SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

DEPARTMENT OF MATHEMATICS

PROPERTIES OF FOURIER TRANSFORM

1. F.T is linear

i.e, $F[a f(x) + b g(x)] = a F[f(x)] + b F[g(x)]$

$= a F(s) + b G(s)$

where a and b are constants

Proof:

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$F[a f(x) + b g(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (a f(x) + b g(x)) e^{isx} dx$$

$$= a\, F[f(x)] + b F[g(x)]$$

$$= a F(s) + b G(s)$$

2. Change of Scale property:

If $F[f(x)] = F(s)$ then

$$F[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right)$$

Proof:

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(ax) e^{isx} dx$$

Take $ax = t \Rightarrow x = \frac{t}{a} \Rightarrow dx = \frac{dt}{a}$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{it(s/a)} \frac{dt}{a}$$

$$= \frac{1}{a} F\left(\frac{s}{a}\right)$$

Shifting theorem

3. If $F[f(x)] = F(s)$ then

(i) $F(e^{iax} f(x)) = F(s+a)$

Proof:

$$F[f(x)e^{iax}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{ix(s+a)}\, dx$$

$$= F(s+a)$$

4. If $F[f(x)] = F(s)$ then (ii) $F[f(x-a)] = e^{isa} F(s)$

(Shifting theorem)

proof:

$$F[f(x-a)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x-a)\, e^{isx}\, dx$$

Put $x-a = t \Rightarrow x = t+a \Rightarrow dx = dt$

$$F[f(x-a)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{ist}\, e^{isa}\, dt$$

$$= e^{isa} F(s).$$

5. Modulation theorem:

If $F[f(x)] = F(s)$ then $F[f(x) \cos ax]$

$= \frac{1}{2}[F(s-a) + F(s+a)]$

Proof:

$$F[f(x) \cos ax] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \left(\frac{e^{iax} + e^{-iax}}{2} \right) e^{isx}\, dx$$

$\left(\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}\right)$

$$= \frac{1}{2} [F(s+a) + F(s-a)]$$

6. If $F[f(x)] = F(s)$ then

* $F[x f(x)] = -i \frac{d}{ds} F(s)$ * $F[f'(x)] = is F(s)$

* $F[x^n f(x)] = (-i)^n \frac{d^n}{ds^n} F(s)$ * $F\left(\int_a^x f(x)\, dx\right) = \frac{F(s)}{-is}$