SNS COLLEGE OF TECHNOLOGY
Coimbatore

TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT – I

FOURIER SERIES

PART A

1. Pick out the even function x^2, $\sin x$, $1+x$, e^x

   a. x^2 b. x^2, $\sin x$

   c. e^x d. x^2, e^x **Ans : a**

2. The period of tan x is

   a. $\pi/2$ b. π

   c. $3\pi/2$ d. 2π **Ans : b**

3. The value of a_0 in the Fourier series of f(x) = x in (0, 2π) is

   a. 0 b. 2π

   c. 4π d. π **Ans : b**

4. The value of b_n in the expansion of x^2 as a Fourier series in (-π, π) is

   a. $2\pi^3/3$ b. $\pi^3/3$

   c. $3\pi^2/2$ d. 0 **Ans : d**

5. The value of a_n in the Fourier series of f(x) in (0, l) is

   a. $\frac{1}{l}\int_0^l f(x)\cos nx\,dx$ b. $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

   c. $\frac{2}{l}\int_0^l f(x)\cos nx\,dx$ d. $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ **Ans : d**

6. The value of a_0 in the Fourier series of f(x) in (0, l) is

   a. $\frac{2}{l}\int_0^l f(x)\cos nx\,dx$ b. $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

   c. $\frac{2}{l}\int_0^l f(x)dx$ d. $\frac{2}{l}\int_0^l f(x)\sin nx\,dx$ **Ans : c**

7. The root mean square value of f(x) = x in (0, l) is

   a. $l/3$ b. $l/\sqrt{3}$

c. $\sqrt{l/3}$ d. $\sqrt{l}/3$ **Ans : b**

8. The value of the constant term in the Fourier series corresponding to $f(x) = x - x^3$ in (-π, π) is

a. $\pi - \pi^3$ b. π

c. π^3 d. 0 **Ans : d**

9. The value of the constant a_0 in the Fourier series of $(\pi - x)^2/4,\ 0 < x < 2\pi$ is

a. $2\pi^2/3$ b. $\pi^2/3$

c. $\pi^2/6$ d. π^2 **Ans : c**

10. The Fourier series of $f(x) = (\pi - x)^2/4,\ 0 < x < 2\pi$ converges to ________ at x = 0

a. $2\pi^2/3$ b. $\pi^2/3$

c. $\pi^2/4$ d. π^2 **Ans : c**

11. The value of the a_n in the Fourier expansion of K, on (0, 10) is

a. 10 b. 2K/nπ

c. Π d. 0 **Ans : d**

12. Parseval's Identity of f(x) with period 2*l* in (*-l*, *l*)

a. $\frac{1}{2l}\int_{-l}^{l}[f(x)]^2 dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{1}^{\infty}\left[a_n^2 + b_n^2\right]$

b. $\frac{1}{2l}\int_{-l}^{l}[f(x)]^2 dx = \frac{a_0^2}{4} + \sum_{1}^{\infty}\left[a_n^2 + b_n^2\right]$

c. $\frac{1}{l}\int_{-l}^{l}[f(x)]^2 dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{1}^{\infty}\left[a_n^2 + b_n^2\right]$

d. $\int_{-l}^{l}[f(x)]^2 dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{1}^{\infty}\left[a_n^2 + b_n^2\right]$ **Ans : a**

13. If a periodic function f(x) is even in *(-l, l)*, then a_0 is

a. $\frac{1}{l}\int_{0}^{l} f(x)\ dx$ b. $\frac{2}{l}\int_{0}^{l} f(x)\ dx$

c. $\frac{2}{l}\int_{-l}^{l} f(x)\ dx$ d. 0 **Ans : b**

14. The function $f(x) = \frac{1}{1-x}$ is

a. Continuous at x = 1 b. Discontinuous at x = 1

c. Continuous for all x d. Discontinuous at x = 0 **Ans : b**

15. The value of the b_n in the Fourier expansion of $f(x) = |\sin x|$, in (-π, π) is

a. 0 b. 2π

c. π d. 4π **Ans : a**

16. The value of the b_n in the Fourier expansion of $f(x) = x\cos x$, in $(-\pi, \pi)$ is

a. 0 b. $2\pi/3$

c. $4\pi/3$ d. 4π **Ans : a**

17. e value of the constant a_0 in the Fourier expansion of $f(x) = x - x^2$, in $(-\pi, \pi)$ is

a. $\pi - \pi^3$ b. $\pi^2/2 - \pi^4/4$

c. $\pi/2 - \pi^4/4$ d. 0 **Ans : d**

18. The value of the Fourier series of $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$ at x = 0 is

a. 2π b. 0

c. $\sqrt{2}$ d. 2 **Ans : b**

19. The value of the constant a_0 in the Fourier expansion of $f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$

a. π b. 2π

c. 3π d. 4π **Ans : a**

20. The value of the b_n in the Fourier expansion of $f(x) = |x|$, in $(-\pi, \pi)$ is

a. π b. 2π

c. 3π d. 0 **Ans : d**

21. To which value, the Fourier series of $f(x) = \begin{cases} l - x, & 0 < x < l \\ 0, & l < x < 2l \end{cases}$ converges at x = 0

a. l b. $l/3$

c. $l/2$ d. $2l$ **Ans : a**

22. The value of the constant a_n in the Fourier expansion of $f(x) = e^x$, $0 < x < \pi$ is

a. 2π b. $2/\pi$

c. $1/\pi$ d. 0 **Ans : d**

23. The value of the constant a_n in the Fourier expansion of $f(x) = \cos x$, $0 < x < \pi$ is

a. 2π b. $2/\pi$

c. $1/\pi$ d. 0 **Ans : d**

24. The root mean square value of $f(x) = x^2$, $-\pi < x < \pi$ is

a. $\pi^4/5$ b. $\pi^3/5$

c. $\pi^2/5$ d. $\pi/5$ **Ans : a**

25. The root mean square value of unity in $(0, \pi)$ is

a. $1/2$ b. $1/3$

c. 0 d. 1 **Ans : d**

26. The value of the b_n in the Fourier expansion of unity in $(0, \pi)$ for even values of "n" is

a. 2π b. $2/\pi$

c. $1/\pi$ d. 0 **Ans : d**

27. In the Harmonic analysis the value of a_0 is

a. $\frac{2\sum y}{n}$ b. $\sum y$

c. $2\sum y$ d. $\frac{\sum y}{n}$ **Ans : a**

28. In the Harmonic analysis the value of a_n is

a. $\frac{2\sum y}{n}$ b. $\frac{2\sum y \cos nx}{n}$

c. $\frac{2\sum y \sin nx}{n}$ d. 0 **Ans : b**

29. In the Harmonic analysis the value of b_n is

a. $\frac{2\sum y}{n}$ b. $\frac{2\sum y \cos nx}{n}$

c. $\frac{2\sum y \sin nx}{n}$ d. 0 **Ans : c**

30. If x_0 is a continuous point, then the Fourier series of f(x) converges to

a. $f(x)$ b. $f(x_0)$

c. $f(0)$ d. 0 **Ans : b**

PART B
TWO MARKS

1. Explain periodic function with two examples.

Solution: A function f(x) is said to have a period T if for all x, $f(x+T) = f(x)$,

Where T is a positive constant. The least value of $T > 0$ *is called the period of* $f(x)$.

For examples, $f(x) = \text{Sin}x$

$f(x+2\pi) = \text{Sin}(x+2\pi) = \text{Sin}x$

Here, $f(x) = f(x+2\pi)$

2. State Dirichlet's condition for a given function to expend in Fourier series.

Solution: Any function f(x) can be developed as a Fourier series, provided

i) f(x) is periodic, single valued & finite.
ii) f(x) has a finite number of discontinuities in any one period
iii) f(x) has a finite number of maxima and minima

3. State general Fourier series.

solution: The Fourier series of $f(x)$ in $c \le x \le c + 2l$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

Where a_0, an &b_n are called Fourier coefficients(or) Euler constants

4. Find the coefficient of b_n of $\cos 5x$ in the Fourier cosine series of the function

$f(x) = \sin 5x$ in the in the interval $(0, \pi)$.

Solution: The Fourier Cosine series is

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx = \frac{2}{\pi}\int_0^{\pi} \cos 5x \cos nx dx$$

$$= \frac{2}{\pi}\int_0^{\pi} [\cos(5+n)x + \cos(5-n)x]d$$

$$= \frac{2}{\pi}\left[\frac{\sin(5+n)x}{5+n} + \frac{\sin(5-n)x}{5-n}\right]_0^{\pi} = 0; \text{Therefore}, b_n = 0$$

5. Find the constant a_0 of the Fourier series for the function of $f(x) = x$ in $0 \le x \le 2\pi$

Solution:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx = \frac{1}{\pi}\int_0^{2\pi} xdx = \frac{1}{\pi}\left(\frac{x^2}{2}\right)_0^{2\pi} = 2\pi$$

6. Obtain the first term of the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi}\int_{0}^{\pi} x^2 dx = \frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^{\pi} = \frac{2}{3}\pi^2$$

7. If $f(x) = x^3, -\pi < x < \pi$. *Find the constant term of its Fourier series.*

Solution:

$f(x) = x^3$, is an odd function. Therefore, the fourier constants $a_0 = 0$

8. Find the constant term in the Fourier series expansion of $f(x) = x$ in $(-\pi, \pi)$

Solution:

$a_0 = 0$ since $f(x)$ is an odd function $(-\pi, \pi)$

9. Find the sum of the Fourier series of $f(x) = x + x^2$ in $-\pi < x < \pi$ at $x = \pi$.

Solution:

Given $f(x) = x + x^2$

The sum of Fourier series is equal to the arithmetic mean of the value of $f(x)$ at $x = \pi$ and $x = -\pi$.

Sum of Fourier series $= \frac{f(\pi) + f(-\pi)}{2} = \frac{\pi + \pi^2 - \pi + \pi^2}{2} = \pi^2$

10. What is the constant term a_0 and the coefficient of $\cos nx$, a_n in the Fourier series of $f(x) = x - x^3$ in $(-\pi, \pi)$.

Solution:

$$f(x) = x - x^3 \Rightarrow f(-x) = -x + x^3$$

$$= -(x - x^3) = -f(x)$$

Therefore, f(x) is an odd function of x in (-π, π). Therefore, the Fourier series of f(x) Contains sine terms only. Therefore, $a_0 = 0$ and $a_n = 0$

11. Find the root mean square value of the function f(x)= x^2 in the interval (0,1).

Solution:

$$\text{RMS value} = \sqrt{\frac{1}{l}\int_0^1 x^2\,dx} = \sqrt{\frac{1}{l}\left(\frac{x^3}{3}\right)_0^1} = \sqrt{\frac{1}{l}\left[\frac{l^3}{3}\right]} = \frac{1}{\sqrt{3}}$$

22. Define root mean square value of a function f(x) in a < x < b.

Solution:

$$\text{R.M.S.value } \bar{y} = \sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2\,dx}$$

23 .State Parseval`s identity for full range expansion of f(x) as Fourier series in (0,2l).

Solution:

$$\frac{1}{l}\int_0^{2l} (f(x))^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) \text{ where } a_0, a_n \text{ and } b_n \text{ are}$$

Fourier coefficients in the expansion of $f(x)$ as a Fourier series .

25. What do you mean by Harmonic Analysis?

Solution:

The process of finding the Fourier series for a function given by numerical value is known as harmonic analysis. In harmonic analysis

$$a_0 = 2 \text{ (mean value of y in } (0,2\pi))$$

$$a_n = 2 \text{ (mean value of y Cos nx in } (0,2\pi))$$

$$b_n = 2\text{(mean value of y Sin nx in } (0,2\pi))$$