Causal and Non-causal system :-

A system is said to be causal system if its output depends upon present and past inputs for all the values of t

Non-causal system :-

A system is said to be non-causal system if its output depends upon future input also.

1) $y(t) = \sin x(t)$

$y(0) = \sin x(0)$

$y(1) = \sin x(1)$

$y(-1) = \sin x(-1)$

o/p depends upon present i/p

causal system

2) $y(t) = x(-t)$

$y(0) = x(0)$

$y(1) = x(-1)$

$y(-1) = x(1)$

o/p depends upon future i/p

Non-causal system

3) $y(t) = x(2n)$

$y(0) = x(0)$

$y(1) = x(2)$

$y(-1) = x(-2)$

o/p depends upon future i/p

Non-causal system

4) $y(n) = 5x(n) + 6$

$y(0) = 5x(0) + 6$

$y(1) = 5x(1) + 6$

$y(-1) = 5x(-1) + 6$

o/p depends upon present i/p

causal system

5) $y(n) = e^{x(n)}$

$y(0) = e^{x(0)}$

$y(1) = e^{x(1)}$

$y(-1) = e^{x(-1)}$

o/p depends on present input

causal system

Static and Dynamic System :-

Static System :- [System Without Memory]

when the output of the system depends upon only the present input then the system is called Static system.

Dynamic system [System Without Memory] :- A system is said to be dynamic, if the output depends upon Past and future inputs. If the system involves any differentiation (or) integration that systems are also called as dynamic system

① $y(n) = x(n) \sin \omega_0 n$

$y(0) = x(0) \sin \omega_0 n$

$y(1) = x(1) \sin \omega_0 n$

$y(-1) = x(-1) \sin \omega_0 n$

o/p depends upon present i/p

Static system

② $y(t) = x(-t)$

$y(0) = x(0)$

$y(1) = x(-1)$

$y(-1) = x(1)$

o/p depends upon future i/p

Dynamic system

③ $y(n) = x(-n+2)$

$y(0) = x(2)$

$y(1) = x(-1+2) = x(1)$

$y(-1) = x(1+2) = x(3)$

o/p depends upon future i/p

Dynamic system

④ $y(t) = \sin x(t)$

$y(0) = \sin x(0)$

$y(1) = \sin x(1)$

$y(-1) = \sin x(-1)$

o/p depends upon present i/p

static system

Stable and Unstable system :-

★ A system is said to be unstable if and only if every bounded input produces the bounded output.

A stable system is also known as BIBO stable system.

★ A system is said to be stable, if the bounded input $x(t)$ produces the bounded output $y(t)$

if $|x(t)| \le M_x < \infty$ for all t

then $|y(t)| \le M_y < \infty$ for all t

condition for stability :- $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

Inverse system (or) Invertability :-

A system is said to be inverse system if there is unique output for every unique input

$$x(t) \rightarrow \boxed{\text{System}} \xrightarrow{y(t)} \boxed{\text{Inverse system}} \rightarrow x(t)$$

check for stability :-

① $h(n) = n\,u(n)$

$$= \sum_{n=-\infty}^{\infty} |n\,u(n)|$$

$$= \sum_{n=0}^{\infty} n$$

$$= 0 + 1 + 2 + \dots \infty$$

$$= \infty$$

② $h(n) = 2^n u(n-3)$

$$u(n-3) = \begin{cases} 1, & n \ge 3 \\ 0, & n < 3 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} |h(n)|$$

$$= \sum_{n=-\infty}^{\infty} \left[2^n u(n-3)\right]$$

$$= \sum_{n=3}^{\infty} |2^n| \Rightarrow 2^3 + 2^4 + \dots 2^\infty$$

$$= \infty$$

③ $h(n) = 3^n u(-n)$

$$= \sum_{n=-\infty}^{\infty} |h(n)| \qquad u(-n) = \begin{cases} 1, & n \ge -\infty \\ 0, & n < 0 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} |3^n u(-n)| \Rightarrow \sum_{n=-\infty}^{\infty} 3^n$$

$$= \sum_{n=0}^{\infty} 3^{-n} \Rightarrow \sum_{n=0}^{\infty} (1/3)^n \Rightarrow 1 + \frac{1}{3} + (1/3)^2 + \cdots$$

$$= \frac{1}{1-1/3} \Rightarrow 3/2 \quad \text{[Stable system]}$$

Verify the properties of the system :-

① $y(n) = x(n)\, u(n)$

$y_1(n) = x_1(n)\, u(n)$

$y_2(n) = x_2(n)\, u(n)$

$y_3(n) = a_1 x_1(n)\, u(n) + a_2 x_1(n)\, u(n)$

$y_3'(n) = u(n)[a_1 x_1(n) + a_2 x_2(n)]$

$= a_1 x_1(n)\, u(n) + a_2 x_2(n)\, u(n)$

$y_3(n) = y_3'(n) \rightarrow$ linear system

$y(n) = x(n)\, u(n)$

$y(n, n_1) = x(n-n_1)\, u(n)$

$y(n-n_1) = x(n-n_1)\, u(n-n_1)$

$y(n, n_1) \neq y(n-n_1)$

Time Variant system

$y(n) = x(n)\, u(n)$

$y(0) = x(0)\, u(n)$

$y(1) = x(1)\, u(n)$

$y(-1) = x(-1)\, u(n)$

o/p depends upon present i/p

causal system

$y(n) = x(n)\, u(n)$

$y(0) = x(0)\, u(n)$

$y(1) = x(1)\, u(n)$

$y(-1) = x(-1)\, u(n)$

o/p depends upon present i/p

static system

★ Linear, Time Variant, causal & static.