Prove the following Boolean identies

i). $a + (a' \cdot b) = a + b$

ii) $a \cdot (a' + b) = a \cdot b$

iii) $(a \cdot b) + (a \cdot b') = a$

Proof:

i). $a + (a' \cdot b) = (a + a') \cdot (a + b)$
$= 1 \cdot (a + b)$
$= a + b$

ii). $a \cdot (a' + b) = (a \cdot a') + (a \cdot b)$
$= 0 + (a \cdot b)$
$= a \cdot b$

iii). $(a \cdot b) + (a \cdot b') = a \cdot (b + b')$
$= a \cdot (1)$
$= a$

Simplify $a' \cdot b' \cdot c + a \cdot b' \cdot c + a' \cdot b' \cdot c'$

Soln.

$a' \cdot b' \cdot c + a \cdot b' \cdot c + a' \cdot b' \cdot c'$
$= a' \cdot b' \cdot c + a' \cdot b' \cdot c' + a \cdot b' \cdot c$
$= a' \cdot b' \cdot (c + c') + a b' \cdot c$
$= a' \cdot b' \cdot (1) + a \cdot b' \cdot c$
$= b' \cdot (a' + (a \cdot c))$
$= b' \cdot ((a' + a) \cdot (a' + c))$
$= b' \cdot [1 \cdot (a' + c)]$
$= b' (a' + c)$

Hw In any BA, ST
$(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$

Atom:

Let $(B, \wedge, \vee, 0, 1)$ be a BA.

A non zero elt. $a \in B$ is called an atom if it is an immediate successor of zero elt.

i.e., $0 \leq b \leq a \Rightarrow b = 0$ or $b = a$.

Stone's Theorem:

Let B be a finite BA and A be set of all atoms of B. The B.A B is isomorphic to the BA $P(A)$, where $P(A)$ is the power set of A.

Corollary:

Every finite B.A. $(B, \wedge, \vee, 0, 1)$ has 2^n elts. for some +ve integer n.

2 element Boolean Algebra:

+	0	1
0	0	1
1	1	1

·	0	1
0	0	0
1	0	1

x	$\bar{x}$
0	1
1	0

$1+1=1$ $\quad 1\cdot 1=1$

$1+0=1$ $\quad 1\cdot 0=0$

$0+1=1$ $\quad 0\cdot 1=0$

$0+0=0$ $\quad 0\cdot 0=0$

$a+1=1$ $\quad a+a=a$

$a\cdot 0=0$ $\quad a\cdot a=a$

1\. prove that $a+ab=a$

Sol:

LHS $a+ab=a(1+b)$ distributive law

$=a(1)$

$a+ab=a$

2\. $a+\bar{a}b=a+b$

Sol:

LHS, $a+\bar{a}b=a+b$

$a+\bar{a}b=a+ab+\bar{a}b$ from $a=a+ab$

$=a+b(a+\bar{a})$

$=a+b(1)$

$a+\bar{a}b=a+b$

3 $(a+b)(a+c)=a+bc$

Sol:

LHS $(a+b)(a+c)=aa+ac+ba+bc$

$=a+ac+ba+bc$

$=a(1+c)+ba+bc$

$=a(1)+ba+bc$ from

$=a+ba+bc$ (or) $a+b$

$=a(1+b)+bc$

$=a(1)+bc = a+bc$

1. In any Boolean Algebra, show that
$(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$

Sol:

LHS

$(a+b')(b+c')(c+a')$

$= (ab+ac'+b'b+b'c')(c+a')$

$= (abc+aba'+acc'+ac'a'+b'bc+b'ba'+b'c'c+b'c'a')$

$= abc+0+0+0+0+0+0+b'c'a'$ $[\because aa'=0, bb'=0, cc'=0]$

$= abc+a'b'c'$

RHS.

$(a'+b)(b'+c)(c'+a)$

$= (a'b'+a'c+bb'+bc)(c'+a)$

$= a'b'c'+a'b'a+a'cc'+a'ca+bb'c'+bb'a+bcc'+bca.$

$= a'b'c'+0+0+0+0+0+0+bca$

$= abc+a'b'c'$

$(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$

In a Boolean Algebra, prove that

(i) $a \cdot a = a$ and $a+a=a$

(ii) $a \cdot 0 = 0$ and $a+1=1$

Sol: (i) $a \cdot a = a$

Now

$a = a \cdot 1$

$= a \cdot (a + a')$

$= (a \cdot a) + (a \cdot a')$

$= a \cdot a + 0$

$a = a \cdot a$

$\boxed{a \cdot a = a}$

Take dual on both sides

$\boxed{a + a = a}$

(ii) $a \cdot 0 = (a \cdot 0) + 0$

$= (a \cdot 0) + (a \cdot a')$

$= a \cdot (0 + a')$

$= a \cdot a'$

$= 0$

$\boxed{a \cdot 0 = 0}$

Take dual on bothsides

$\boxed{a + 1 = 1}$

Evaluate the expression $x = a \cdot [\overline{(b+c)} + \overline{d}]$ for $a = 0,\ b = 0,\ c = 1\ \&\ d = 1$.

Sol:

$x = 0 \cdot [\overline{(0+1)} + \overline{1}]$

$= 0 \cdot [\overline{1} + 0]$

$= 0 \cdot \overline{1}$

$= 0 \cdot 0$

$\boxed{x = 0}$

Reduce the expression

(i) $a \cdot \bar{a} b$

$a \cdot \bar{a} b = 0$ $\quad \therefore a \cdot \bar{a} = 0$

(ii) $a(a+c)$

$a(a+c) = aa + ac$
$= a + ac$
$= a(1+c) = a(1) = a.$

(iii) $z(y+z)(x+y+z)$

$z(y+z)(x+y+z) = (zy + zz)(x+y+z)$
$= (yz + z)(x+y+z)$
$= z(y+1)(x+y+z)$
$= z.(1)(x+y+z)$
$= z(x+y+z)$
$= zx + zy + zz$
$= zx + zy + z$ $\quad x + y = x$
$= z(x+y+1)$
$= z(1)$
$= z$

$\boxed{z(y+z)(x+y+z) = z.}$

Absorption law in Boolean Algebra.

Statement:

If a and b are two elements of boolean algebra, prove that

$a + (a \cdot b) = a$
$a \cdot (a+b) = a$

proof:

Now

$$a+(a\cdot b) = (a\cdot 1)+(a\cdot b)$$
$$= a(1+b)$$
$$= a\cdot 1$$
$$= a$$

and

$$a\cdot(a+b) = (a+a)\cdot(a+b)$$
$$= a.a + a\cdot b + a\cdot a + a\cdot b$$
$$= a + a\cdot b + a + a\cdot b$$
$$= a(1+b) + a(1+b)$$
$$= a(1) + a(1)$$
$$= a+a$$
$$= a.$$

Sub boolean Algebra

Let $(B, \wedge, \vee, -, 0, 1)$ be a boolean algebra and $S \subseteq B$. If S contains the elements 0 and 1 and it is closed under the operations $\wedge$, $\vee$ and $-$, then $(S, \wedge, \vee, -, 0, 1)$ is called sub boolean algebra.

Prove that D_{110}, the set of all +ve divisors of +ve integer 110, is a boolean algebra and find all sub algebras.

Sol:

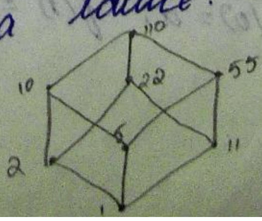
$$D_{110} = \{1, 2, 5, 10, 11, 22, 55, 110\}$$

Since D satisfies reflexive, antisymmetric, Transitive property, D is the partial order relation on D_{110} (D_{110}, D) it is a poset.

Here $a \wedge b = GLB(a, b)$ $\forall a, b \in$

$a \vee b = LUB(a, b)$

$(D_{110}, \wedge, \vee)$ is a lattice. Its hasse diagram is



Here least element (0 element) is 1

Greatest element (1 element) is 110

$LCM(1,110) = 110$
$GCD(1,110) = 1$
$1' = 110$ $1' = 1$?
$2' = 55$ $11' = 10$
$5' = 22$

Here each and every element has a complement

$\therefore$ It is complemented lattice.

from the hasse diagram It is clear that,

It is a distributive lattice

(D_{110}, D) is a boolean Algebra.

The subboolean Algebra's are

1. $\{0, 1\} = \{1, 110\}$

2. $\{1, 2, 5, 10, 11, 22, 55, 110\}$

3. $\{a, a', 0, 1\}$, $\forall a \in S$.