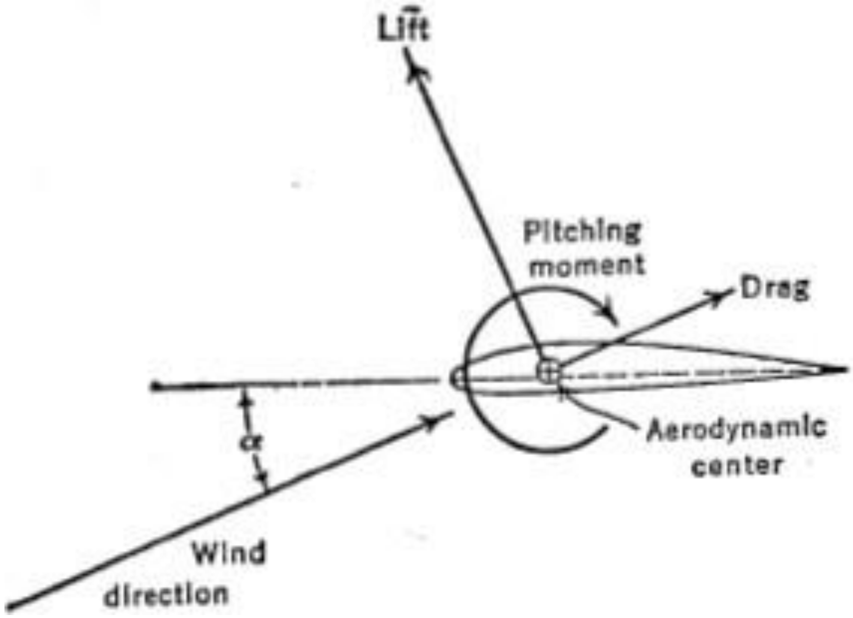
The longitudinal stability theory will first be developed for the simplified case of an airplane in gliding flight with controls locked (stick-fixed) and propellers windmilling. The theories thus obtained will be expanded later to account for the effects of power and free controls.

The study of the longitudinal equilibrium and static stability of the airplane, then, requires an investigation into the moments about the airplane's Y axis through the c.g. and their variation with the airplane's lift coefficient. Equilibrium demands that the summation of these moments equal zero, and static stability demands that a diving moment accompany an increase in lift coefficient and a stalling moment accompany a decrease in lift coefficient from equilibrium.

Throughout this section, it will be assumed that any wing or tail surface can be represented by a mean aerodynamic chord, the forces and moments on which represent all the forces and moments operating on the surface. It will also be assumed that there exists an aerodynamic center on this mean aerodynamic chord about which the wing pitching moment coefficient is invariant with lift coefficient. The forces and moments acting on any wing or tail surface can be repre-



The lift and drag are by definition always perpendicular and parallel to the wind, respectively. It is therefore inconvenient to use these forces to obtain moments, for their arms to the center of gravity vary with angle of attack. For this reason all forces are resolved into normal and chordwise forces whose axes remain fixed with the airplane and whose arms are therefore constant.

Resolving the wing forces perpendicular and parallel to the airplane reference:

$$N = L \cos (\alpha - i_w) + D \sin (\alpha - i_w)$$

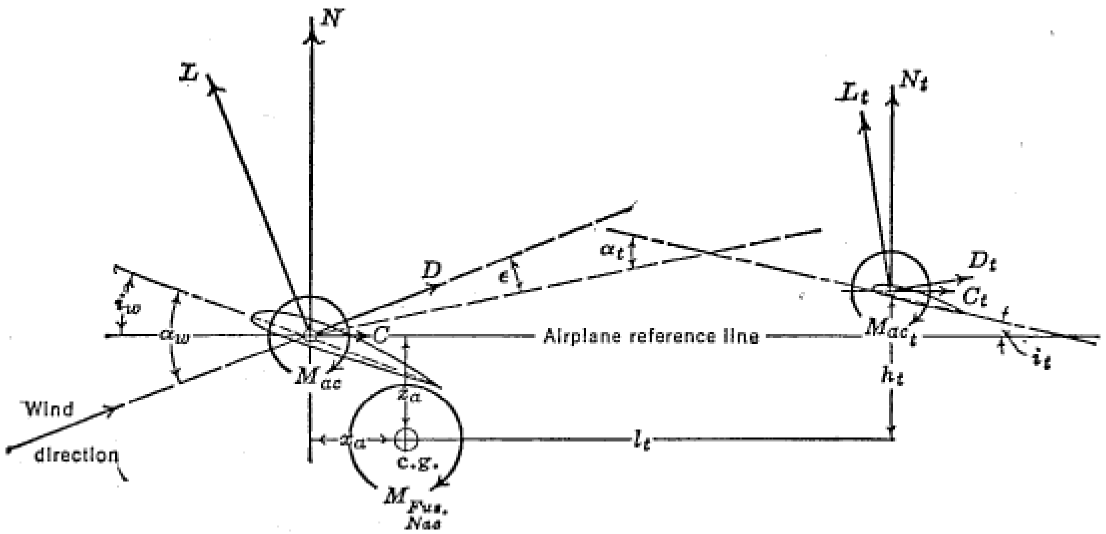
$$C = D \cos (\alpha - i_w) - L \sin (\alpha - i_w)$$

the following summation of moments about the airplane's c.g. is obtained:

$$M_{cg} = Nx_a + Cz_a + M_{ac} + M_{Fus} + M_{Nac} + M_{act} + C_t h_t - N_t l_t$$

For equilibrium it is necessary for M_{cg} to equal zero. It has been found convenient to place this equation in coefficient form by dividing through by $qS_w c$, where q is the dynamic pressure in pounds per square foot, S_w the wing area in square feet, and c the mean aerodynamic chord in feet. The ratio q_t/q, is called the tail efficiency, η_t, and for power-off flight is less than unity, because of the loss of energy as the air interacts with parts of the wing wake and fuselage boundary layer.

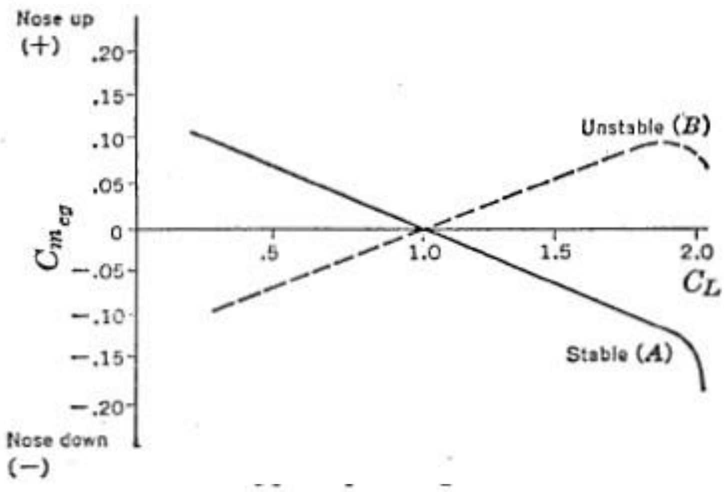
$$C_{m_{cg}} = C_N \frac{x_a}{c} + C_c \frac{z_a}{c} + C_{m_{ac}} + C_{m_{\substack{Fus\\Nac}}} + C_{m_{act}} \frac{S_t}{S_w} \frac{c_t}{c} \eta_t + C_{c_t} \frac{S_t}{S_w} \frac{h_t}{c} \eta_t - C_{N_t} \frac{S_t}{S_w} \frac{l_t}{c} \eta_t$$



The fifth and sixth terms of equation (5–3) have been found to be negligible when compared with the other terms, and will therefore be eliminated from further consideration. This leaves

$$C_{m_{cg}} = C_N \frac{x_a}{c} + C_c \frac{z_a}{c} + C_{m_{ac}} + C_{m_{\substack{Fus\\Nac}}} - C_{N_t} \frac{S_t}{S_w} \frac{l_t}{c} \eta_t$$

This is the equilibrium equation in pitch which must sum up to $C_{m_{cg}} = 0$ for equilibrium in a given flight condition. The pitching moment coefficient will be shown to be a function of the lift coefficient, and the slope of the curve of pitching moment coefficient plotted against lift coefficient is used for evaluating the static stability of the airplane in pitch.



The slope of this curve of C_m versus C_L is given by the derivative dC_m/dC_L, and a negative sign for this derivative is required for static longitudinal stability. The slope of the pitching moment curve dC_m/dC_L can be obtained analytically by differentiating equation (5–4) with respect to C_L:

$$\frac{dC_m}{dC_L} = \underbrace{\frac{dC_N}{dC_L}\frac{x_a}{c} + \frac{dC_c}{dC_L}\frac{z_a}{c} + \frac{dC_{m_{ac}}}{dC_L}}_{\text{Contr. of wing}} + \underbrace{\left(\frac{dC_m}{dC_L}\right)_{\substack{Fus\\Nac}}}_{\text{Contr. of fuselage and nacelles}} - \underbrace{\frac{dC_{N_t}}{dC_L}\frac{S_t}{S_w}\frac{l_t}{c}}_{\text{Contr. of horizontal tail}}\eta_t$$

The contribution of the various parts of the airplane to the total airplane stability can be broken down as shown in equation (5–5). These various contributions will be studied separately.