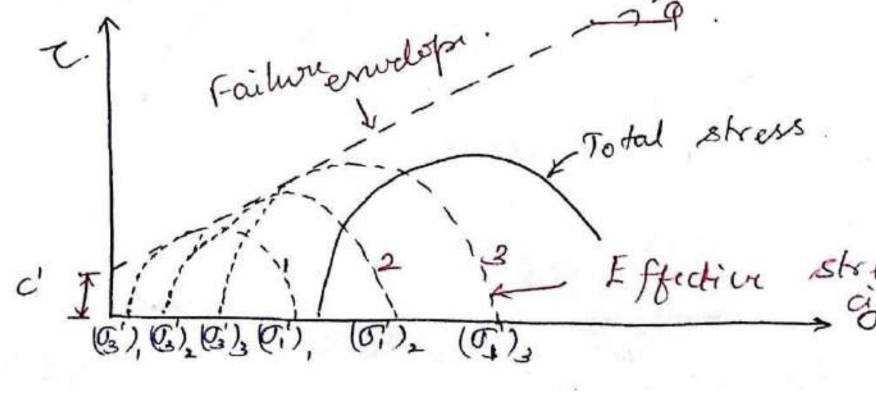
Deviator stress $\sigma_d = \sigma_1' - \sigma_3'$

Effective stress envelope cannot be obtained from this test.

b) Undrained test on partly saturated cohesive soil:

In case of Earth Embankments, which are compacted at optimum water content, the soil remains partly saturated and it is necessary to conduct undrained test to determine Shear parameters of soil.





As all pressure increased, deviator stress @ failure also increases, though this increase in deviator stress becomes smaller as the air in soil voids is compressed & dissolved.

The increase in deviator stress later ceases when large stresses cause full saturation.

Due to this, failure envelope in terms of total stress is non-linear.

Failure envelope in terms of effective stress is very closely a straight line over wide range of pressure.

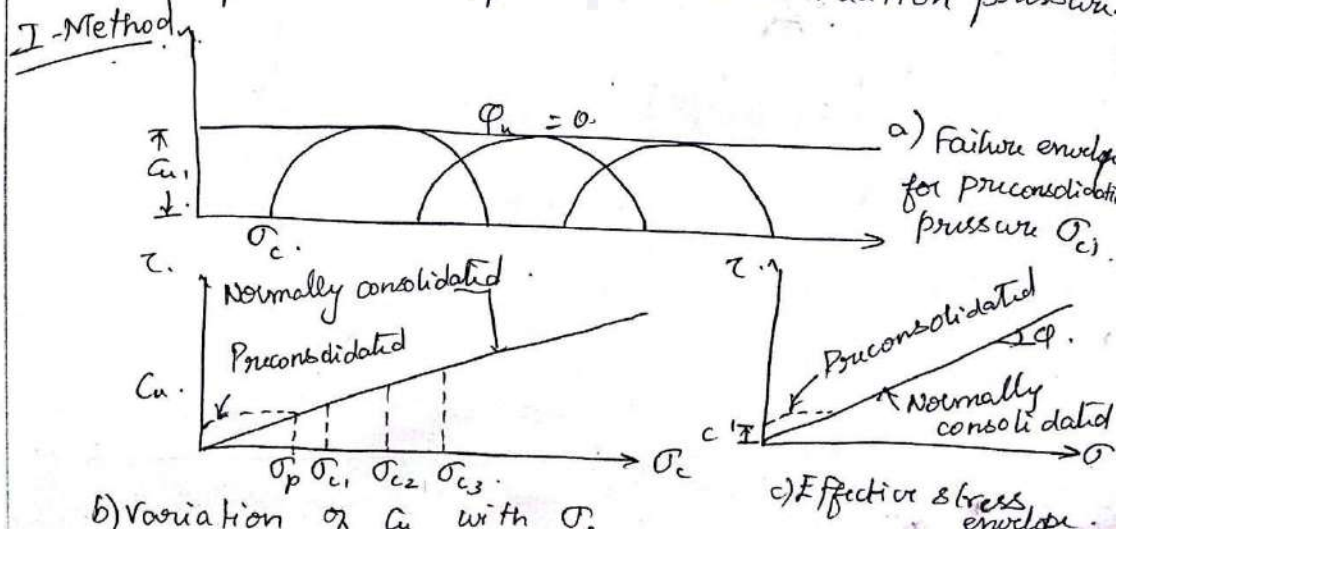
c) Consolidated undrained test on saturated cohesive soils:

Performed by two methods.

i) Moulded specimens are first consolidated under same cell pressure & then sheared under undrained conditions with different cell pressure by increasing axial stress.

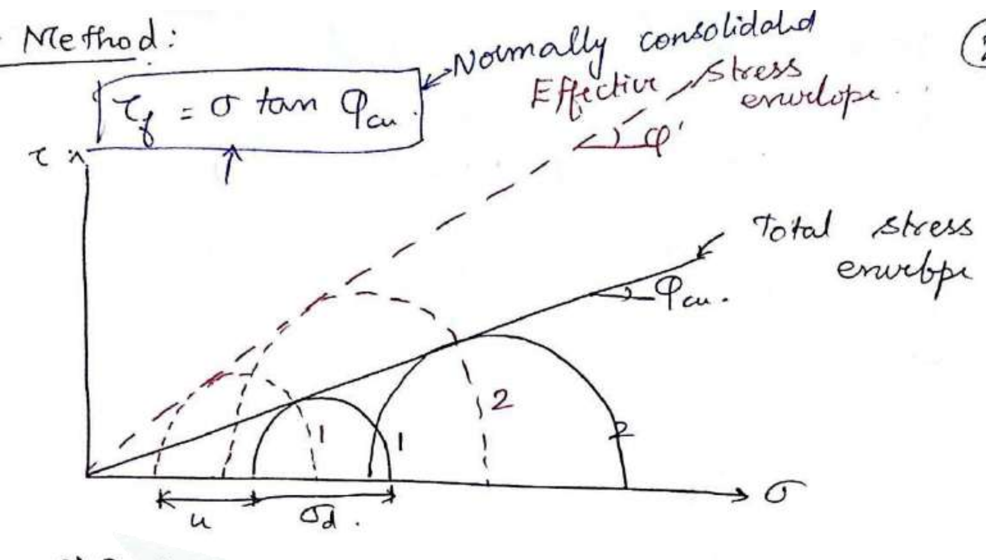
ii) Remoulded specimens are sheared under a cell pressure equal to consolidation pressure.

I-Method.

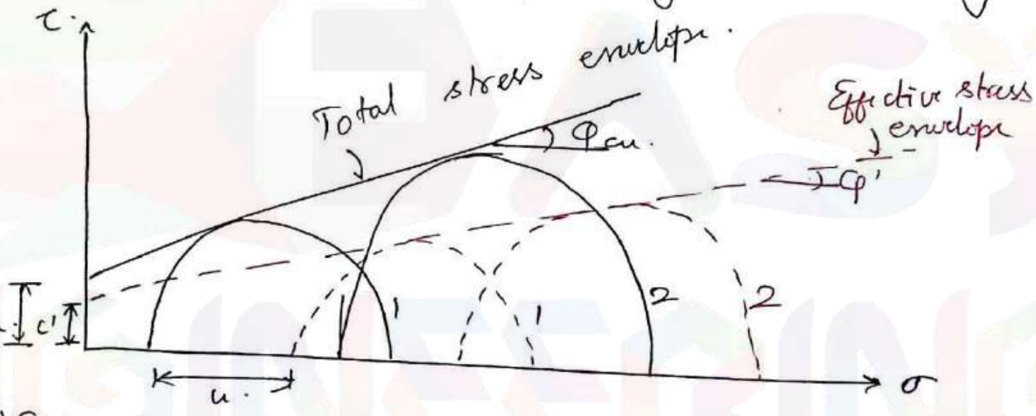


II - Method:

$$\boxed{\tau_f = \sigma \tan \varphi_{cu}}$$



a) Failure envelope for normally consolidated clay.



b) Failure envelopes for preconsolidated clay.

$$\boxed{\tau_f = C_{cu} + \sigma \tan \varphi_{cu}}$$ ← Preconsolidated.

d) Consolidated undrained test on partly saturated cohesive soils:

– To examine the effect on c' & φ' of flooding foundation strata & earth fill materials, by applying back pressure to pore space to ensure full saturation.

Shear strength - independent of change in cell pressure.

Incomplete saturation will mean that unique value of C_{cu} & φ_{cu} will only be obtained if no change in cell pressure is made after consolidation stage, before sample is sheared.

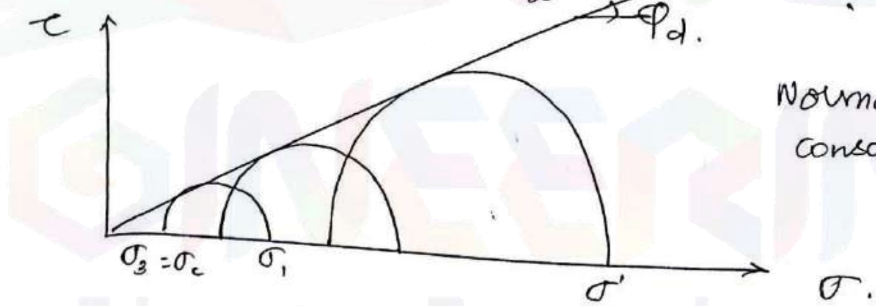
C' & φ' are determined by measuring pore pressure & getting value of effective stress @ failure.

e) Drained tests :

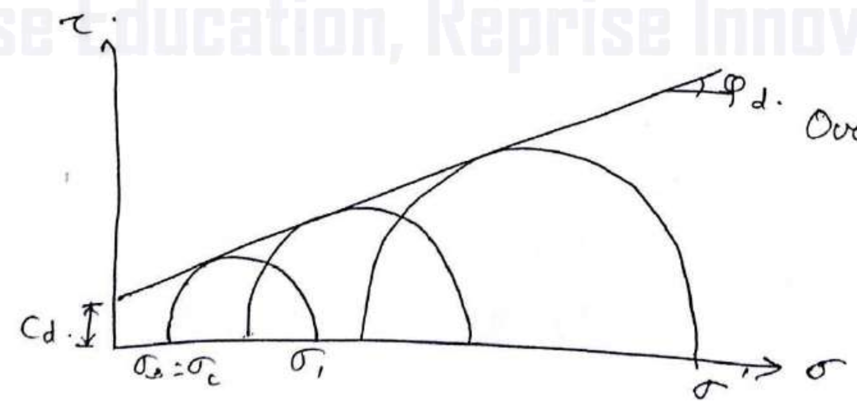
Specimen is first consolidated under cell pressure ($\sigma_c = \sigma_3$) & is then sheared slowly so that pore pressure developed during shearing is dissipated.

$\sigma_3' = \sigma_c$ $\quad \sigma_c'$ - Axial stress.

$u = 0$, Total stress = Effective stress.



Normally consolidated clay.



Overconsolidated clay.

LIQUEFACTION:

(21)

* It is a phenomenon in which loose saturated sand loses a large percentage of its shear strength & develops characteristics similar to those of a liquid.

* It is usually induced by cyclic loading of relatively high frequency, resulting in undrained conditions in sand. Cyclic loading may be caused by vibrations from machinery & by earth tremors.

* Loose sand tends to compact under cyclic loading. The decrease in volume causes an increase in pore water pressure which cannot dissipate under undrained conditions.

* Indeed, there may be cumulative increase in pore water pressure under successive cycles of loading. If pore water pressure = Maximum total stress component, normally overburden pressure, value of effective stress will be zero. & sand will exist in a liquid state with negligible shear strength.

* Even if effective stress doesnot fall to zero, the reduction in shear strength may be sufficient to cause failure.

* Liquefaction may develop at any depth in sand deposit where a critical combination of insitu density and cyclic deformation occurs.

- Higher void ratio, Lower confining pressure - Liquefaction
- Larger strains produced by cyclic loading, Lower no. of cycles required for liquefaction

Initiation of liquefaction

The fact that soil deposit is susceptible to liquefaction does not mean than liquefaction will necessarily occur in given Earthquake. It's occurrence requires a disturbance that is strong enough to initiate or trigger it. Evaluation of nature of that disturbance is one of the most critical parts of liquefaction hazard evaluation.

Cyclic mobility is an earthquake related phenomena, flow liquefaction can be initiated in variety of ways.

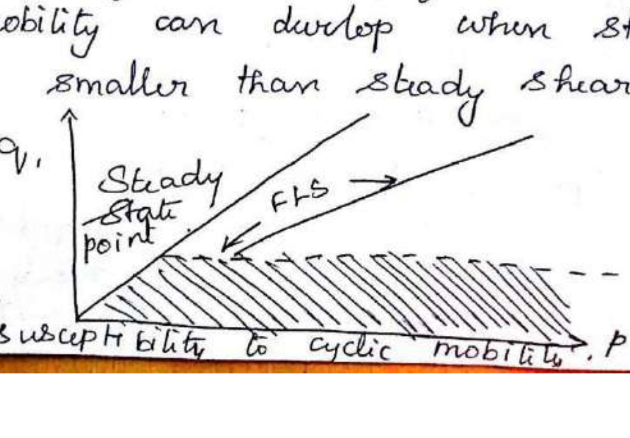
Static liquefaction - Monotonic loading.

Flow liquefaction Surface:

The effective stress conditions at the ~~intimation~~ initiation of flow liquefaction can be described in stress path space by a 3D surface that will be referred to as flow liquefaction surface (FLS)

CYCLIC MOBILITY:

Although flow liquefaction cannot occur, cyclic mobility can develop when static shear stress is smaller than steady shear strengths.

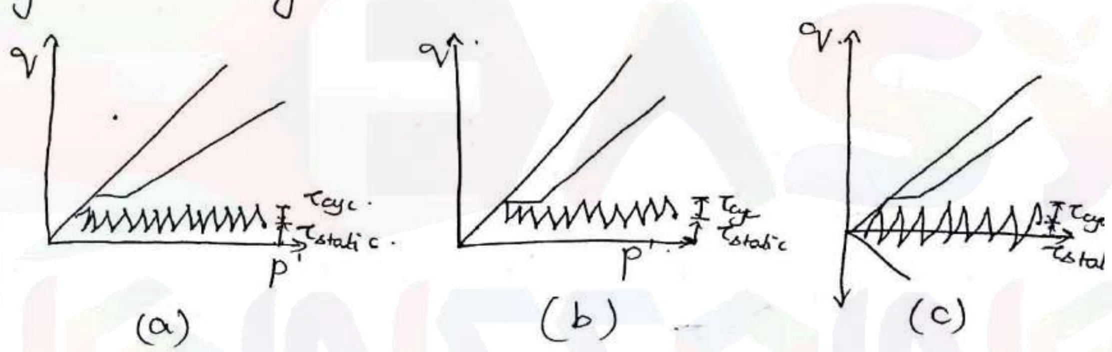


Zone of susceptibility to cyclic mobility.

If initial conditions plot within shaded zone, cyclic mobility can occur.

Cyclic mobility can occur in both loose & dense soils [the shaded region extends from very low to very high effective confining pressures and corresponds to states that would plot both above & below SSL].

Three combinations of initial conditions & cyclic loading conditions generally produce cyclic mobility.



a) No stress reversal and no exceedance of steady state strength.

b) No stress reversal with momentary periods of steady state strength exceedance.

c) Stress reversal with ~~no~~ exceedance of steady state strength.

First – when $\tau_{static} - \tau_{cyc} > 0$ [No stress reversal] & $\tau_{static} + \tau_{cyc} > S_{su}$ (Steady state strength is exceeded momentarily).

Again cyclic loading will cause effective stress path to move to left, when it touches FLS momentary periods of instability will occur.

Significant permanent strain may develop during these periods, particularly τ_{static} is greater than quasi-static shear strength, but straining will generally cease at end of cyclic loading when shear stress returns to τ_{static}.

Final condition: $\tau_{static} - \tau_{cyc} > 0$ (Stress reversal occurs).

$\tau_{static} + \tau_{cyc} > S_{su}$ (Steady state strength is not exceeded).

In this case, the direction of shear stress changes so that each cycle includes both compressional & extensional loading.

Effective stress path moves relatively quickly to left [because excess pore pressure builds up quickly - Rate of pore pressure generation increases with increasing degree of stress reversal] and eventually oscillates along compression & extension portions of drained failure envelope.

Each time, effective stress path passes through origin, the specimen is in an instantaneous state of zero effective stress.

State of zero effective stress - Initial liquefaction - No shear strength.

If monotonic loading is applied at state of initial liquefaction, specimen will dilate until steady state strength is mobilized. Significant permanent strains may accumulate during cyclic loading, but flow failure cannot occur.

Note that initial liquefaction can only occur when stress reversals occur. (23)

Incontrast to flow liquefaction, there is no clear cut point at which cyclic mobility is initiated.

Permanent strains & permanent deformation, they produce accumulate incrementally. Their magnitude depends on static shear stress & duration at nearly level sites, permanent deformations may be small.

For moderately sloping sites or gently sloping sites subjected to ground motions of long duration, cyclic mobility can produce damaging levels of soil deformation.

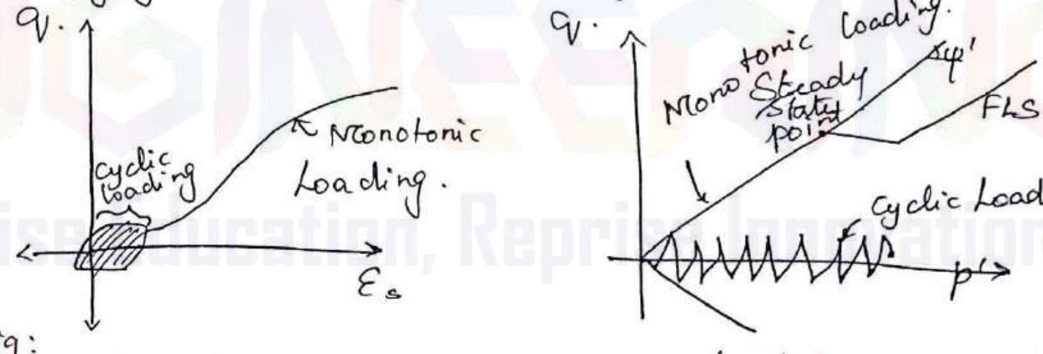


Fig: Dilative behavior of specimen loaded monotonically after occurrence of cyclic mobility. Cyclic loading with stress reversal causes effective confining pressure to decrease rapidly, eventually reaching momentary values of zero. Subsequent monotonic loading, causes dilation as steady state strength is mobilized.