

(An Autonomous Institution)



DEPARTMENT OF MATHEMATICS

Problems: IL-TINU

FOURIER TRANSFORMS

Fourier transforms:

Fourier transform of f(x) is defined as

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

Inverse Fourier transforms:

Inverse Fourier transform of is defined

as,

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F[f(x)] e^{-iSx} ds$$

* Fourier transform and Inverse Fourier transform are jointly called as fourier transform pair.

Parseval's Identity:

If F(S) is a Fourier transform of

$$f(x)$$
, then

$$\int_{-\infty}^{\infty} \left[F(s) \right]^2 ds = \int_{-\infty}^{\infty} \left[-f(x) \right]^2 dx$$

$$\star = \frac{-i0}{e} = \cos \theta - i \sin \theta,$$

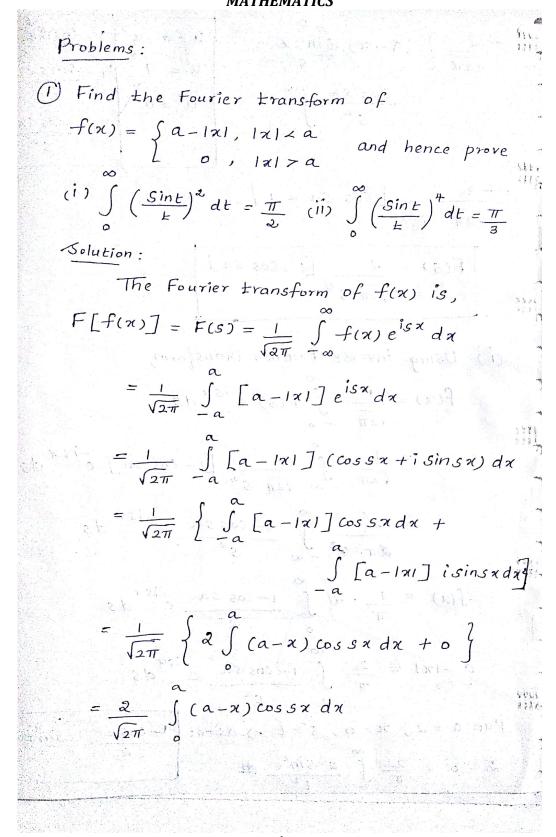
$$+\int_{-\infty}^{\infty} f(x) dx = 0$$
, if $f(x)$ is odd

*
$$\int f(x) dx = 0$$
, if $f(x)$ is odd.
* $\int f(x) dx = 2 \int f(x) dx$, if $f(x)$ is even.





(An Autonomous Institution) DEPARTMENT OF MATHEMATICS





(An Autonomous Institution) DEPARTMENT OF MATHEMATICS

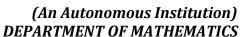


$$= \frac{2}{\sqrt{2\pi}} \left\{ (a - x) \frac{sinsx}{s} \right\}$$

$$= \frac{2}{\sqrt{2\pi}} \left[(a - x) \frac{sinsx}{s} \right]$$

$$= \frac{2}{\sqrt{2\pi}} \left[(a - x) \frac{s$$







$$2 = \frac{H}{\pi} \int_{0}^{\infty} \frac{\sin^{2}t}{t^{2}} dt$$

$$\int_{0}^{\infty} \frac{\sin^{2}t}{t^{2}} dt = \frac{\pi}{2}$$

(ii) Using Parseval's identity,
$$\int_{-\infty}^{\infty} \left[F(s)\right]^{2} ds = \int_{-\infty}^{\infty} \left[-f(\pi)\right]^{2} d\pi$$

$$\int_{-\infty}^{\infty} \left[\frac{2}{\sqrt{2\pi}} \int_{s^{2}}^{s^{2}} \left[1-\cos sa\right]\right]^{2} ds = \int_{-a}^{a} \left(a-1\pi I\right)^{2} d\pi$$

$$\frac{2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\left(1-\cos sa\right)^{2}}{s^{4}} ds = 2 \int_{0}^{a} \left(a-\pi\right)^{2} d\pi$$

$$\frac{2}{\sqrt{2\pi}} \times 2 \int_{0}^{\infty} \frac{\left(1-\cos sa\right)^{2}}{s^{4}} ds = 2 \left[\frac{\left(a-\pi\right)^{3}}{-3}\right]^{a}$$

$$\frac{4}{\pi} \int_{0}^{\infty} \frac{\left(1-\cos sa\right)^{2}}{s^{4}} ds = -\frac{2}{3} \left[0-a^{3}\right]$$

$$\frac{4}{\pi} \int_{0}^{\infty} \frac{\left(1-\cos sa\right)^{2}}{s^{4}} ds = \frac{2a^{3}}{3}$$

$$\frac{4}{\pi} \int_{0}^{\infty} \frac{\left(1 - \cos 2t\right)^{2}}{t^{4}} dt = \frac{2}{3} \times 2^{3}$$

$$\frac{4}{\pi} \int_{0}^{\infty} \frac{(2 \sin^{2} t)^{2}}{t^{4}} dt = \frac{16}{3}$$



(An Autonomous Institution)
DEPARTMENT OF MATHEMATICS



$$\frac{16}{\pi} \int_{0}^{\infty} \frac{\sin^{4}t}{t^{4}} dt = \frac{16}{3}$$

$$\int_{0}^{\infty} \frac{\sin^{4}t}{t^{4}} dt = \frac{16}{3} \times \frac{\pi}{1/4}$$

$$\int_{0}^{\infty} \left(\frac{\sin t}{t}\right)^{4} dt = \frac{\pi}{3}$$

$$\int_{0}^{\infty} \left(\frac{\sin t}{t}\right)^{4} dt = \frac{\pi}{3}$$

$$\int_{0}^{\infty} \left(\frac{\sin t}{t}\right)^{4} dt = \frac{\pi}{3}$$