# UNIT-II

## Analysis of Continuous Time Signals

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$ Fourier Transform

$\omega = 2\pi f$,

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt \quad — (1)$$

Inverse fourier transform.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(F)\, e^{-j2\pi ft}\, df \quad — (2)$$

$F[x(t)] = X(F)$

$F^{-1}[x(F)] = X(t)$

## Properties:

* **Linearity**

$x_1(t) \xleftrightarrow{F.T} X_1(F)$

$x_2(t) \xleftrightarrow{F.T} X_2(F)$

$$C_1 x_1(t) + C_2 x_2(t) \xleftrightarrow{F.T} C_1 x_1(F) + C_2 X_2(F)$$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$X(F) = \int_{-\infty}^{\infty} C_1 x_1(t) + C_2 x_2(t)\; e^{-j2\pi ft}\, dt$$

$$= c_1 \int_{-\infty}^{\infty} x_1(t) e^{-j2\pi f t} + c_2 \int_{-\infty}^{\infty} x_2(t) e^{-j2\pi f t}$$

$$= c_1 x_1(F) + c_2 x_2(F).$$

## * Time Shifting

$$x(t) \xleftrightarrow{F.T} x(F)$$

then $x(t-t_0) \xleftrightarrow{F.T} x(F)\, e^{-j2\pi f t_0}$

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$x(F) = \int_{-\infty}^{\infty} x(t-t_0) e^{-j2\pi f t} dt$$

Assume,

$$t - t_0 = m$$

$$t = m - t_0$$

$$x(F) = \int_{-\infty}^{\infty} x(m)\, e^{-j2\pi f(m+t_0)} dm$$

$$= \int_{-\infty}^{\infty} x(m) e^{-j2\pi f(m+t_0)} dm \,.\, e^{-j2\pi f t_0}$$

$$= x(F)\, e^{-j2\pi f t_0}$$

* **Frequency Shift**

$$x(t) \xleftrightarrow{F.T} x(F)$$

$$e^{j2\pi f_c t}\, x(t) \longleftrightarrow x(F - F_c)$$

$$x(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} e^{+j2\pi ft}\, x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi (F-F_c)t}\, dt$$

$$= x(F - F_c)$$

* **Area Under $x(F)$:**

$$x(t) \longleftrightarrow x(F)$$

$$x(t) = \int_{-\infty}^{\infty} x(F)\, dF = x(0)$$

$$x(t) = \int_{-\infty}^{\infty} x(F)\, e^{j2\pi Ft}\, dF$$

$t = 0$

$$x(0) = \int_{-\infty}^{\infty} x(F)\, dF$$

## * Time Scaling

$$x(t) \longleftrightarrow X(F)$$

$$x(at) = \frac{1}{|a|} X(F/a)$$

$a \rightarrow$ Scaling factor

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} x(at)\, e^{-j2\pi ft}\, dt$$

i) $a > 0$

$at = m$, $t = m/a$, $dt = \frac{dm}{a}$

$$X(F) = \int_{-\infty}^{\infty} x(at)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} x(m) e^{-j2\pi f\left(\frac{m}{a}\right)}\, dm/a$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} x(m) e^{-j2\pi (f/a) m}\, dm$$

$$= \frac{1}{a} X\left(\frac{F}{a}\right) \quad \text{--- (1)}$$

ii) $a < 0$

$$X(F) = \frac{1}{-a} X\left(\frac{F}{a}\right) \text{ --- (2)}$$

Combining (1) & (2)

$$X(F) = \frac{1}{|a|} X\left(\frac{F}{a}\right)$$

## * Duality or Symmetric Property

$$x(t) \longleftrightarrow x(F)$$

$$x(t) \rightleftarrows x(-F)$$

Inverse Fourier Transform

$$x(t) = \int_{-\infty}^{\infty} x(F)\, e^{j2\pi ft}\, dF$$

Replace $t = -t$

$$x(-t) = \int_{-\infty}^{\infty} x(F)\, e^{j2\pi f(-t)}\, dF$$

$$x(-F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dF$$

$$x(t) = x(-F)$$

## * Area under $x(F)$

$$x(t) \longleftrightarrow x(F)$$

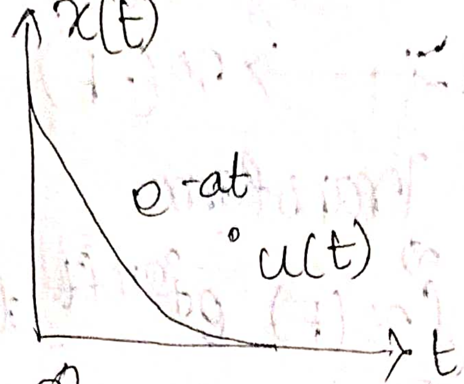
$$x(t) = \int_{-\infty}^{\infty} x(t)\, dt = x(0)$$

$$x(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$F = 0$

$$x(0) = \int_{-\infty}^{\infty} x(F)\, dF$$

Q) Find the fourier transform of decaying exponential function.




$$x(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} \cdot u(t) e^{-j2\pi ft} dt$$

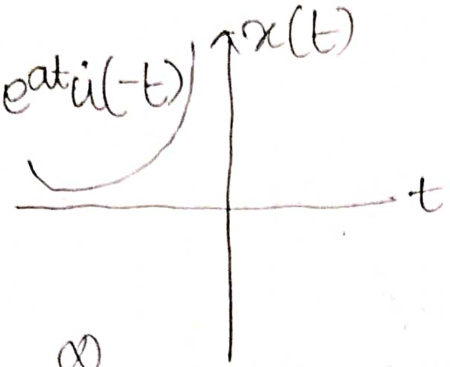
$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j2\pi ft} dt$$

$$= \int_{0}^{\infty} e^{-(a+j2\pi f)t} dt$$

$$= \left[ \frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)t} \right]_0^{\infty}$$

$$X(F) = \frac{1}{a+j2\pi f}$$

Q) Find the fourier transform of rising exponential function




$$x(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} e^{at} u(-t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} e^{at} \cdot e^{j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j2\pi f)t}\, dt$$

$$= \left[ \frac{e^{(a-j2\pi f)t}}{(a-j2\pi f)} \right]_{-\infty}^{0}$$

$$X(F) = \frac{1}{a-j2\pi f} \quad //$$