Time to climb:

The rate of climb, by definition is the vertical component of the airplane's velocity, which is simply the time rate of change of altitude $\frac{dh}{dt}$. Hence,

$$\frac{dh}{dt} = R/C. \quad (or) \quad dt = \frac{dh}{R/C.} \longrightarrow (1)$$

$\therefore$ R/C is a function of altitude and dt is the small increment in time required to climb the small height dh @ a given instantaneous altitude.

The time to climb from one altitude h_1 to another h_2 is obtained by integrating equ (1) between the two altitudes.

$$t = \int_{h_1}^{h_2} \frac{dh}{R/C} \longrightarrow (2)$$

Normally, the performance characteristic labeled time to climb is considered from sea level, where $h_1 = 0$. Hence, the time to climb from sea level to any given altitude h_2 is form equ (2)

$$t = \int_{h_0}^{h_2} \frac{dh}{R/C.} \longrightarrow (2)$$

the maximum rate of climb is used at each altitude then t becomes the minimum time to climb to altitude h_2.

$$t_{min} = \int_0^{h_2} \frac{dh}{(R/C)_{max.}}$$