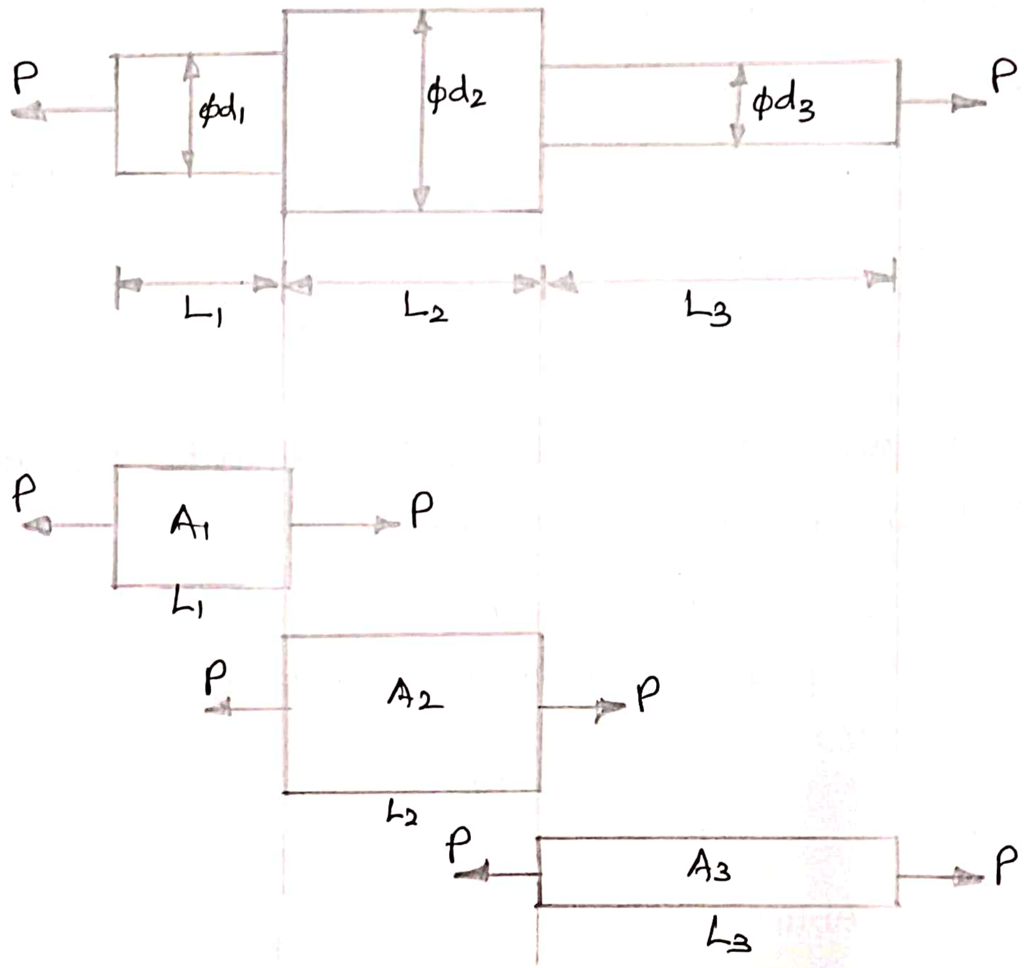
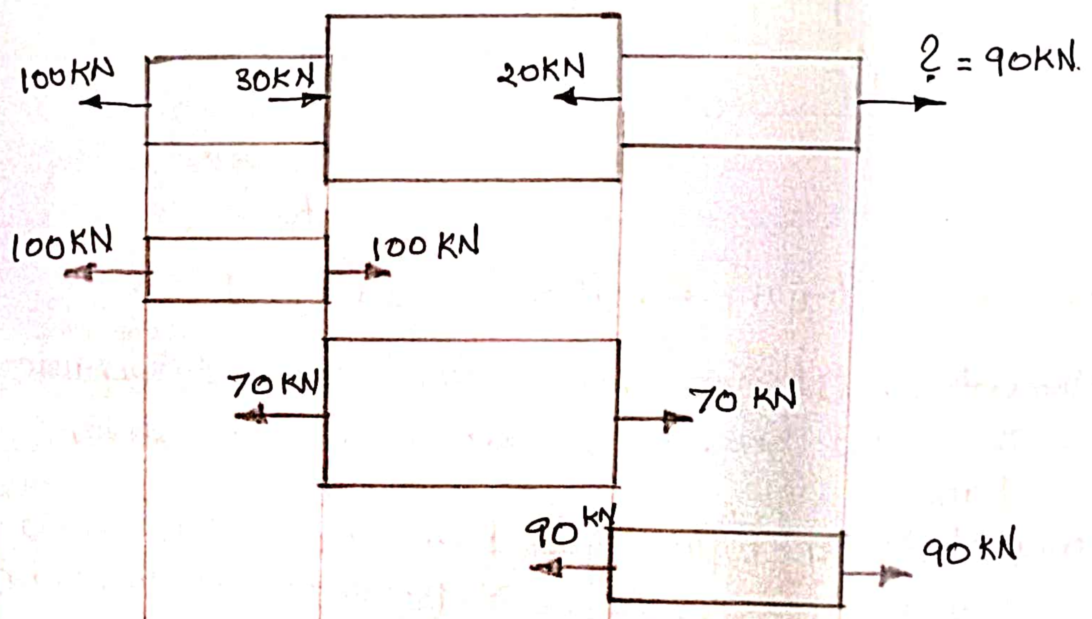
* Axial deformation of a stepped bar

$$\delta = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \cdots$$

*





1- 14

Statically Determinate structure

If a structure can be solved for the forces, displacements, strains and stresses using only the static equations of Equilibrium ($\Sigma F_x = 0$, $\Sigma F_y = 0$ $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$ $\Sigma M_z = 0$), then the structure is a statically determinate structure.

$\Sigma F_x = 0$ Forces acting on the x-direction (Summation of)

$\Sigma M_x = 0$ Summation of moments about the x-axis.

Statically Indeterminate structure.

If a structure can not be solved.....

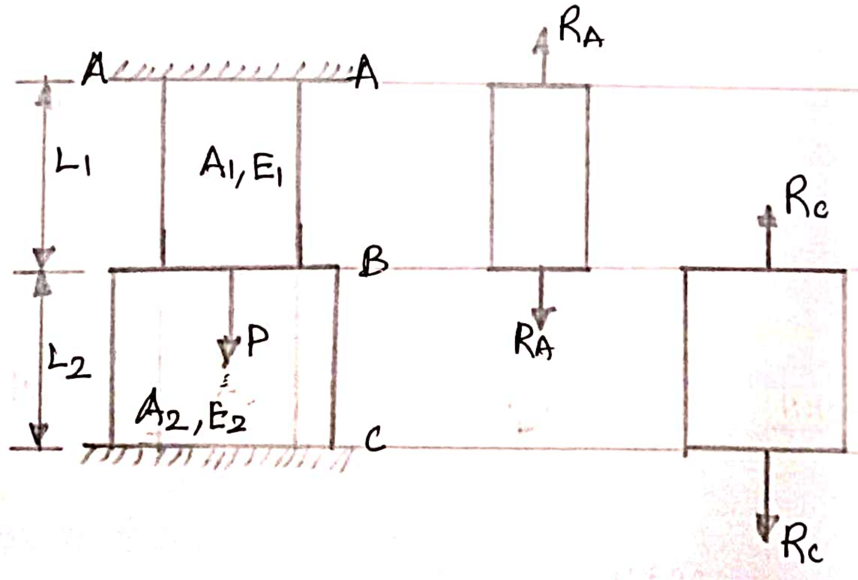
How do you solve a statically indeterminate structure.

Using compatibility Equation

Displacement compatibility Equation

strain compatibility equation.

*.

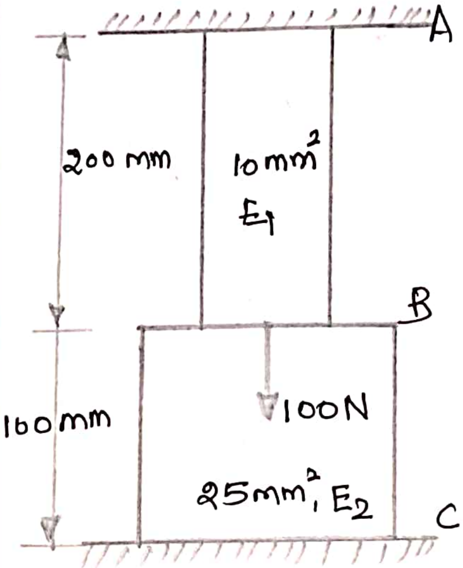


Force Equilibrium Equation ⇒ $R_A + R_C = P$

Moment equilibrium equation does not exist because There are no moments produced in the given bar.

Compatibility Equation ⇒ Elongation of portion AB = Shortening of portion BC

$$\frac{R_A L_1}{A_1 E_1} = \frac{R_C L_2}{A_2 E_2}$$



Take $E_1 = 200$ GPa

$$E_1 = 200 \times 10^3 \frac{N}{mm^2}$$

$E_2 = 100$ GPa

$$E_2 = 100 \times 10^3 \frac{N}{mm^2}.$$

Force Equilibrium Equation $R_A + R_C = 100$ N. — ①

Displacement compatibility equation:

Elongation of Portion AB = Shortening of portion BC

$$\frac{R_A \, L_1}{A_1 E_1} = \frac{R_C L_2}{A_2 E_2}$$

$$\frac{R_A \cancel{200}}{10 \times \cancel{200}} = \frac{R_C \cancel{100}}{25 \times \cancel{100}}$$

$$R_A = R_C \frac{10}{25} = 0.4 R_C \quad \text{—— ②}$$

Substituting ② in ①

$R_A + R_C = 100$ N

$0.4 R_C + R_C = 100$ N

$1.4 R_C = 100$ N

$R_C = 71.428\underline{57}$ N 143

$R_A = 100 - 71.42857$ N

$R_A = 28.571428$ N 57

$$\delta_{AB} = \frac{R_A L_1}{A_1 E_1}$$

$$\delta_{AB} = \frac{28.57 \times 200}{10 \times 200}$$

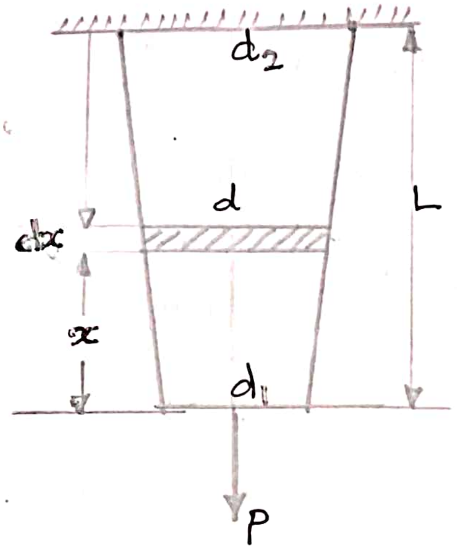
$\delta_{AB} = 2.8571$ mm.

$$\delta_{BC} = \frac{R_C L_2}{A_2 E_2}$$

$$\delta_{BC} = \frac{71.428 \times 100}{25 \times 100}$$

$\delta_{BC} = 2.8571$ mm.

1-16 Elongation of a tapered bar.



diameter at a distance x

$$d = d_1 + \left(\frac{d_2 - d_1}{L}\right) x$$

At $x = 0$,
$d = d_1$

At $x = L$,
$d = d_2$

Cross-sectional Area at distance x $= \frac{\pi}{4} d^2$

$$A = \frac{\pi}{4}\left[d_1 + \left(\frac{d_2 - d_1}{L}\right) x\right]^2$$

Let $g = \frac{d_2 - d_1}{L}$

$$A = \frac{\pi}{4}(d_1 + gx)^2$$

Elongation of the shaded region $= \frac{P\,dx}{AE}$

$$= \frac{P\,dx}{\frac{\pi}{4}(d_1 + gx)^2 E}$$

Elongation of the tapered bar $= \int_0^L \frac{P\,dx}{\frac{\pi}{4}(d_1 + gx)^2 E}$

$$= \frac{4P}{\pi E}\int_0^L (d_1 + gx)^{-2}\,dx = \frac{4P}{\pi E}\left[\frac{(d_1 + gx)^{-2+1}}{(-2+1)\cdot g}\right]_0^l$$

$$= \frac{4P}{\pi E}\frac{(-1)}{g}\left[\frac{1}{(d_1 + gx)}\right]_0^l$$

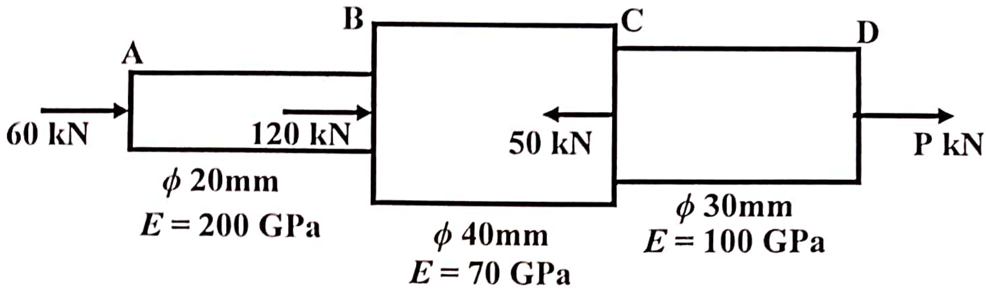
$$= \frac{4P}{\pi E}(-1)\frac{1}{\frac{d_2 - d_1}{L}}\left[\frac{1}{d_1 + \frac{d_2 - d_1}{L}\cdot L} - \frac{1}{d_1 + \frac{d_2 - d_1}{L}\cdot 0}\right]$$

$$= \frac{4P}{\pi E}(-1)\frac{L}{d_2 - d_1}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] = \frac{4P}{\pi E}(-1)\frac{L}{d_2 - d_1}\left[\frac{d_1 - d_2}{d_1 d_2}\right]$$

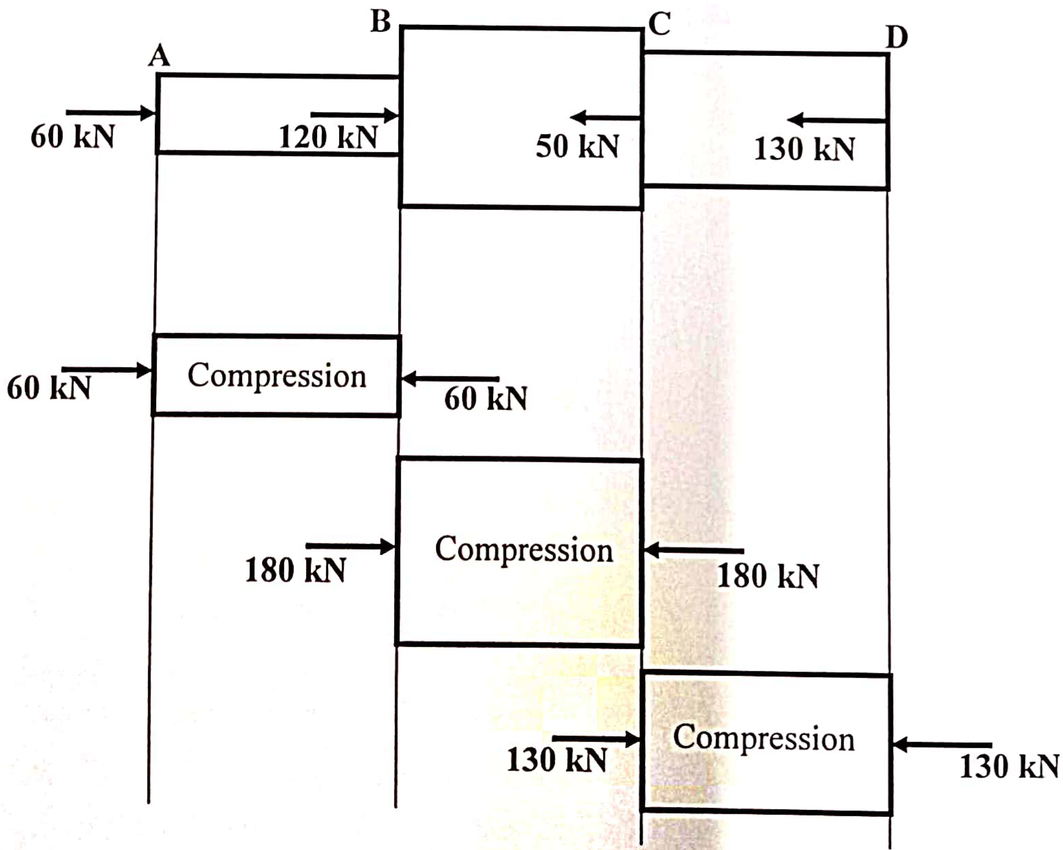
$$= \frac{4P}{\pi E}\frac{L}{d_2 - d_1}\cdot\frac{d_2 - d_1}{d_1 d_2} = \frac{4PL}{\pi E d_1 d_2}$$

UNIT – 1.2.1 STEPPED BAR – 3 REGIONS PROBLEM

A stepped bar is shown in Figure. AB = 800 mm, BC = 500 mm and CD = 900 mm. Find the force P, stresses and strains in all the regions and the total elongation of the stepped bar. The stepped bar is in static force equilibrium.



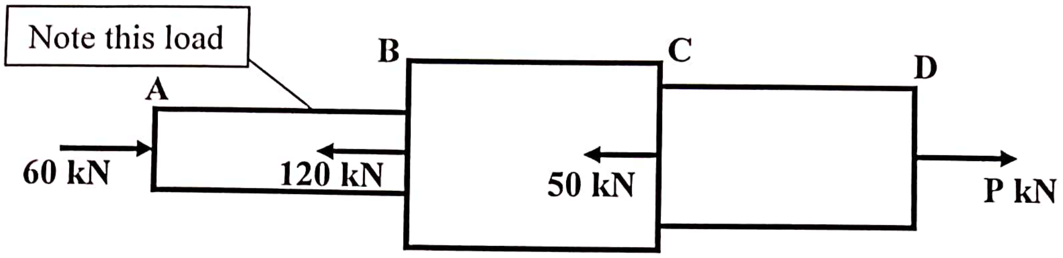
For static force equilibrium condition of the stepped bar, $\sum F_x = 0$, i.e., summation of forces acting along x-direction = 0.
Forces acting to the right are considered positive.
Forces acting to the left are considered negative.
Let us take P as positive. If, after calculation, P is negative, then P is acting to the left.



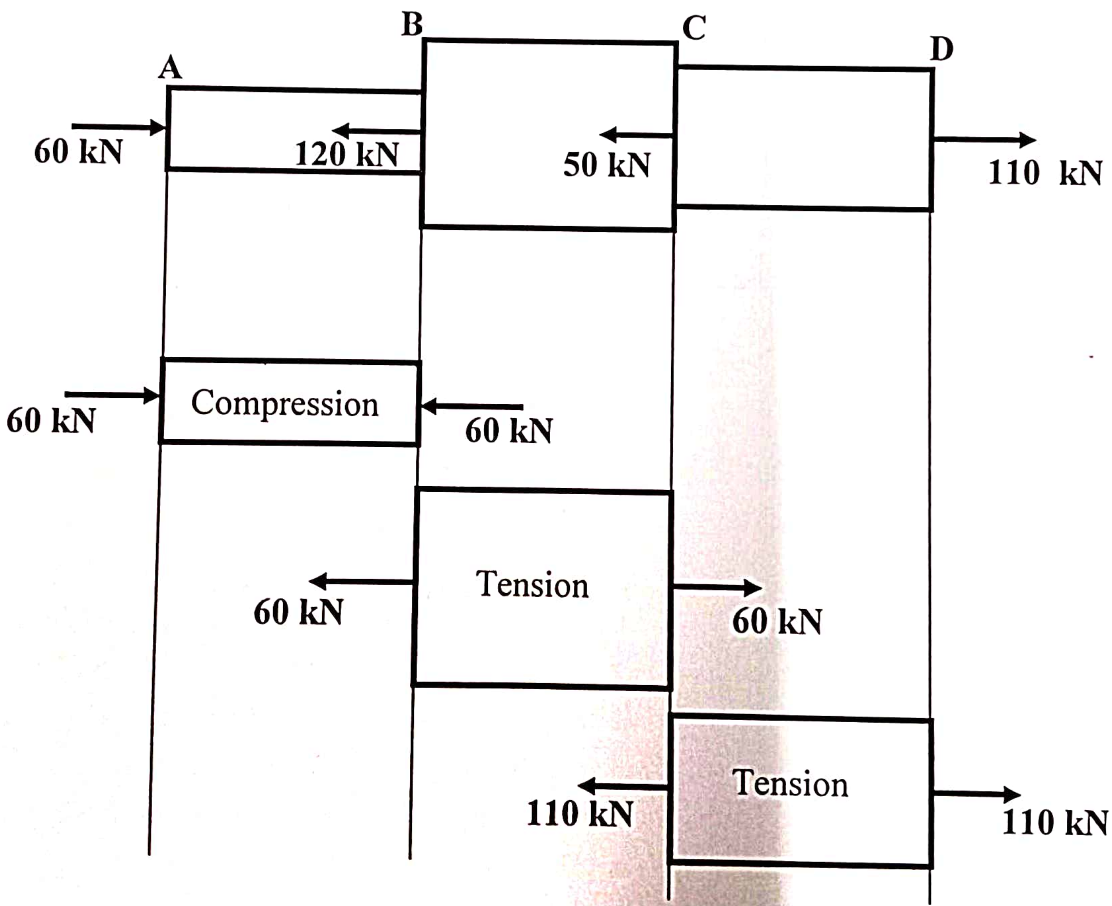
$$\delta L = \sum_{1}^{3} \frac{P_i L_i}{A_i E_i} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3}$$

$$\delta L = \frac{(-60\times10^3)\times800}{\frac{\pi}{4}\times20^2\times200\times10^3} + \frac{(-180\times10^3)\times500}{\frac{\pi}{4}\times40^2\times70\times10^3} + \frac{(-130\times10^3)\times900}{\frac{\pi}{4}\times30^2\times100\times10^3} = -3.44229 \text{ mm}$$

A slightly different problem – but a completely different answer:

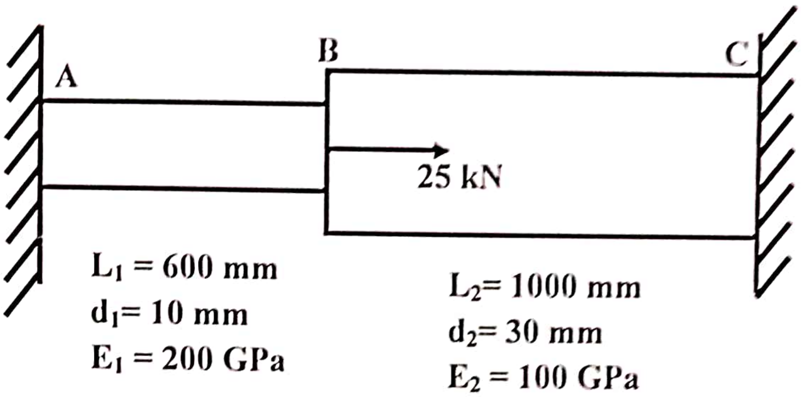


$$60 - 120 - 50 + P = 0 \quad \Rightarrow \quad P = 110 \text{ kN}$$

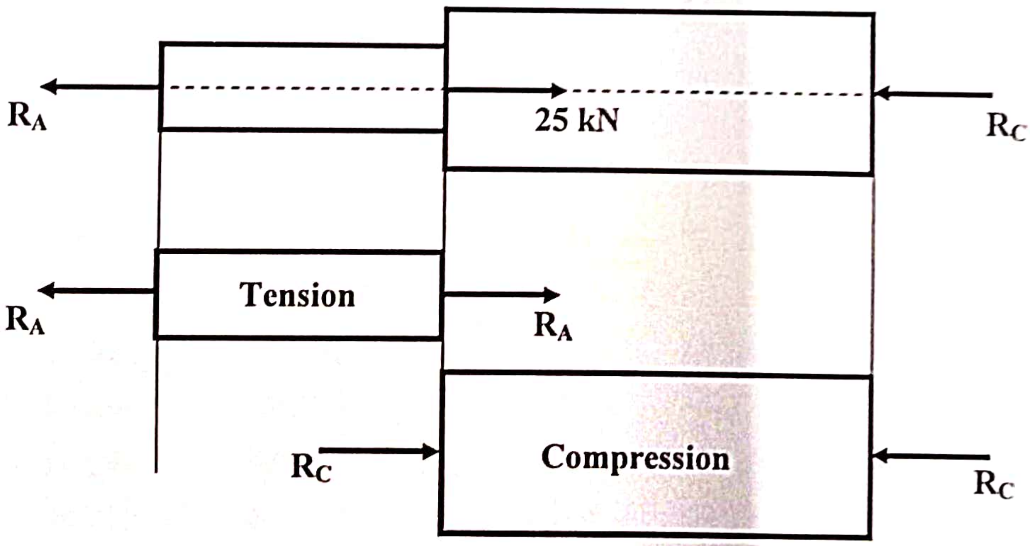


UNIT - 1.3.1 INDETERMINATE STEPPED BAR PROBLEM

A stepped bar ABC has its both ends A and C fixed as shown in the Figure. The length, diameter and Young's modulus of region AB are 600 mm, 10 mm and 200 GPa, respectively and that of region BC is 1000 mm, 30 mm and 100 GPa, respectively. An Axial load of 25 kN acts at B along the positive *x*-direction. All the applied loads and reactions act through the centroids of the cross-sections and form a collinear force system. Find the stresses, strains and deformation in the regions AB and BC.



From the loading diagram, we can infer that region AB undergoes tension, i.e., region AB elongates and region BC undergoes compression, i.e., region BC shortens. Also we can infer that some portion of the total load is used to elongate region AB and the remaining portion of the total load is used to shorten region BC.
Let us call the support reaction force developed at A as R_A and that at C as R_B.



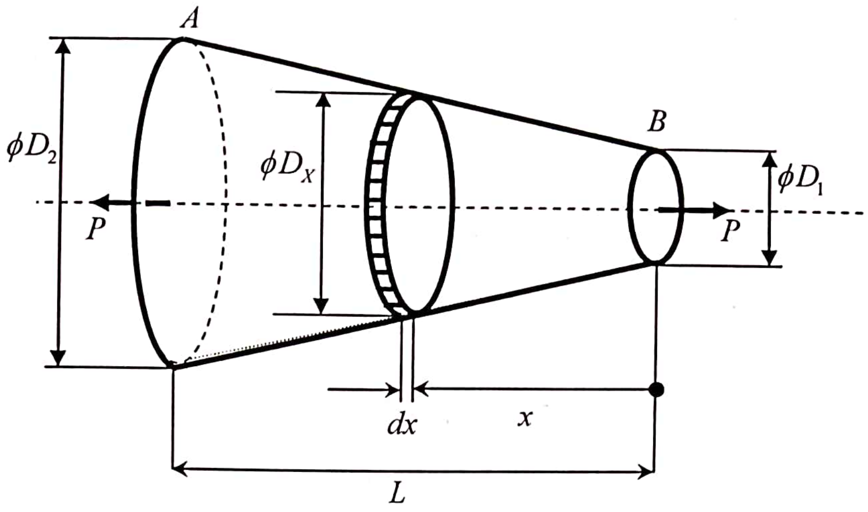
From the free body diagrams, we can infer the static force equilibrium equation as that $R_A + R_C = 25$ kN ....(1)

Equation (1) cannot be solved since there are two unknowns. Hence, the given stepped bar is an indeterminate bar. To solve this indeterminate bar, we need "compatibility" equation or "compatibility" condition which is given by

"Elongation of region AB = Shortening of portion BC"

$$\delta_{AB} = \delta_{BC} \Rightarrow \frac{P_1L_1}{A_1E_1} = \frac{P_2L_2}{A_2E_2} \Rightarrow \frac{R_A \times 600}{\frac{\pi}{4}\times 10^2 \times 200\times 10^3} = \frac{R_C \times 1000}{\frac{\pi}{4}\times 30^2 \times 100\times 10^3} \;.....(2)$$

ELONGATION / SHORTENING OF A UNIFORMLY TAPERED BAR



Consider a uniformly tapered bar of length L and having diameter D_1 at one end and diameter D_2 at the other end, subjected to an axial centric tensile load P. We have to determine the elongation of this tapered bar. The distance of any point along the centerline of the bar is measured from B and is designated as x. At the right end B, $x = 0$ and at the left end A, $x = L$. First we find the elongation of a small strip of thickness dx at a distance x from end B. we then integrate this term from "zero to L" to find the total elongation of the tapered bar. Since dx is very very small, this dx length is assumed to be a prismatic region with diameter D_X.

The diameter at any section at a distance x from the end $B = D_x = D_1 + \left(\dfrac{D_2 - D_1}{L}\right)x = D_1 + gx$

where $g = \left(\dfrac{D_2 - D_1}{L}\right) =$ diameter gradient = slope = $\dfrac{dy}{dx}$. (Straight line equation $y = mx + c$).

Check: At $x = 0$, $D_x = D_1$; At $x = L$, $D_x = D_2$;

Elongation of the strip of thickness $dx = \dfrac{P\,dx}{A_x E} = \dfrac{P\,dx}{\frac{\pi}{4} D_x^2 E} = \dfrac{4P\,dx}{\pi D_x^2 E}$

Total elongation of the entire tapered bar $= \delta = \displaystyle\int_0^L \frac{4P\,dx}{\pi D_x^2 E} = \frac{4P}{\pi E}\int_0^L \frac{dx}{(D_1 + gx)^2}$

(Why did we take the term $\left(\dfrac{4P}{\pi E}\right)$ outside the integral?)

$$\delta = \frac{4P}{\pi E}\int_0^L (D_1 + gx)^{-2}\,dx \qquad \text{We know that } \int_S^T (a+bx)^n\,dx = \left[\frac{1}{b}\frac{(a+bx)^{n+1}}{n+1}\right]_S^T$$

Applying the above integration formula to the present case, we get

$$\delta = \frac{4P}{\pi E}\left[\frac{1}{g}\frac{(D_1 + gx)^{-2+1}}{(-2+1)}\right]_0^L = -\frac{4P}{\pi E g}\left[\frac{1}{(D_1 + gx)}\right]_0^L$$

(Why did we take the term g outside the boundary term?)

Substituting the upper limit and lower limit values for x,

$$\delta = -\frac{4P}{\pi Eg}\left[\frac{1}{(D_1+gL)} - \frac{1}{(D_1+0)}\right] = -\frac{4P}{\pi Eg}\left[\frac{1}{\left(D_1 + \frac{D_2-D_1}{L}L\right)} - \frac{1}{D_1}\right]$$

$$\delta = -\frac{4P}{\pi Eg}\left[\frac{1}{D_2} - \frac{1}{D_1}\right] = -\frac{4P}{\pi Eg}\left[\frac{D_1-D_2}{D_1D_2}\right] = \frac{4P}{\pi E\frac{D_2-D_1}{L}}\left[\frac{D_2-D_1}{D_1D_2}\right] = \frac{4PL}{\pi ED_1D_2}$$

(Negative sign is used to change (D_1-D_2) to (D_2-D_1) since c. Refer Figure.)

In case of a prismatic bar, $D_1 = D_2 = D$,

then $\delta = \frac{4PL}{\pi ED^2} = \frac{PL}{\left(\frac{\pi D^2}{4}\right)E} = \frac{PL}{AE}$ where $A = \frac{\pi D^2}{4}$

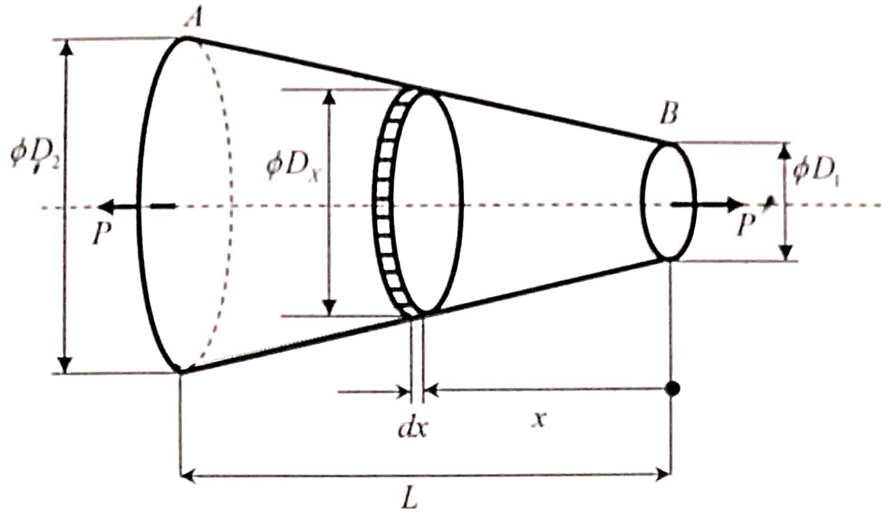
Normal stress σ_A at the left end A $= \frac{P}{\frac{\pi}{4}D_2^2}$; Normal stress σ_B at the right end B $= \frac{P}{\frac{\pi}{4}D_1^2}$.

Since $D_2 > D_1$, $\sigma_B > \sigma_A$.

Applications of taper

1. In tapered bearings.
2. To secure cutting tools or tool holders in the spindle of a machine tool or power tool.
3. In Conically tapered joints.
4. Luer Taper, a standardized fitting system used for making leak-free connections between slightly conical syringe tips and needles
5. Tapered thread, a conical screw thread made of a helicoidal ridge wrapped around a cone
6. Taper ratio in a wing $= \frac{\text{Tip chord}}{\text{Root chord}}$.

UNIT – 1.5.1 ELONGATION / SHORTENING OF A UNIFORMLY TAPERED BAR



Consider a uniformly tapered bar of length L and having diameter D_1 at one end and diameter D_2 at the other end, subjected to an axial centric tensile load P. We have to determine the elongation of this tapered bar. The distance of any point along the centerline of the bar is measured from B and is designated as x. At the right end B, $x = 0$ and at the left end A, $x = L$. First we find the elongation of a small strip of thickness dx at a distance x from end B. we then integrate this term from "zero to L" to find the total elongation of the tapered bar. Since dx is very very small, this dx length is assumed to be a prismatic region with diameter D_X.

The diameter at any section at a distance x from the end B = $D_x = D_1 + \left(\frac{D_2 - D_1}{L}\right)x = D_1 + gx$

where $g = \left(\frac{D_2 - D_1}{L}\right)$ = diameter gradient = slope = $\frac{dy}{dx}$. (Straight line equation $y = mx + c$).

Check: At $x = 0$, $D_x = D_1$; At $x = L$, $D_x = D_2$;

Elongation of the strip of thickness $dx = \frac{P\,dx}{A_x E} = \frac{P\,dx}{\frac{\pi}{4} D_x^2 E} = \frac{4P\,dx}{\pi D_x^2 E}$

Total elongation of the entire tapered bar = $\delta = \int_0^L \frac{4P\,dx}{\pi D_x^2 E} = \frac{4P}{\pi E}\int_0^L \frac{dx}{(D_1 + gx)^2}$

(Why did we take the term $\left(\frac{4P}{\pi E}\right)$ outside the integral?)

$\delta = \frac{4P}{\pi E}\int_0^L (D_1 + gx)^{-2}\,dx$ We know that $\int_S^T (a+bx)^n\,dx = \left[\frac{1}{b}\frac{(a+bx)^{n+1}}{n+1}\right]_S^T$

Applying the above integration formula to the present case, we get

$$\delta = \frac{4P}{\pi E}\left[\frac{1}{g}\frac{(D_1 + gx)^{-2+1}}{(-2+1)}\right]_0^L = -\frac{4P}{\pi E g}\left[\frac{1}{(D_1 + gx)}\right]_0^L$$

(Why did we take the term g outside the boundary term?)

Substituting the upper limit and lower limit values for x,

$$\delta = -\frac{4P}{\pi Eg}\left[\frac{1}{(D_1+gL)}-\frac{1}{(D_1+0)}\right] = -\frac{4P}{\pi Eg}\left[\frac{1}{\left(D_1+\frac{D_2-D_1}{L}L\right)}-\frac{1}{D_1}\right]$$

$$\delta = -\frac{4P}{\pi Eg}\left[\frac{1}{D_2}-\frac{1}{D_1}\right] = -\frac{4P}{\pi Eg}\left[\frac{D_1-D_2}{D_1 D_2}\right] = \frac{4P}{\pi E\frac{D_2-D_1}{L}}\left[\frac{D_2-D_1}{D_1 D_2}\right] = \frac{4PL}{\pi E D_1 D_2}$$

$$\left.\begin{array}{r}\text{Elongation of a tapered bar}\\ \text{due to axial loading}\end{array}\right\} = \delta = \frac{PL}{\frac{\pi}{4}D_1 D_2 E}$$

(Negative sign is used to change $(D_1 - D_2)$ to $(D_2 - D_1)$ since c. Refer Figure.)

In case of a prismatic bar, $D_1 = D_2 = D$,

then $\delta = \frac{4PL}{\pi E D^2} = \frac{PL}{\left(\frac{\pi D^2}{4}\right)E} = \frac{PL}{AE}$ where $A = \frac{\pi D^2}{4}$

Normal stress σ_A at the left end A $= \frac{P}{\frac{\pi}{4}D_2^2}$; Normal stress σ_B at the right end B $= \frac{P}{\frac{\pi}{4}D_1^2}$.

Since $D_2 > D_1$, $\sigma_B > \sigma_A$.

Applications of taper

1. In tapered bearings.
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5. Tapered thread, a conical screw thread made of a helicoidal ridge wrapped around a cone
6. Taper ratio in a wing $= \frac{\text{Tip chord}}{\text{Root chord}}$.

Problem: 5

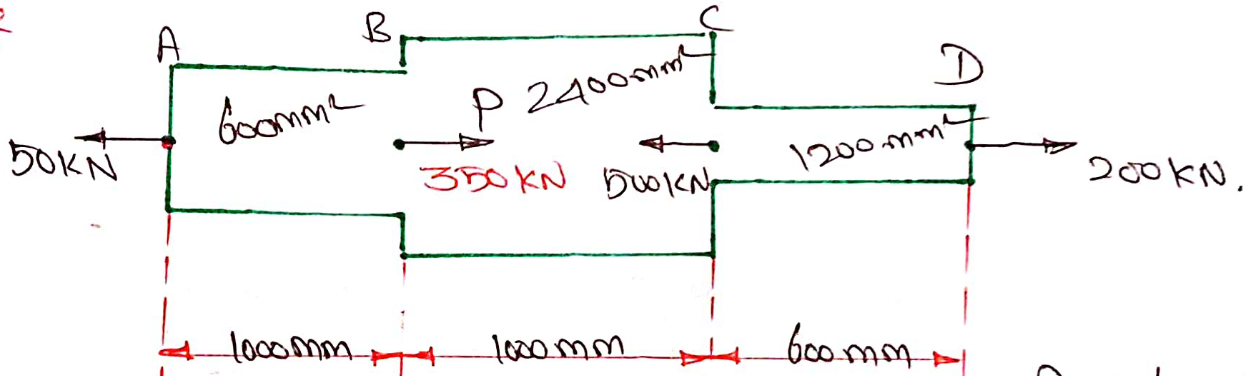
A member ABCD is subjected to point loads as shown in Fig. calculate:-

(i) Force P Necessary for equilibrium.

(ii) Total elongation of the bar. (δL)

Take $E = 210\ GN/m^2$.

To Solve

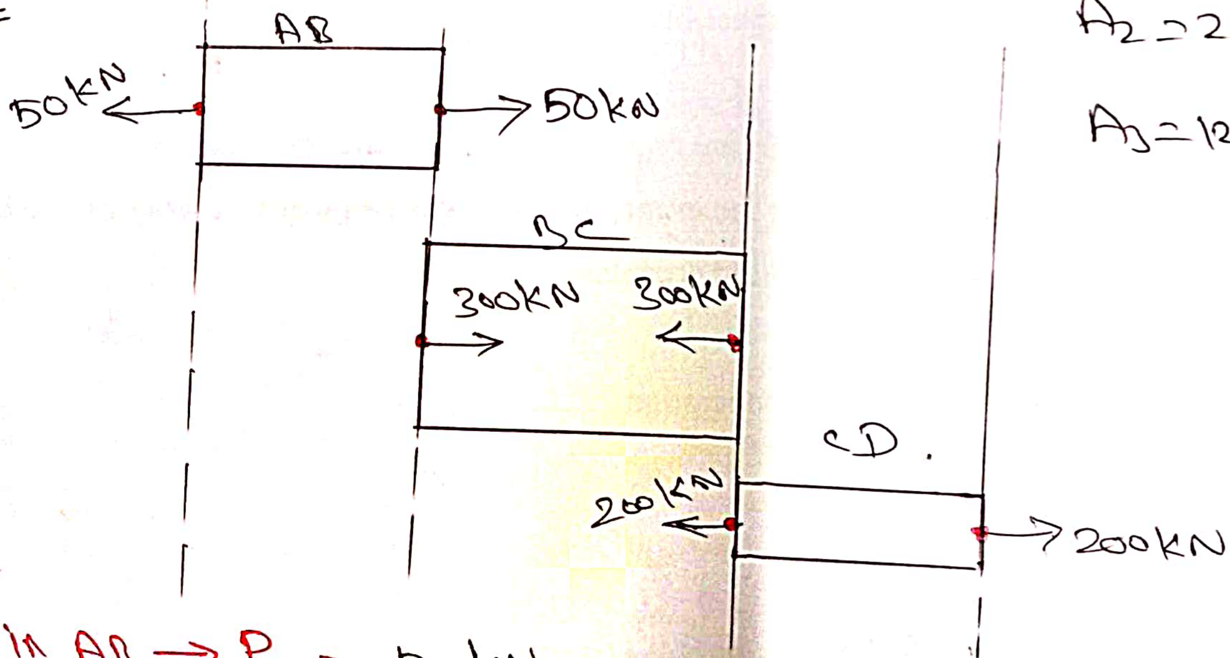


$A_1 = 600\ mm^2$

$A_2 = 2400\ mm^2$

$A_3 = 1200\ mm^2$

FBD



load in AB $\Rightarrow P_1 = 50$ kN

Load in BC $\Rightarrow P_2 = 300$ kN

Load in CD $\Rightarrow P_3 = 200$ kN.

Then,

LM bar $\quad \delta l_1 = \dfrac{P_1 l_1}{AE} \longrightarrow$ increase (+) Tension.

MN $\rightarrow \quad \delta l_2 = \dfrac{P_2 l_2}{AE} \longrightarrow$ decrease (−) Compression.

NP $\rightarrow \quad \delta l_3 = \dfrac{P_3 l_3}{AE} \longrightarrow$ decrese (−) compression

Net change in length.

$$\delta l = \cancel{\delta l_1 + \ldots}$$

$$= \delta l_1 - \delta l_2 - \delta l_3$$

$$= \frac{P_1 l_1}{AE} - \frac{P_2 l_2}{AE} - \frac{P_3 l_3}{AE}$$

$$= \frac{1}{AE}\left(P_1 l_1 - P_2 l_2 - P_3 l_3\right)$$

$$= \frac{10^3}{1000 \times 10^{-6} \times 100 \times 10^{9}} \left(\overset{P_1}{50} \times 0.6 - \overset{P_2}{30} \times 1 - \overset{P_3}{10} \times 1.2\right)$$

$$= -0.12 \text{ mm}.$$

Negative sign indicates that the bar is shortend by 0.12 mm.

Result: Total elongation of bar = 0.12 mm.

Problem: 6

For the bar show in fig calculate the reaction produced by the lower support on the bar. Take $E = 200\ GN/m^2$ GPa
Find also the stresses in the bars.

Given Data:-

$E = 200\ GN/m^2$ (or) GPa.

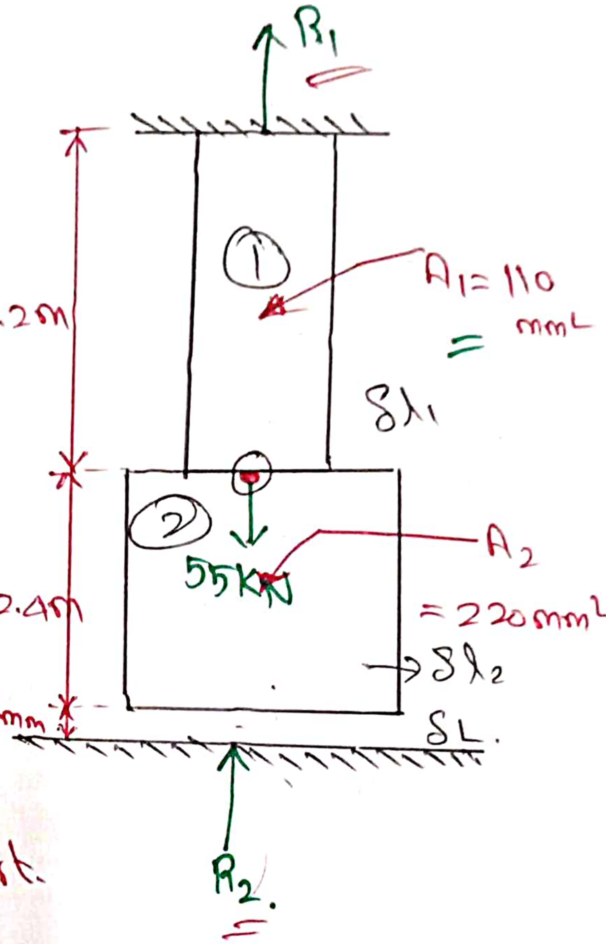
$A_1 = 110\ mm^2$

$A_2 = 220\ mm^2$

$l_1 = 1.2\ m$

$l_2 = 2.4\ m$

$\delta_L = 1.2\ mm$



To Find

R_1 = Reaction at the upper support

R_2 = Reaction at the lower support.

σ_1 = Stress in bar 1

σ_2 = stress in bar 2

Sol

$R_1 + R_2 = 55\ KN$

$= 55000\ N.$

For. bar 1 Total force $= R_1 = (55000 - R_2)$ $(55KN - R_2)$ (T)

For bar 2 Total force $\Rightarrow R_2$ (Compressive)

1 $\quad \delta l_1 = \dfrac{P_1 l_1}{A_1 E} = \dfrac{(55000 - R_2) \times 1.2}{(110 \times 10^{-6}) \times 200 \times 10^9}$

Bar (2) $\delta l_2 = \frac{P_2 l_2}{A_2 E}$

$$= \frac{R_2 \times 2.4}{220 \times 10^{-6} \times 200 \times 10^{9}}$$

Compatibility equation

$$\delta l_1 - \delta l_2 = 1.2\ mm = 0.0012\ m.$$

$$\left(\frac{(55000 - R_2) \times 1.2}{110 \times 10^{-6} \times 200 \times 10^{9}}\right) - \left(\frac{R_2 \times 2.4}{220 \times 10^{-6} \times 200 \times 10^{9}}\right) = 0.0012\ m.$$

$$2(55000 - R_2) \times 1.2 - 2.4 R_2 = 52800$$

$$\boxed{R_2 = 16500\ N} \text{ (or) } \boxed{16.5\ kN}$$

$$R_1 = 55 - 16.5$$

$$\boxed{R_1 = 38.5\ kN}$$

Stress in bar (1)

$$\sigma_1 = \frac{R_1}{A_1} = \frac{38.5\ kN}{110 \times 10^{-6}} = 0.35 \times 10^{6}\ kN/m^2$$

Stress in bar (2)

$$\sigma_2 = \frac{R_2}{A_2} = \frac{16.5\ kN}{220 \times 10^{-6}} = 0.075 \times 10^{6}\ kN/m^2$$

Problem: (9)

A Rectangular bar made of steel is 2.8 m long and 15 mm thick. The rod is subjected to an axial tensile load of 40 kN width various from 75 mm to 30 mm. Find the extension of bar, E = 210 GPa.
(dL)

Solution:-

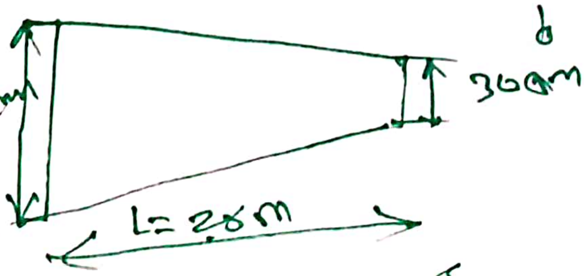
$L = 2.8\,m = 2800\,mm$

$t = 15\,mm$

$P = 40\,kN = 40 \times 10^3\,N$

$a = 75\,mm$, $b = 30\,mm$

$E = 2 \times 10^5\,N/mm^2$



$E = 210\,GPa$

$= 210 \times 10^9 \times \frac{N}{m^2}$

$= 210 \times 10^9 \times \frac{N}{(1000\,mm)^2}$

$= 210 \times 10^9 \times \frac{N}{10^6\,mm^2}$

$= 210 \times 10^3\,N/mm^2$

$= 2.1 \times 10^5$

To solve:

$$\boxed{dL = \frac{PL}{Et(a-b)} \log_e\left(\frac{a}{b}\right)}$$

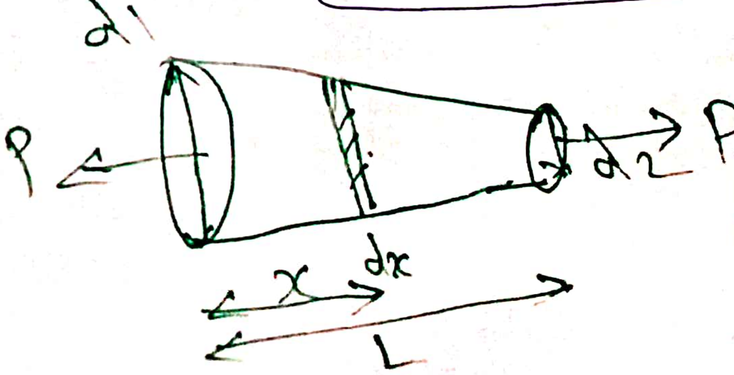
$$= \frac{40 \times 10^3 \times 2800}{2 \times 10^5 \times 15 (75-30)} \log_e\left(\frac{75}{30}\right)$$

$$\frac{112000}{1350 \times 10^2} \times \ln\left(\frac{75}{30}\right)$$

$$\boxed{0.8296 \times 0.916}$$

$$= 0.8296 \times 0.9162$$

$$\boxed{dL = 0.76015\,mm}$$



$$\boxed{dL = \frac{4PL}{\pi E d_1 d_2}}$$

Extension of small elemental length dx

$= \text{Strain} \times \underline{\text{length of } dx}$ $\qquad \epsilon = \frac{dL}{L}$

$= \frac{\text{stress}}{E} \times dx$

$= \frac{\left[\frac{P}{(a-kx)t}\right]}{E} \times dx$ $\qquad \because \sigma = \frac{P}{A}$, $\sigma = E\epsilon_1$

$= \frac{P}{E(a-kx)t} \times dx$

$\therefore$ Total extension can be obtained by integrating

$dL = \int_0^L \left(\frac{P}{E(a-kx)t}\right) dx$ $\qquad$ as $\int \frac{1}{x} = \log_e x$

$\therefore \int \frac{1}{a-bx} = \log_e (a-bx) \times -\frac{1}{b}$

$= \frac{P}{Et} \int_0^L \frac{1}{(a-kx)} dx$

$\log m - \log n = \log \frac{m}{n}$

$= \frac{P}{Et} \log_e (a-kx)_0^L \left(-\frac{1}{k}\right)$

$= \frac{P}{Et}\left[\log_e(a-kL) - \log_e a\right] \times \left(-\frac{1}{k}\right)$

$= \frac{P}{Etk}\left[\log_e a - \log_e (a-kL)\right]$ $\qquad \therefore \log_e x = \ln x$

$= \frac{P}{Etk} \log_e \left(\frac{a}{a-kL}\right)$

Put $k = \left(\frac{a-b}{L}\right)$

$$\therefore \quad = \frac{P}{Et\left(\frac{a-b}{L}\right)} \times \log_e \left[\frac{a}{a - \left(\frac{a-b}{\cancel{L}}\right)\cancel{x}} \right]$$

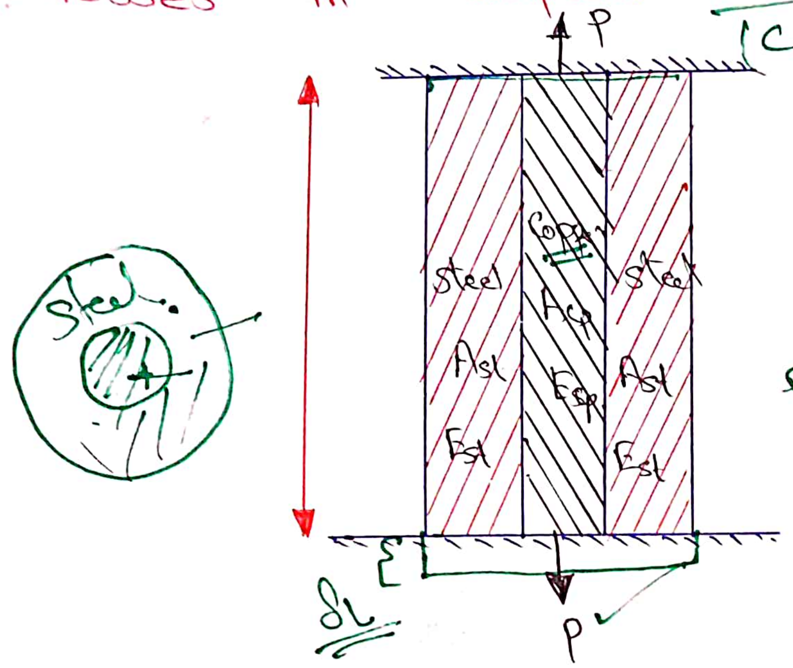
$$= \frac{PL}{Et(a-b)} \log_e \left(\frac{a}{\cancel{a} - \cancel{a} + b} \right)$$

$$\boxed{dL = \frac{PL}{Et(a-b)} \log_e \left(\frac{a}{b} \right)}$$

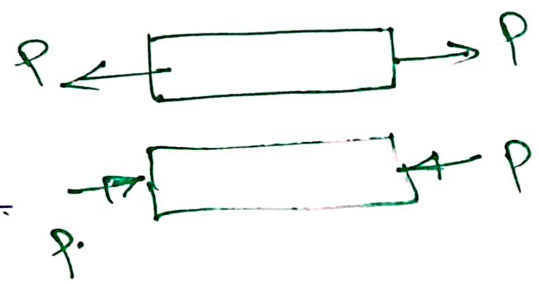
$$\frac{PL}{AE}$$

Stresses in Compound BARS - (Indeterminate Structure)

(C, T)



$$P = P_{st} + P_{cu} \rightarrow (1)$$



∴ Elongation of Steel Bar = Elongation of Copper bar

$\delta L = \frac{PL}{AE}$

$$\frac{P_{st} L_{st}}{A_{st} E_{st}} = \frac{P_{cp} L_{cp}}{A_{cp} E_{cp}}$$

Since both lengths are the same and since

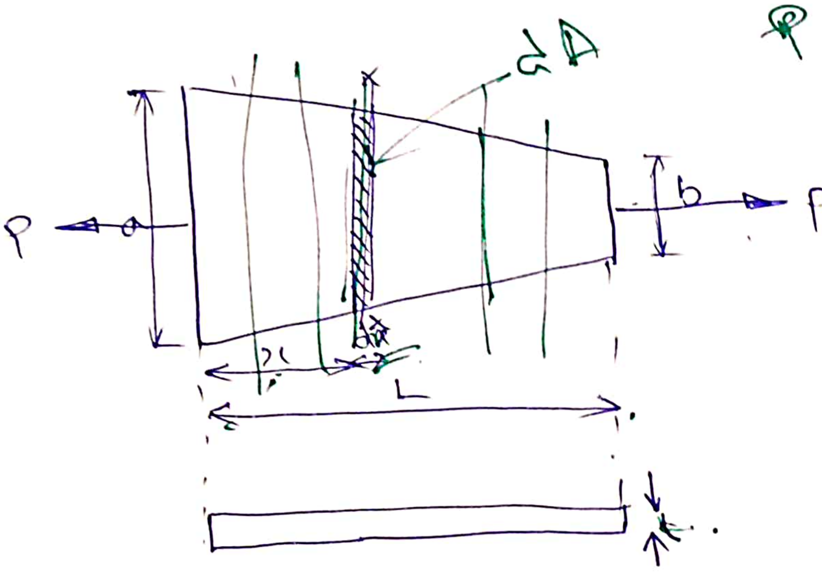
$$\sigma = \frac{P}{A}$$

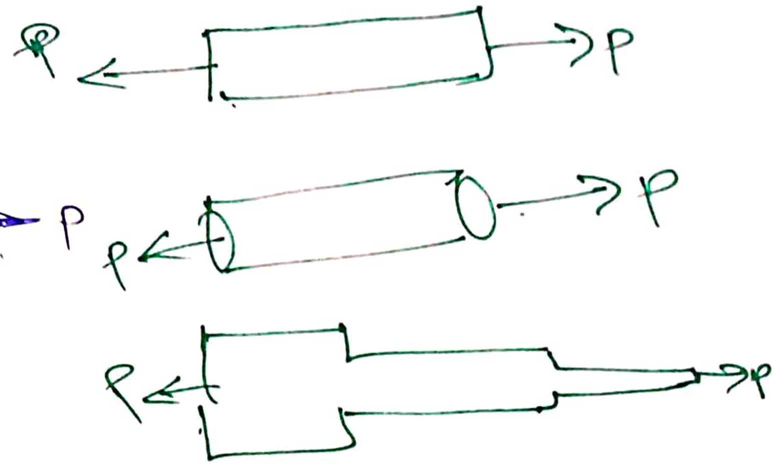
$$\frac{\sigma_{st}}{E_{st}} = \frac{\sigma_{cp}}{E_{cp}} \rightarrow (2)$$

∴ equation (1) can be written as

$$\boxed{P = \sigma_{st} A_{st} + \sigma_{cu} A_{cu}}$$

UNIFORMLY TAPERING RECTANGULAR BAR.





Let P = Axial Load

L = length of bar

a = width of bigger end

b = width of smaller end

E = young's modulus.

t = thickness of bar.

∴ Let's consider section 'x-x' at a distance 'x'

the width of bar at section.

$$= a - \frac{(a-b)}{L}x$$

where $k = \frac{a-b}{L}$.

$$= a - kx$$

thickness of bar at 'x-x' $= t$

Area of the section 'x-x' = width × thickness

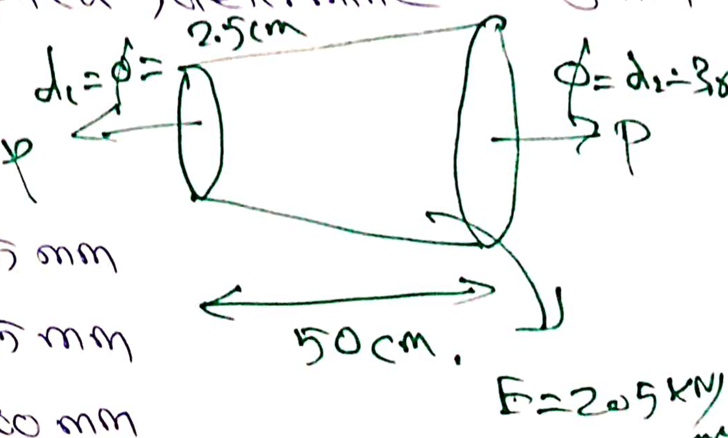
$$dA = (a-kx) \times t.$$

∴ stress on section 'x-x'

$$\sigma = \frac{P}{A} = \frac{P}{(a-kx)t}.$$

Problem (10)

A Rounded bar of steel taper uniformly from dia. 2.5 cm to 3.5 cm in length 50 cm. If an axial force 60000 N is applied, determine elongation of bar, $E = 205\ kN/mm^2$.



Solution:

$d_1 = 2.5\ cm = 25\ mm$

$d_2 = 3.5\ cm = 35\ mm$

$L = 50\ cm = 500\ mm$

$E = 205\ kN/mm^2 = 2.05 \times 10^5\ N/mm^2$

To Solve

$$\boxed{dL = \frac{4PL}{\pi E d_1 d_2}} = \frac{4 \times 60 \times 10^3 \times 500}{\pi \times 205 \times 10^3 \times 25 \times 35}$$

$$= \frac{120000}{563523.18}$$

$$= 0.2129 45$$

$= 0.213\ mm.$

$$\boxed{dL = 0.213\ mm}$$

Problem: (11)

A Copper rod of 40 mm diameter is surrounded Tightly by a Cast iron Tube of 80 mm external diameter, the ends being firmly Fastened together. when put to a Compressive load of 30 kN, What load will be shared by each? Also determine the amount by which the Compound bar shortens if it is 2 meter long. Young's modulus of Copper and Cast iron are, 75 GPa and 175 GPa, respectively.

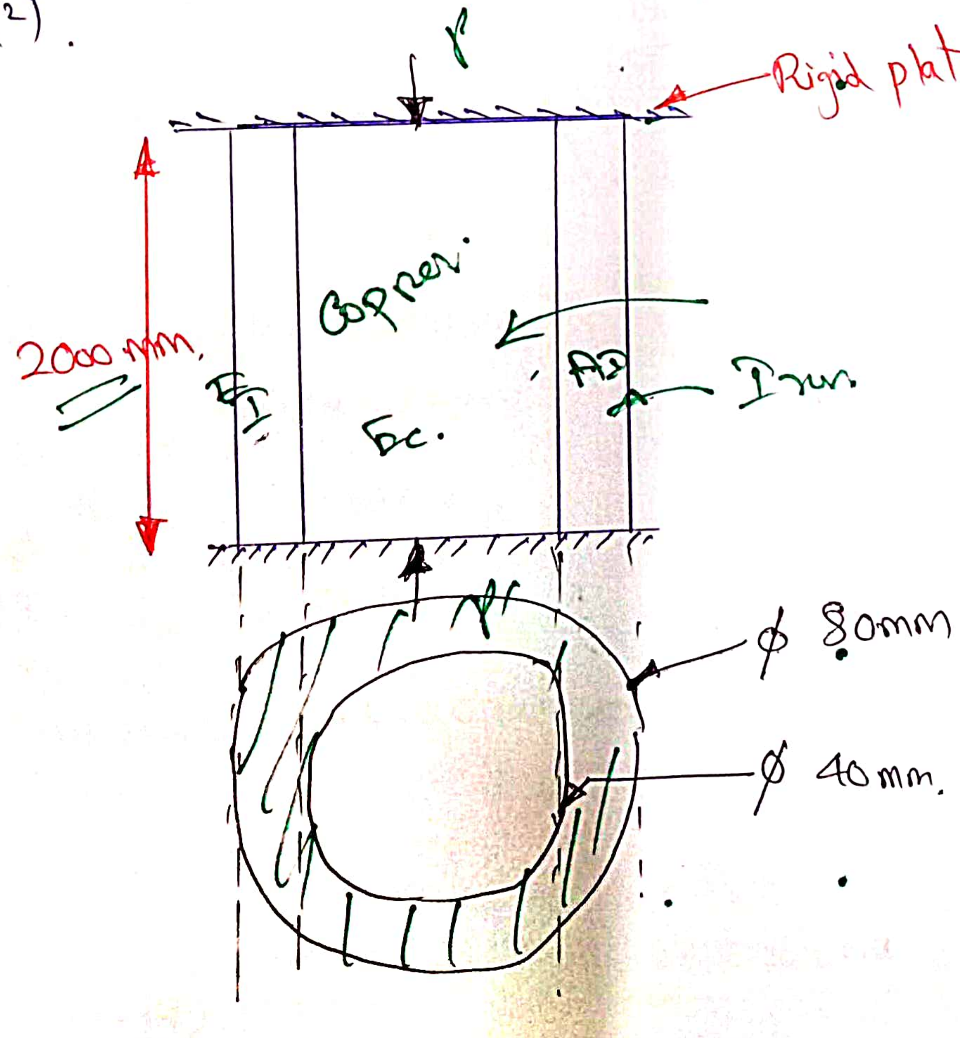
Given Data:-

- Tightly means that : external diameter of Copper rod = internal diameter of Cast iron.

$E_{CI} = 75\ GPa = 75 \times 10^3\ N/mm^2$

$E_{Co} = 175\ GPa = 175 \times 10^3\ N/mm^2$

cross sectional Area of Copper rod $\frac{\pi}{4} d^2$ and Cast iron is $= \frac{\pi}{4}(D^2 - d^2)$.

- 

$$A_c = \frac{\pi}{4} \times (40\,mm)^2 = 1256.63706\ mm^2$$

$$A_I = \frac{\pi}{4} \times (80^2 - 40^2) = 3769.9118\ mm^2.$$

Total load of 30 KN is shared by Copper and iron tube

$$\boxed{P = P_c + P_I}$$

$$30 \times 10^3\ N = P_{Copper} + P_{Iron} \longrightarrow (1)$$

∴ equ. (1) cannot be solved since there are Two unknowns, Hence the given structure is an indeterminate structure

∴ We need Another equation - called the "Compatibility" equation or "Compatibility" condition.

Compatibility condition states that in this problem, both the Copper rod and the cast iron tube shorten by the same amount

$$\begin{matrix} \delta L_c & & \delta L_I \\ \left\{\begin{matrix}\text{Shortening of} \\ \text{Copper rod}\end{matrix}\right\} & = & \left\{\begin{matrix}\text{Shortening of} \\ \text{cast iron tube}\end{matrix}\right\} \end{matrix}$$

$$\frac{P_c L_c}{A_c E_c} = \frac{P_I L_I}{A_I E_I}.$$

$$\frac{P_{Copper} \times 2000}{1256.63 \times 75 \times 10^3} = \frac{P_{CI} \times 2000}{3769.9118 \times 175 \times 10^3}$$

$$\frac{94247.25}{659734.4} = 0.1428\ P_{CI}$$

$$\therefore P_{copper} = \frac{1256.63 \times 75 \times \cancel{10^3} \times 2000}{3769.91118 \times 175 \times \cancel{10^3}} \times P_{CI}$$

$$= \cancel{[illegible]}\ P_{CI}$$

Sub. equ ② in equ ①.

$$\underline{30 \times 10^3 N} = 0.142857\, P_I + P_I.$$

$\therefore P_I (1 + 0.142857) = 30 \times 10^3 N$

$$P_I = \frac{30 \times 10^3 N}{1.142857} = 26,250 N$$

$$\boxed{P_I = 26.25 kN}$$

Using P_I we can find P_c.

$$\underline{P_{copper}} = (30 \times 10^3 - 26,250)$$
$$= 3750 N$$

$$\boxed{P_c = 3.75 kN}$$

Shortening of copper rod $= \frac{P_c \times 2000}{1256.63706 \times 75 \times 10^3}$

δL

$$= \frac{3.75 \times 2000}{1256.63706 \times 75 \times 10^3} = \frac{7500}{94247.\ldots \times 10\ldots}$$

$$= \underline{0.0795775 \text{ mm}}$$

$\therefore$ Shortening of cast iron is also be the same.

Both copper rod and cast iron also undergoes same normal compressive strain. $\boxed{\varepsilon_c = \varepsilon_I}$

$\frac{\delta L}{L} =$

Normal Compressive strain $= \dfrac{\delta L}{L}$

$$= \frac{0.0795775}{2000}$$

$$= 3.978875 \times 10^{-5}$$

$$= 39.788 \times 10^{-6}\ \varepsilon$$

$$= 39.788\ \mu\varepsilon$$

∴ Normal stress (Compressive) in copper rod

$$\sigma_c = \frac{P_{cr}}{A_c} = \frac{3{,}750\ N}{1256.6370\ mm^2}$$

$$= 2.984155\ N/mm^2$$

∴ Normal stress (compressive) in cast-iron tube

$$\sigma_{CI} = \frac{P_I}{A_I} = \frac{26{,}250N}{3769.9118} = 6.963028\ N/mm^2.$$