SNS COLLEGE OF TECHNOLOGY

AN AUTONOMOUS INSTITUTION

COIMBATORE – 35

Department of

Electronics and Communication Engineering

UNIT III

IMPULSE RESPONSE OF LTI CONTINUOUS TIME SYSTEMS

Impulse Response

Let a system be described by

$$a_2\,y''(t)+a_1\,y'(t)+a_0\,y(t)=x(t)$$

and let the excitation be a unit impulse at time $t=0$. Then the zero-state response y is the impulse response h.

$$a_2\,h''(t)+a_1\,h'(t)+a_0\,h(t)=\delta(t)$$

Since the impulse occurs at time $t=0$ and nothing has excited the system before that time, the impulse response before time $t=0$ is zero (because this is a causal system). After time $t=0$ the impulse has occurred and gone away. Therefore there is no longer an excitation and the impulse response is the homogeneous solution of the differential equation.

Impulse Response

$$a_2 \, h''(t) + a_1 \, h'(t) + a_0 \, h(t) = \delta(t)$$

What happens at time, $t = 0$? The equation must be satisfied at all times. So the left side of the equation must be a unit impulse. We already know that the left side is zero before time $t = 0$ because the system has never been excited. We know that the left side is zero after time $t = 0$ because it is the solution of the homogeneous equation whose right side is zero. These two facts are both consistent with an impulse. The impulse response might have in it an impulse or derivatives of an impulse since all of these occur only at time, $t = 0$. What the impulse response does have in it depends on the form of the differential equation.

Impulse Response

Continuous-time LTI systems are described by differential equations of the general form,

$$a_n \, y^{(n)}(t) + a_{n-1} \, y^{(n-1)}(t) + \cdots + a_1 \, y'(t) + a_0 \, y(t)$$
$$= b_m \, x^{(m)}(t) + b_{m-1} \, x^{(m-1)}(t) + \cdots + b_1 \, x'(t) + b_0 \, x(t)$$

For all times, $t < 0$:

If the excitation $x(t)$ is an impulse, then for all time $t < 0$ it is zero. The response $y(t)$ is zero before time $t = 0$ because there has never been an excitation before that time.

Impulse Response

For all time $t > 0$:

The excitation is zero. The response is the homogeneous solution of the differential equation.

At time $t = 0$:

The excitation is an impulse. In general it would be possible for the response to contain an impulse plus derivatives of an impulse because these all occur at time $t = 0$ and are zero before and after that time. Whether or not the response contains an impulse or derivatives of an impulse at time $t = 0$ depends on the form of the differential equation

$$a_n \mathrm{y}^{(n)}(t) + a_{n-1}\mathrm{y}^{(n-1)}(t) + \cdots + a_1 \mathrm{y}'(t) + a_0 \mathrm{y}(t)$$
$$= b_m \mathrm{x}^{(m)}(t) + b_{m-1}\mathrm{x}^{(m-1)}(t) + \cdots + b_1 \mathrm{x}'(t) + b_0 \mathrm{x}(t)$$

Impulse Response

$$a_n \mathrm{y}^{(n)}(t) + a_{n-1}\mathrm{y}^{(n-1)}(t) + \cdots + a_1 \mathrm{y}'(t) + a_0 \mathrm{y}(t)$$
$$= b_m \mathrm{x}^{(m)}(t) + b_{m-1}\mathrm{x}^{(m-1)}(t) + \cdots + b_1 \mathrm{x}'(t) + b_0 \mathrm{x}(t)$$

Case 1: $m < n$

If the response contained an impulse at time $t = 0$ then the nth derivative of the response would contain the nth derivative of an impulse. Since the excitation contains only the mth derivative of an impulse and $m < n$, the differential equation cannot be satisfied at time $t = 0$. Therefore the response cannot contain an impulse or any derivatives of an impulse.

Impulse Response

$$a_n \, y^{(n)}(t) + a_{n-1} \, y^{(n-1)}(t) + \cdots + a_1 \, y'(t) + a_0 \, y(t)$$
$$= b_m \, x^{(m)}(t) + b_{m-1} \, x^{(m-1)}(t) + \cdots + b_1 \, x'(t) + b_0 \, x(t)$$

Case 2: $m = n$

In this case the highest derivative of the excitation and response are the same and the response could contain an impulse at time $t = 0$ but no derivatives of an impulse.

Case 3: $m > n$

In this case, the response could contain an impulse at time $t = 0$ plus derivatives of an impulse up to the $(m - n)$th derivative.

Case 3 is rare in the analysis of practical systems.

Impulse Response

Example

To find the constant K integrate $h'(t)+3h(t)=\delta(t)$ over the infinitesimal range 0^- to 0^+.

$$\int_{0^-}^{0^+} h'(t)dt + 3\int_{0^-}^{0^+} h(t) = \int_{0^-}^{0^+} \delta(t)$$

$$\underbrace{h(0^+)}_{=K} - \underbrace{h(0^-)}_{=0} + 3\int_{0^-}^{0^+} Ke^{-3t}\,u(t)dt = \underbrace{u(0^+)}_{=1} - \underbrace{u(0^-)}_{=0}$$

$$K + 3K\left[\frac{e^{-3t}}{-3}\right]_0^{0^+} = K + 3K\underbrace{\left[(-1/3)-(-1/3)\right]}_{=0} = 1$$

$$K = 1 \Rightarrow h(t) = e^{-3t}\,u(t)$$

Impulse Response

Example

To check the solution, put it into the differential equation to see whether it is satisfied.

$$\frac{d}{dt}\left(e^{-3t}\,\mathrm{u}(t)\right)+3e^{-3t}\,\mathrm{u}(t)=\delta(t)$$

$$e^{-3t}\delta(t)-3e^{-3t}\,\mathrm{u}(t)+3e^{-3t}\,\mathrm{u}(t)=\delta(t)$$

$$\underbrace{e^{-3t}\delta(t)}_{=e^{0}\delta(t)=\delta(t)}=\delta(t)\Rightarrow\delta(t)=\delta(t)\quad \text{Check.}$$

Unit Impulse Response & Unit Step Response

In any LTI system let an excitation $x(t)$ produce the response $y(t)$. Then the excitation $\frac{d}{dt}(x(t))$ will produce the response $\frac{d}{dt}(y(t))$. It follows then that the unit impulse response $h(t)$ is the first derivative of the unit step response $h_{-1}(t)$ and, conversely that the unit step response $h_{-1}(t)$ is the integral of the unit impulse response $h(t)$.

Stability and Impulse Response

A system is BIBO stable if its impulse response is **absolutely integrable.** That is if

$$\int_{-\infty}^{\infty} \left| h(t) \right| dt \text{ is finite.}$$

THANK YOU