Energy & Power Signal

Energy - Joule
Power - Watt

Total energy is finite & non-zero is Energy Signal.

$$0 < E < \infty$$

CT: $$E = \int_{-\infty}^{+\infty} (x(t))^2 \, dt$$

$$E = \lim_{T \to \infty} \int_{-T}^{T} (x(t))^2 \, dt$$

DT: $$E = \sum_{n=-\infty}^{+\infty} (x(n))^2 \, dn$$

$$E = \lim_{N \to \infty} \sum_{-N}^{N} (x(n))^2$$

A signal is said to be power signal if its normalised power is finite & non-zero.

$$0 < P < \infty$$

$$RMS = \sqrt{Power}$$

CT: $$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

DT: $$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(t)|^2$$

Prove that power of a energy signal is zero aber infinite time.

Assume,

$$CT;\ P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (x(t))^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \lim_{T\to\infty} \int_{-T}^{T} (x(t))^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \cdot E$$

$$= \frac{1}{\infty} \cdot E$$

$$= 0$$

Prove that energy of power signal is infinite over infinite time.

Assume,

$$CT;\ E = \lim_{T\to\infty} \int_{-T}^{T} (x(t))^2\, dt$$

Multiply & Divide by 2T,

$$= \lim_{T\to\infty} 2T \cdot \frac{1}{2T} \int_{-T}^{T} (x(t))^2\, dt.$$

$$= \lim_{T\to\infty} 2T \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (x(t))^2\, dt$$

$$= \lim_{T\to\infty} 2T \cdot P \Rightarrow \infty \cdot P \Rightarrow \infty$$

Q) $x(t) = \cos t$, find energy & power of the Signal.

(X)

$$E = \lim_{T\to\infty} \int_{-T}^{T} (x(t))^2\, dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} (\cos t)^2\, dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} \cos^2 t\, dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} \frac{1+\cos 2t}{2}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2} \int_{-T}^{T} (1+\cos 2t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2} \left[\int_{-T}^{T} 1 \cdot dt + \int_{-T}^{T} \cos 2t\, dt \right]$$

$$= \lim_{T\to\infty} \frac{1}{2} \left[[t]_{-T}^{T} + 0 \right]$$

$$= \lim_{T\to\infty} \frac{1}{2} \left[T - (-T) \right]$$

$$= \lim_{T\to\infty} \frac{1}{2} (2T)$$

$$= \lim_{T\to\infty} (T)$$

$$\boxed{E = \infty \text{ Joules}}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} (x(t))^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 t \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{1 + \cos 2t}{2} \, dt$$

$$= \lim_{T \to \infty} \frac{1}{4T} \int_{-T}^{T} (1 + \cos 2t) \, dt$$

$$= \lim_{T \to \infty} \frac{1}{4T} \left[\int_{-T}^{T} 1 \cdot dt + \int_{-T}^{T} \cos 2t \cdot dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{4T} \left[(t)_{-t}^{t} + 0 \right]$$

$$= \lim_{T \to \infty} \frac{1}{4T} \left[T - (-T) \right]$$

$$= \lim_{T \to \infty} \frac{1}{4T} [2T]$$

$$= \lim_{T \to \infty} \frac{1}{2}$$

$$\boxed{P = \frac{1}{2} \text{ Watt}}$$

Q) Determine the RMS & power

$x(t) = e^{j\alpha t} \cos \Omega_0 t$

$$\left\{ e^{j\alpha t} = 1 \right\}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |e^{j\alpha t} \cos \Omega_0 t|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 \Omega_0 t \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{1 + \cos 2\Omega_0 t}{2} dt$$

$$= \lim_{T \to \infty} \frac{1}{4T} \int_{-T}^{T} (1 + \cos 2\Omega_0 t) \, dt$$

$$= \lim_{T \to \infty} \frac{1}{4T} \left[\int_{-T}^{T} 1 . dt + \int_{-T}^{T} \cos 2\Omega_0 t . dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{4T} \left[(t)_{-T}^{T} + 0 \right] = \lim_{T \to \infty} \frac{1}{4T} [T - (-T$$

$$= \lim_{T \to \infty} \frac{1}{4T} [2T]$$

$$= \lim_{T \to \infty} \frac{1}{2}$$

$$= \frac{1}{2} \text{ Watts} \; //$$

$$\text{RMS} = \sqrt{\text{Power}}$$

$$= \sqrt{\frac{1}{2}}$$

Q) $x(n) = e^{j\left(\frac{n\pi}{2} + n\frac{\pi}{8}\right)}$

Calculate energy & power.

$$\sum_{n=N_1}^{N_2} 1 = N_2 - N_1 + 1$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left| e^{j\left(\frac{n\pi}{2} + \frac{n\pi}{8}\right)} \right|^2$$

$$\lim_{N \to \infty} \sum_{n=-N}^{N} (1)$$

$$\Rightarrow N - (-N) + 1$$

$$\Rightarrow \lim_{N \to \infty} 2N+1$$

$E \Rightarrow \infty$ Joules

$$\text{Power: } \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j\left(\frac{n\pi}{2} + \frac{n\pi}{8}\right)} \right|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (1)$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} (2N+1)$$

$\boxed{P = 1 \text{ Watt}}$

Q) What is the total energy of DT signal $x(n)$ which takes the value unity (1) at $n = -1, 0, 1$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=-1}^{1} 1$$

$$\lim_{n \to \infty} 1-(-1)+1$$

$$\boxed{E = 3 \text{ Joules}}$$

Q) What is the energy of CT signal & p

$$x(t) = e^{j(2t+\pi/4)}$$

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} |e^{j(2t+\pi/4)}|^2 \, dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} 1 . dt$$

$$= \lim_{T \to \infty} [t]_{-T}^{T}$$

$$= \lim_{T \to \infty} 2T$$

$$= \infty \text{ Joules.}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |e^{j(2t + \pi/4)}|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[(t) \right]_{-T}^{T}$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[2T \right]$$

$$\boxed{P = 1 \text{ Watt}}$$