The Stick-free Neutral Point

From the discussion of control surface hinge moments, the total hinge moment coefficient of the elevator may be given as:

$$C_{h_e} = C_{h_0} + C_{h_\alpha}\alpha_s + C_{h_{\delta_e}}\delta_e + C_{h_{\delta_t}}\delta_t \qquad (6\text{–}19)$$

from which the floating angle of the elevator can be obtained by equating C_{h_e} to zero and solving for δ_e. For the case of $C_{h_0} = 0$ and $\delta_t = 0$

$$\delta_{e_{\text{Float}}} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha_s \qquad (6\text{–}20)$$

where α_s is the angle of attack of the stabilizer.

In Chapter 5 the elevator angle required for equilibrium at each lift coefficient was obtained, and the derivative $d\delta_e/dC_L$ given as the criterion for stick-fixed longitudinal stability. Now, if the elevator floating angle at each lift coefficient just happened to be the same as

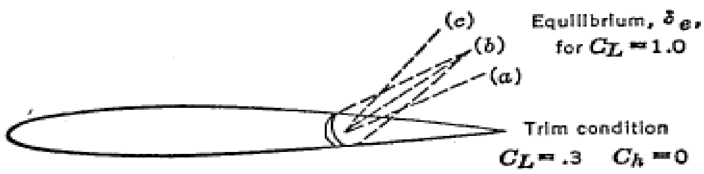

FIGURE 6–14. Typical floating conditions of the elevator.

the elevator angle required for equilibrium, the pilot would not have to apply any force to hold any other air speed than the original trim speed, and the airplane would be termed neutrally stable stick-free. If for an increase in lift coefficient from trim the elevator floats up, but at a smaller angle than required of the elevator for equilibrium, the pilot will have to supply a pull force at the stick to hold the equilibrium elevator angle, and the airplane would be considered stick-free stable. Finally, if the floating angle exceeded the equilibrium angle required for an increase in C_L from trim, the pilot would have to supply a push force at the stick which would be termed stick-free instability. The three conditions are shown schematically in Figure 6–14.

To evaluate the static longitudinal stability stick-free, it is necessary to determine the value of the derivative $(dC_m/dC_L)_{\text{Free}}$. This is obviously the value of $(dC_m/dC_L)_{\text{Fix}}$ plus the destabilizing effect of freeing the elevators.

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} + \left(\frac{dC_m}{dC_L}\right)_{\substack{\text{Free}\\\text{elevator}}} \qquad (6\text{–}21)$$

The moment coefficient due to the tail was developed in Chapter 5 and can be restated as follows:

$$C_{m_t} = -C_{L_t}\bar{V}\eta_t \tag{6–22}$$

and

$$C_{m_\delta} = -\frac{dC_{L_t}}{d\delta_e}\bar{V}\eta_t \tag{6–23}$$

The stability contribution of the free elevator is therefore

$$\left(\frac{dC_m}{dC_L}\right)_{\substack{\text{Free}\\ \text{elevator}}} = \left(\frac{d\delta_e}{dC_L}\right)_{C_{he}=0} \times C_{m_\delta} \tag{6–24}$$

The value of the floating angle derivative $(d\delta_e/dC_L)_{C_{he}=0}$ comes from equation (6–20).

$$(\delta_e)_{C_{he}=0} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha_s \tag{6–25}$$

$$\left(\frac{d\delta_e}{dC_L}\right)_{C_{he}=0} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\frac{d\alpha_s}{dC_L} \tag{6–26}$$

The angle of attack of the stabilizer, α_s, was given in Chapter 5 as follows:

$$\alpha_s = \alpha_w - \epsilon - i_w + i_t \tag{6–27}$$

therefore

$$\frac{d\alpha_s}{dC_L} = \frac{d\alpha}{dC_L}\left(1 - \frac{d\epsilon}{d\alpha}\right) = \frac{(1 - d\epsilon/d\alpha)}{a_w} \tag{6–28}$$

also

$$\frac{dC_{L_t}}{d\delta_e} = \left(\frac{dC_L}{d\alpha}\right)_t \tau = a_t\tau \tag{6–29}$$

Substituting (6–23), (6–26), (6–28), and (6–29) into (6–24),

$$\left(\frac{dC_m}{dC_L}\right)_{\substack{\text{Free}\\ \text{elevator}}} = \frac{C_{h_\alpha}}{C_{h_\delta}}\frac{a_t}{a_w}\bar{V}\eta_t\tau\left(1 - \frac{d\epsilon}{d\alpha}\right) \tag{6–30}$$

finally

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} + \frac{C_{h_\alpha}}{C_{h_\delta}}\frac{a_t}{a_w}\bar{V}\eta_t\tau\left(1 - \frac{d\epsilon}{d\alpha}\right) \tag{6–31}$$

The stability criterion $(dC_m/dC_L)_{\text{Fix}}$ was developed in Chapter 5 as follows for the props-off case:

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} = \frac{x_a}{c} + \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus\\ Nac}} - \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right) \tag{6–32}$$

Substituting (6–32) into (6–31) and rearranging,

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \frac{x_a}{c} + \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus\\Nac}} - \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\left(1 - \frac{C_{h_\alpha}}{C_{h_\delta}}\tau\right) \quad (6\text{–}33)$$

This is the equation for static longitudinal stability, stick-free, props off. The stick-free stability, props windmilling, can be written as follows:

$$\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} = \frac{x_a}{c} + \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus\\Nac}} - \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\left(1 - \frac{C_{h_\alpha}}{C_{h_\delta}}\tau\right)$$

$$+ \frac{N(dC_N/d\alpha)_{PT=0}\,(d\beta/d\alpha)l_pS_p}{S_wca_w} + \frac{a_t}{a_w}\frac{\bar{V}\eta_t}{.07}\left(\frac{d\beta}{d\alpha}\right)\left(\frac{dC_N}{d\alpha}\right)_{PT=0} \quad (6\text{–}34)$$

The effect of freeing the elevator enters the tail term as the multiplying factor $\left(1 - \tau\frac{C_{h_\alpha}}{C_{h_\delta}}\right)$. For an airplane equipped with an elevator having no change in hinge moment with angle of attack ($C_{h_\alpha} = 0$), this term becomes unity, and the stick-fixed and stick-free stabilities are equal. However, if the elevator has a large floating tendency (the ratio $C_{h_\alpha}/C_{h_\delta}$ large and positive), the stability contribution of the horizontal tail can be reduced materially. For instance, for a ratio of $C_{h_\alpha}/C_{h_\delta} = 2$ and a normal value of τ equal to .5, the floating of the elevator can obviate the whole tail contribution to stability. From this brief analysis the importance of careful elevator balance design to insure proper hinge moment characteristics and thereby good stick-free stability can be readily appreciated.

The stick-free neutral point, N_0', can be obtained directly from equation (6–33) for the propeller-off condition.

The Stick-free Neutral Point

$$N_0' = x_{ac} - \left(\frac{dC_m}{dC_L}\right)_{\substack{Fus\\Nac}} + \frac{a_t}{a_w}\bar{V}\eta_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\left(1 - \frac{C_{h_\alpha}}{C_{h_\delta}}\tau\right) \quad (6\text{–}35)$$

The difference between the stick-fixed and stick-free neutral points is as follows:

$$N_0 - N_0' = \frac{a_t\bar{V}\eta_t C_{h_\alpha}}{a_w C_{h_\delta}}\tau\left(1 - \frac{d\epsilon}{d\alpha}\right) = -\frac{C_{m_\delta}}{a_w}\frac{C_{h_\alpha}}{C_{h_\delta}}\left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (6\text{–}36)$$

A typical value of this difference is .02 to .05 per cent m.a.c.

In practice, curves of C_m versus C_L can be obtained in the wind tunnel for the stick-free case by mounting the elevator on ball bearings and allowing it to float freely throughout the test run. Typical curves of wind-tunnel results of elevator free tests in comparison to elevator fixed curves are given in Figure 6–15.

Actually this technique is not usually resorted to. In actual wind-tunnel practice elevator hinge moments are obtained during pitching moment versus lift coefficient runs, with varying fixed values of the

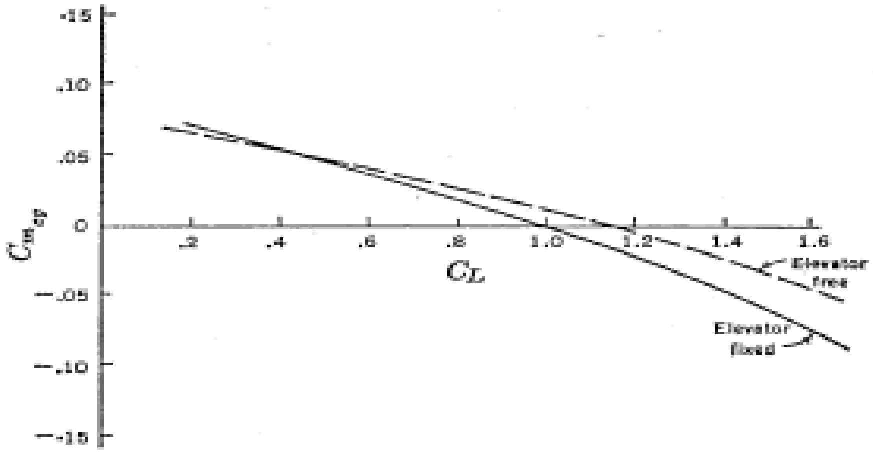

FIGURE 6–15. Typical reduction of stability due to freeing elevator.

elevator angle. From these data the elevator hinge moment coefficients, C_{h_e}, are plotted versus airplane lift coefficient for various elevator angles. The stick-free characteristics are nearly always deduced from such curves. Elevator hinge moment runs are shown in Figure 6–16 for a typical case.