Subject Code & Name: **16AE309 Airplane Stability and Control** Date: 27.**01.20**

DAY: **18** TOPIC: ***STICK FORCE GRADIENT***

Stick Force Gradients in Unaccelerated Flight

The criterion $(dC_m/dC_L)_{\text{Free}}$ is important because of its basic influence on the variation of the stick force, F_s, required of the pilot to change the airplane's speed from a given trim speed, $F_s = 0$. For desirable flying qualities it is necessary that the pilot be required to apply pull forces on the stick for flight at speeds below the trim speed and push forces for flight at speeds above the trim speed, and the stick-free stability criterion $(dC_m/dC_L)_{\text{Free}}$ has a determining influence on the gradient of the stick force versus speed curve through trim. The theory of this will be explained in the remainder of this chapter.

The pilot's force at the top of the stick is determined by the hinge moment at the elevator and the gearing between the elevator and the pilot's control. A typical arrangement of stick and elevator is shown in Figure 6–17.

The gearing between the stick and the elevator may be obtained by equating the work done at the top of the stick to the work done at the elevator.

Work done at top of stick = work done at elevator

$$\frac{F_s \times l_s \times \delta_s}{2} = \frac{HM \times \delta_e}{2} \tag{6–37}$$

where l_s is length of the stick in feet, and δ_s and δ_e are the angular rotations of the stick and elevator, respectively, in radians. Equation (6–37) may be reduced as follows:

$$F_s = HM \frac{\delta_e}{l_s \delta_s} \tag{6–38}$$

The term $(\delta_e/l_s\delta_s)$ is called the elevator gearing, G, and is sometimes written as $d\delta_e/ds$, where $s = l_s\delta_s$, the linear movement of the top of the stick.

The stick force required of the pilot can be written as follows, considering a push force as positive:

$$F_s = -G \cdot HM \tag{6–39}$$

Equation (6–39) can be written in coefficient form

$$F_s = -GC_{h_e}S_e c_e q\eta_t \tag{6–40}$$

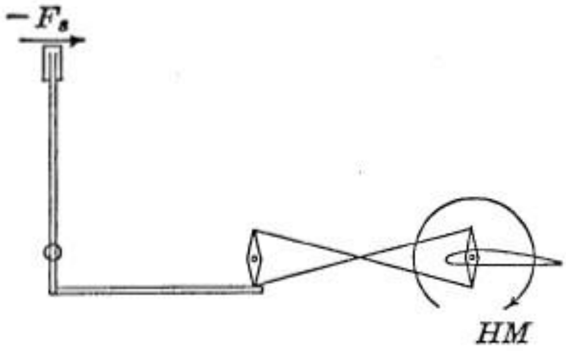

FIGURE 6–17. Elevator-stick gearing.

where the hinge moment coefficient can be written as

$$C_{h_e} = C_{h0} + C_{h\alpha}\alpha_s + C_{h\delta_e}\delta_e + C_{h\delta_t}\delta_t \tag{6–41}$$

Substituting (6–41) into (6–40),

$$F_s = -GS_e c_e q\eta_t(C_{h0} + C_{h\alpha}\alpha_s + C_{h\delta_e}\delta_e + C_{h\delta_t}\delta_t) \tag{6–42}$$

Now

$$\alpha_s = \alpha_w - \epsilon - i_w + i_t$$

or (6–43)

$$\alpha_s = \alpha_0 + \frac{C_L}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - i_w + i_t$$

The elevator angle, δ_e, can be expressed in terms of the elevator angle at zero lift, the stick-fixed stability parameter $(dC_m/dC_L)_{\text{Fix}}$, and the elevator effectiveness, $C_{m\delta}$, as given in equation (5–72).

$$\delta_e = \delta_{e0} - \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} \frac{C_L}{C_{m\delta}} \tag{6–44}$$

If the values of δ_e and α_s are substituted into equation (6–42), the final stick force equation results.

$$F_s = -GS_e c_e \frac{1}{2}\rho V^2 \eta_t \left[C_{h_0} + C_{h_\alpha}\left(\alpha_0 + \frac{C_L}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - i_w + i_t\right) + C_{h_\delta}\left(\delta_{e0} - \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} \frac{C_L}{C_{m_\delta}}\right) + C_{h_{\delta t}}\delta_t\right] \quad (6\text{–}45)$$

letting $K = -GS_e c_e \eta_t$

and $A = C_{h_0} + C_{h_\alpha}(\alpha_0 - i_w + i_t) + C_{h_\delta}\delta_{e0}$

Equation (6–45) becomes

$$F_s = K\frac{1}{2}\rho V^2 \left[A + \frac{C_{h_\alpha} C_L}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{C_{h_\delta}}{C_{m_\delta}} C_L \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} + C_{h_{\delta t}}\delta_t\right] \quad (6\text{–}46)$$

Rearranging

$$F_s = K\frac{1}{2}\rho V^2 \left[A + C_{h_{\delta t}}\delta_t - C_L\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \frac{C_{h_\delta}}{C_{m_\delta}}\right] \quad (6\text{–}47)$$

for unaccelerated flight, $C_L = \dfrac{2W/S}{\rho V^2}$, substitution into (6–47) gives

$$F_s = K\frac{1}{2}\rho V^2 (A + C_{h_{\delta t}}\delta_t) - K\frac{W}{S}\frac{C_{h_\delta}}{C_{m_\delta}}\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \quad (6\text{–}48)$$

Equation (6–48) brings out the interesting fact that the stick force variation with speed is dependent on the first term only and independent in general of the stability level. The slope of the stick force versus speed curve is simply

$$\frac{dF_s}{dV} = K\rho V(A + C_{h_{\delta t}}\delta_t) \quad (6\text{–}49)$$

A plot of elevator stick force, F_s, versus velocity is shown in Figure 6–18 and is made up of a constant force springing from the second or stability term of equation (6–48) plus a variable force proportional to the velocity squared, introduced through the constant A and the tab term $C_{h_{\delta t}}\delta_t$.

For a given center of gravity, then, a stable or negative value of the stability criterion (stick-free) will introduce a constant pull force, while an unstable value will introduce a push force. It can be seen from

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Figure 6–18 that an airplane possessing stick-free stability will require a nose-up tab setting to trim out the stick force ($F_s = 0$) for a given trim speed, and the resultant variation of stick force with air speed will be stable. If $(dC_m/dC_L)_{\text{Free}}$ is unstable, then in order to trim the airplane out at the given trim speed a nose-down tab is required, giving an unstable variation of stick force with air speed. In other words the tab creates the required slope, but the static stability criterion stick-free is essential to allow the tab to move in a stable direction for trim. It is important to notice again that a stable slope is of interest only if equilibrium or trim is established.

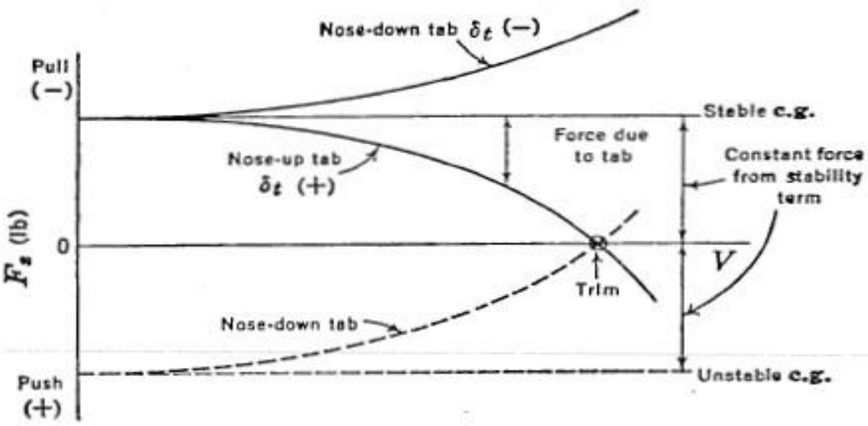

FIGURE 6–18. Stick force build-up.

From this discussion it can be seen that the stability criterion $(dC_m/dC_L)_{\text{Free}}$ plays an important but rather complex role in establishment of the flight condition of a stable stick force variation with speed.

It is interesting to note how explicitly $(dC_m/dC_L)_{\text{Free}}$ can be brought into the picture, if it is required that the trim tab always be deflected to trim the airplane out ($F_s = 0$) to a given speed (V_{Trim}).

The value of $C_{h\delta_t}\delta_t$ for this trim condition can be obtained from equation (6–48) by substituting V_{Trim} for V and equating F_s to zero.

$$C_{h\delta_t}\delta_t = \frac{2W/S}{\rho V_{\text{Trim}}^2} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} - A \qquad (6\text{–}50)$$

Substituting (6–50) into (6–48) gives

$$F_s = K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \left(\frac{V^2}{V_{\text{Trim}}^2} - 1\right) \qquad (6\text{–}51)$$

and

$$\frac{dF_s}{dV} = 2K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \frac{V}{V_{\text{Trim}}^2} \qquad (6\text{–}52)$$

The slope when $V = V_{\text{Trim}}$ will be

$$\frac{dF_s}{dV} = 2K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right) \frac{1}{V_{\text{Trim}}} \tag{6–53}$$

Equation (6–53) indicates that the slope F_s versus V varies with c.g position if the tab is rolled to maintain the trim speed (V_{Trim}), the slope becoming more stable as the c.g. is moved forward, and more

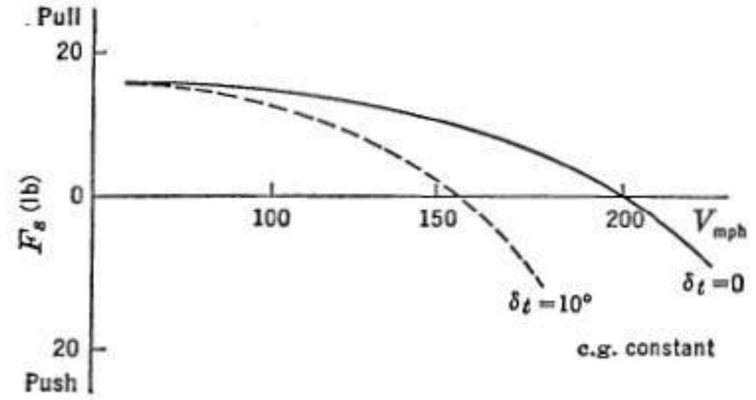

FIGURE 6–19. Stick force versus velocity for different tab angles.

unstable as the c.g. is moved aft. Equation (6–53) also shows that the slope dF_s/dV varies inversely with the trim speed, being higher at the lower speeds. See Figure 6–19.