Unit – 1 prismatic bar problem

A prismatic steel bar of length 2 m and Young's modulus 210 GPa is subjected to an axial push of 70 kN (allowable load). If the shortening of the bar is limited to 0.4 mm, find the suitable radius of the bar. Assume a factor of safety of 3. Also find the stress and strain developed.

Given:

L = 2 m = 2000 mm; E = 210 GPa = 210 $\times 10^3$ N/mm^2; P = 70 kN = 70$\times 10^3$ N;

Factor of safety = 3; shortening in length δ = 0.4 mm. Find the radius of the bar.

Solution:

Since the allowable load is 70 kN and the factor of safety is 3, we have to find the radius of the bar such that the bar withstands 210 kN (= allowable load × factor of safety).

$$\delta = \frac{PL}{AE}; \qquad A = \frac{\pi}{4} d^2 = \frac{PL}{\delta E};$$

$$d^2 = \frac{4}{\pi}\frac{PL}{\delta E} = \frac{4}{\pi}\frac{210\times 10^3 \times 2000}{0.4\times 210\times 10^3};$$ diameter $d = 79.788456$ mm. Radius r =39.894228 mm.

$$\text{Normal compressive stress} = \sigma = \frac{P}{A} = \frac{\text{Allowable load}}{\frac{\pi}{4}\times 79.788456^2} = \frac{70\times 10^3}{5000} = 14 \text{ MPa}$$

Normal compressive strain =

$$\varepsilon = \frac{\delta}{L} = \frac{0.4}{2000} = 2\times 10^{-4} = 200\times 10^{-6} = 200 \text{ micro-strains} = 200\mu\varepsilon$$

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Convert 20 MPa into $\frac{\text{N}}{\text{mm}^2}$.

$$1\text{Pa} = 1\frac{\text{N}}{\text{m}^2};$$

$$20 \text{ MPa} = 20\times 10^6 \frac{\text{N}}{\text{m}^2} = 20\times 10^6 \frac{\text{N}}{\text{m}^2}\frac{\text{m}^2}{(1000 \text{ mm})^2} = 20\times 10^6 \frac{\text{N}}{\text{m}^2}\frac{\text{m}^2}{(10^3 \text{ mm})^2}$$

$$20 \text{ MPa} = 20\times 10^6 \frac{\text{N}}{\text{m}^2}\frac{\text{m}^2}{10^6 (\text{mm})^2} = 20\frac{\text{N}}{\text{mm}^2}$$

Hence, $1 \text{ MPa} = 1 \frac{\text{N}}{\text{mm}^2}$; similarly, $1 \text{ GPa} = 10^3 \frac{\text{N}}{\text{mm}^2}$

UNIT – 1.1.2 PRISMATIC BAR PROBLEM

A prismatic steel bar of length 2 m and Young's modulus 210 GPa is subjected to an axial push of 70 kN (allowable load). If the shortening of the bar is limited to 0.4 mm, find the suitable radius of the bar. Assume a factor of safety of 3. Also find the stress and strain developed.

Given:

L = 2 m = 2000 mm; E = 210 GPa = 210×10^3 N/mm^2; P = 70 kN = 70×10^3 N;

Factor of safety = 3; shortening in length $\delta = 0.4$ mm. Find the radius of the bar.

Solution:

Since the allowable load is 70 kN and the factor of safety is 3, we have to find the radius of the bar such that the bar withstands 210 kN (= allowable load × factor of safety).

$$\delta = \frac{PL}{AE}; \qquad A = \frac{\pi}{4}d^2; \qquad \delta = \frac{PL}{\frac{\pi}{4}d^2 E} = \frac{4PL}{\pi d^2 E}$$

$$d^2 = \frac{4}{\pi}\frac{PL}{\delta E} = \frac{4}{\pi}\frac{210\times10^3\times2000}{0.4\times210\times10^3};$$ diameter $d = 79.788456$ mm. Radius r =39.894228 mm.

$$\text{Normal compressive stress} = \sigma = \frac{P}{A} = \frac{\text{Allowable load}}{\frac{\pi}{4}\times79.788456^2} = \frac{70\times10^3}{5000} = 14\ \text{MPa}$$

$$\text{Normal compressive strain} = \varepsilon = \frac{\delta}{L} = \frac{0.4}{2000} = 2\times10^{-4} = 200\times10^{-6} = 200\ \text{micro-strains} = 200\mu\varepsilon$$

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Convert 20 MPa into $\frac{\text{N}}{\text{mm}^2}$.

$$1\text{Pa} = 1\frac{\text{N}}{\text{m}^2};$$

$$20\ \text{MPa} = 20\times10^6\frac{\text{N}}{\text{m}^2} = 20\times10^6\frac{\text{N}}{\text{m}^2}\frac{\text{m}^2}{(1000\ \text{mm})^2} = 20\times10^6\frac{\text{N}}{\text{m}^2}\frac{\text{m}^2}{(10^3\ \text{mm})^2}$$

$$20\ \text{MPa} = 20\times10^6\frac{\text{N}}{\text{m}^2}\frac{\text{m}^2}{10^6(\text{mm})^2} = 20\frac{\text{N}}{\text{mm}^2}$$

Hence, $1\ \text{MPa} = 1\frac{\text{N}}{\text{mm}^2}$; similarly, $1\ \text{GPa} = 10^3\frac{\text{N}}{\text{mm}^2}$

UNIT – 1.1.1 PRISMATIC BAR PROBLEM

A prismatic steel rod of cross-section 20 mm × 20 mm is to carry an axial tensile load of 10 kN. Calculate the stress, strain and shortening in a length of 500 mm. Take Young's modulus E = 207 GPa.

Given:

Cross-sectional area = A = 20 mm × 20 mm = 400 mm^2.

Axial tensile load $P = 10\text{ kN} = 10\times10^3\text{ N}$

$L = 500\text{ mm};\quad E = 207\text{ GPa} = 207\times10^3\dfrac{\text{N}}{\text{mm}^2};$

Normal tensile stress $= \sigma = \dfrac{P}{A} = \dfrac{10\times10^3\text{ N}}{400\text{ mm}^2} = 25\dfrac{\text{N}}{\text{mm}^2} = 25\text{ MPa}$

Increase in length = axial elongation of the bar $= \delta L = \dfrac{PL}{AE} = \dfrac{10\times10^3\times500}{400\times207\times10^3} = 0.060386\text{ mm}$

Normal tensile strain $= \varepsilon = \dfrac{\delta L}{L} = \dfrac{0.060386}{500} = 1.20773\times10^{-4} = 120.773\times10^{-6} = 120.773\mu\varepsilon$

Area of a solid circle $= \pi r^2 = \pi\left(\dfrac{d}{2}\right)^2 = \dfrac{\pi d^2}{4}$
Area of a hollow circle = area of larger circle – area of smaller circle = $= \dfrac{\pi D^2}{4} - \dfrac{\pi d^2}{4} = \dfrac{\pi\left(D^2 - d^2\right)}{4}$
Area moment of inertia of a solid circle $= \dfrac{\pi d^4}{64}$
Area moment of inertia of a hollow circle = area moment of inertia of larger circle –area moment of inertia of smaller circle $= = \dfrac{\pi D^4}{64} - \dfrac{\pi d^4}{64} = \dfrac{\pi\left(D^4 - d^4\right)}{64}$
Area of a rectangle $= bd$
Area moment of inertia of a rectangle $= \dfrac{bd^3}{12}$

Problem :1

A square steel Rod 30mm × 20mm is section is to carry an axial compression load of 150 KN calculate the shortening in a length of 50 mm Also calculate the stress and strain developed in the Bar. Assume Young's modulus of steel as 210GPa

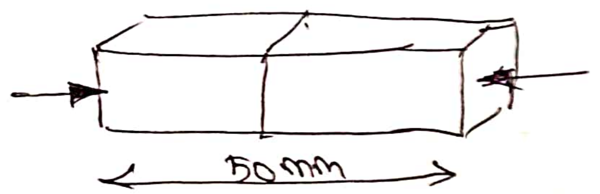
Given Data:

Load $P = 150\,KN = 150\times10^{3}\,N$

Cross sectional area of the bar $= 30\,mm \times 20\,mm$

$A = 600\,mm^2$

Sol



$E = 210\,GPa$

$= 210\times10^{3}\,\frac{N}{mm^2}$

To find Normal stress $\sigma = \frac{P}{A} = \frac{150\times10^{3}\,N}{600\,mm^2} = 250\,\frac{N}{mm^2}$

$\sigma = 250\,MPa$

$\boxed{1\,\frac{N}{mm^2} = 1\,MPa}$

Elongation of the bar $\delta = \frac{PL}{AE}$

$\delta = \frac{150\times10^{3}\times50}{600\times210\times10^{3}} = \frac{7500\times10^{3}}{126000\times10^{3}} = 0.0595$

ormal strain $\varepsilon = \frac{\delta}{L} = \frac{0.0595}{50}$

$= 1.19 \times 10^{-3}$

$$\boxed{\varepsilon = 1.19 \times 10^{-3}}$$

Result:

i) $\sigma = 250\ N/mm^2$

ii) $\delta = 0.0595\ mm$

iii) $\varepsilon = 1.19 \times 10^{-3}$

Problem: 2

A Hollow cast iron cylinder of 4 m long, 300 mm outer diameter and thickness of metal 50 mm is subject to a central load on top when standing straight. The stress produced is 75 000 kN/m². Assume young's modulus of cast iron as 1.5×10^8 kN/m² and find the

(i) magnitude of the load (P)

(ii) Longitudinal strain produced (ε)

(iii) Total decrease in length (δ)

Given Data:-

$L = 4m = 4000$ mm

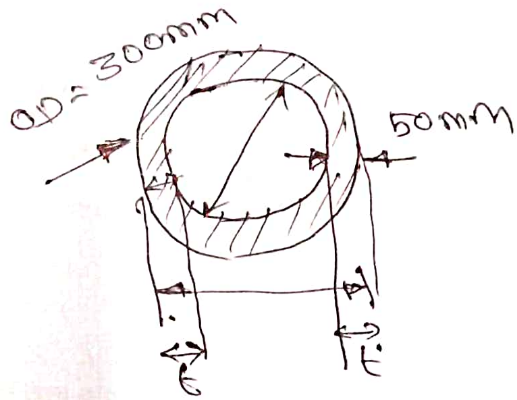
outer dia $= 300$ mm

thickness of tube $= 50$ mm

Inner dia $=$ OD $- 2t$

$= 300 - (2 \times 50)$

ID $= 200$ mm.



To Solve:

$$\left.\begin{array}{l}\text{cross-sectional Area} \\ \text{of hallow bar}\end{array}\right\} = \frac{\pi}{4}(D^2 - d^2)$$

$$A = \frac{\pi}{4}(300^2 - 200^2)$$

$$= \frac{\pi}{4}(50000)$$

$$A = 39269.908 \text{ mm}^2$$

Normal stress developed $= 75000$ kN/m² $= \frac{75000 \times 10^3 \times N}{(1000)^2 \times mm^2}$

$$\Rightarrow 75 \times 10^6 \frac{N}{10^6 \times mm^2} = 75 \text{ N/mm}^2$$

$\sigma = \frac{P}{A} \Rightarrow P = \sigma \times A$

$P = 75 \frac{N}{mm^2} \times 39269.908 \, mm^2$

$= 2945243.1 \, N$

$P = 2945.24 \, kN$

(ii) Normal strain

$\epsilon = \frac{\delta}{L} = \frac{\sigma}{E} = \frac{75}{1.5 \times 10^5} = 5 \times 10^{-4}$

$= 500 \times 10^{-6}$

$= 500 \, \mu\epsilon$

$E = 1.5 \times 10^8 \frac{kN}{m^2}$

$= 1.5 \times 10^5 \, N/mm^2$

(iii)

$\delta = \epsilon L$

$= 500 \times 10^{-6} \times 4 \, m$

$= 2 \times 10^{-3} \, m$

$\delta = 2 \, mm$

Result:

(i) Load (P) $= 2945.24 \, kN$

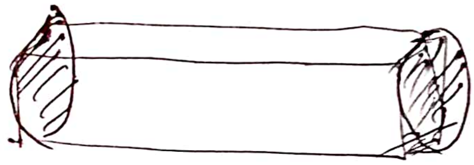
(ii) strain $\epsilon = 500 \, \mu\epsilon$

(iii) decrease in length $\delta = 2 \, mm$.

Problem: 3

A prismatic steel bar of length 2m and young's modulus 210 GPa is subjected to an axial push of 70 KN (allowable load). If the shortening of the bar is limited to 0.4 mm, find the suitable radius of the bar. Assume a factor of safety of 3. Also find the stress and strain developed.

Given Data:-



- $L = 2m$
  $= 2000\ mm$

  $E = 210\ GPa$
  $= 210 \times 10^3\ N/mm^2$

  $P = 70\ kN$
  $= 70 \times 10^3\ N.$

  $Fos = 3$

  $\delta = 0.4\ mm$

- To find the radius of the bar

$$\boxed{\delta = \frac{PL}{AE}}$$

$$A = \frac{PL}{\delta E}$$

$$\frac{\pi}{4} d^2 = \frac{PL}{\delta E}$$

Load = allowable load × Factor of safety
$= 70\ kN \times 3$
$= 210\ kN$
$\boxed{P = 210 \times 10^3\ N}$

$$d^2 = \frac{4PL}{\pi \delta E} = \frac{4 \times 70 \times 10^3 \times 3 \times 2000}{\pi \times 0.4 \times 210 \times 10^3}$$

$d = 79.788456$

$r = 39.89422\ mm$

Stress $\sigma = \frac{P}{A} = \frac{70 \times 10^3}{\frac{\pi}{4} \times d^2} = \frac{70 \times 10^3}{\frac{\pi}{4} \times 79.788^2} = 14 \text{ MPa}.$

$$\boxed{\sigma = 14 \text{ MPa}}$$

(iii) strain:-

$$\varepsilon = \frac{\delta}{L} = \frac{0.4}{2000} = 2 \times 10^{-4} = 200 \times 10^{-6}$$

$$= 200 \, \mu\varepsilon.$$

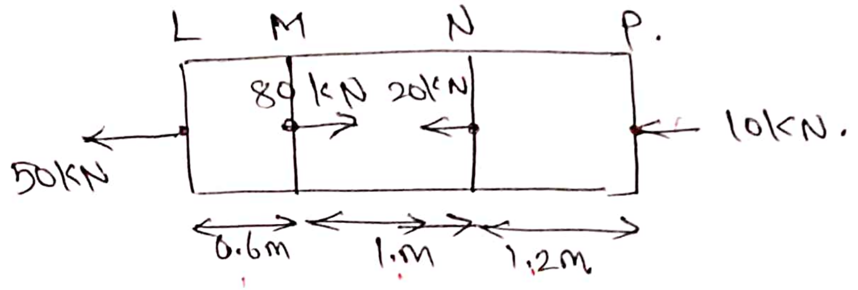
Result:

(i) $r = 39.89422$ mm

(ii) $\sigma = 14$ MPa

(iii) $\varepsilon = 200 \, \mu\varepsilon$

Problem: (4) A brass bar having cross-sectional area of $1000\, mm^2$ is subjected to axial forces.



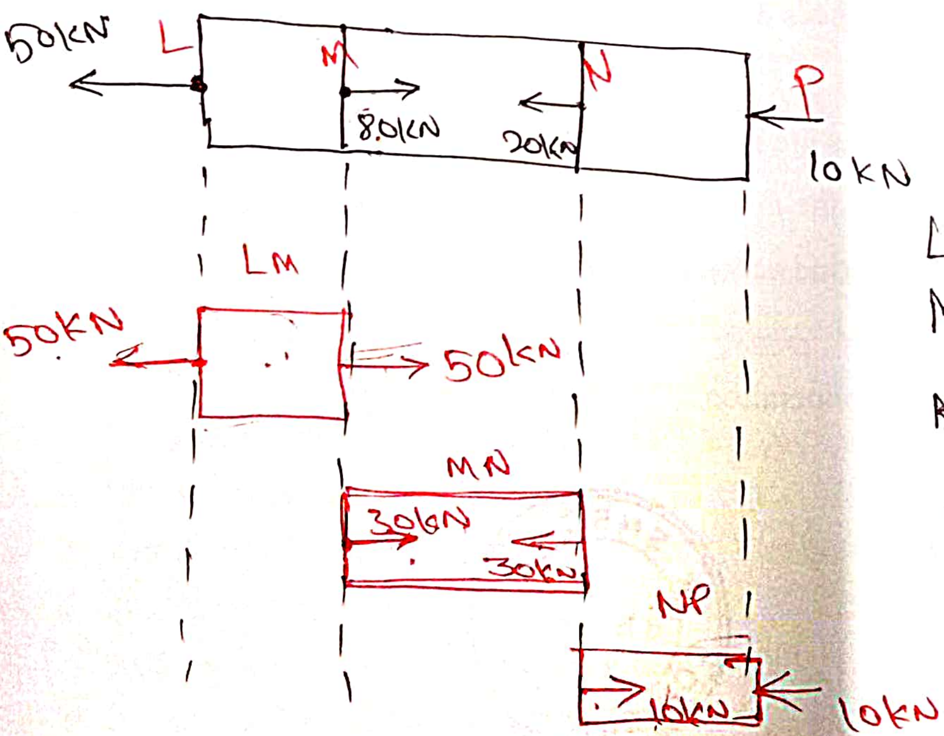
Find the total elongation of the bar. modulus of elasticity of brass $= 100\ GN/m^2$.

Given Data:-

$E = 100\ GN/m^2$

Free body Diagram:-

To Solve



$LM = (80-20-10)$
$= 50$ kN

$MN = (80-50) = 30$ kN.

$\cancel{NP} = (20+10) = 30$ kN

$NP = (80-50-20)$
$= 10$ kN.

Total elongation of the bar

$\delta l_1, \delta l_2, \delta l_3$ be the changes in length of LM, MN, and NP respectively.

(i) Force P Necessary for equilibrium

$$(50\ kN) + (500\ kN) = P + 200\ kN$$

$$550\ kN = P + 200\ kN$$

$$(550 - 200) = P$$

$$\boxed{P = 350\ kN}$$

(ii) Total elongation of the bar.

$$\delta l_1 = \frac{P_1 L_1}{A_1 E} = \frac{50 \times 1000 \times 1}{600 \times 10^{-6} \times 210 \times 10^9}$$

$$= 3.97 \times 10^{-4} m \quad (+)\ \text{Tension}$$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{300 \times 1000 \times 1}{2400 \times 10^{-6} \times 210 \times 10^9}$$

$$= 5.95 \times 10^{-4} m \quad (-)\ \text{Compression}$$

$$\delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{200 \times 1000 \times 0.6}{1200 \times 10^{-6} \times 210 \times 10^9}$$

$$= 4.76 \times 10^{-4} m \quad (+)\ \text{Tension}$$

$$\text{Total } \delta L = \delta l_1 - \delta l_2 + \delta l_3$$

$$= 0.278\ mm.$$

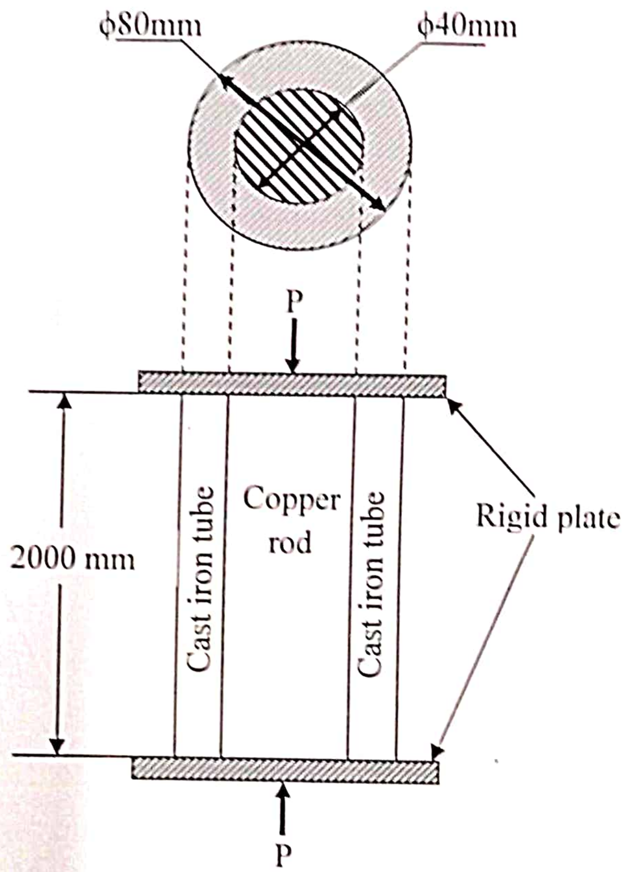
Result

Total elongation of bar = 0.278 mm.

UNIT – 1.6.1 COMPOUND BAR PROBLEM

A copper rod of 40 mm diameter is surrounded *tightly* by a cast iron tube of 80 mm external diameter, the ends being firmly fastened together. When put to a compressive load of 30 kN, what load will be shared by each? Also determine the amount by which the compound bar shortens if it is 2 meters long. Young's modulus of copper and cast-iron are, 75 GPa and 175 GPa, respectively.

"Tightly" means that: external diameter of copper rod = internal diameter of cast iron tube.



Cross-sectional area of copper rod $= \frac{\pi}{4}d^2$

$$A_{copper} = \frac{\pi}{4}\times 40\text{mm}^2 = 1,256.63706 \text{ mm}^2$$

$$A_{CI\ tube} = \frac{\pi}{4}\times\left(80^2 - 40^2\right) = 3,769.91118 \text{ mm}^2$$

The total load of 30 kN is shared by copper rod and cast-iron tube.

$$P = P_{copper} + P_{CI\ tube}$$

$$30\times 10^3 N = P_{copper} + P_{CI\ tube} \quad \text{.................(1)}$$

Equation (1) cannot be solved since there are two unknowns. Hence, the given structure is an indeterminate structure.

We need another equation – called the "compatibility" equation or "compatibility" condition. Compatibility condition states that, in this problem, both the copper rod and the cast iron tube shorten by the same amount, i.e.,

$$\begin{Bmatrix}\text{Shortening of} \\ \text{copper rod}\end{Bmatrix} = \begin{Bmatrix}\text{shortening of} \\ \text{cast-iron tube}\end{Bmatrix}$$

$$\frac{P_{copper}L_{copper}}{A_{copper}E_{copper}} = \frac{P_{CI\ tube}L_{CI\ tube}}{A_{CI\ tube}E_{CI\ tube}}$$

$$\frac{P_{copper}\times 2000}{1,256.63706\times 75\times 10^3} = \frac{P_{CI\ tube}\times 2000}{3,769.91118\times 175\times 10^3}$$

$$P_{copper} = \frac{1,256.63706\times 75\times 10^3}{3,769.91118\times 175\times 10^3}P_{CI\ tube} = 0.142857\ P_{CI\ tube} \quad \text{..................(2)}$$

Substituting equation (2) in equation (1)

$$30\times 10^3 N = 0.142857 P_{CI\ tube} + P_{CI\ tube}$$

$$P_{CI\ tube}\left(1 + 0.142857\right) = 30\times 10^3 N$$

$$P_{CI\ tube} = 26,250 \text{ N} = 26.250 \text{ kN}$$

Then, using equation (1) or equation (2), find P_{copper}.

$P_{copper} = 30\times10^3 - 26,250 = 3,750 \text{ N} = 3.750 \text{ kN}$

Shortening of copper rod $= \dfrac{P_{copper}\times2000}{1,256.63706\times75\times10^3} = 0.0795775$ mm.

Shortening of cast iron tube is also the same.

Both copper rod and cast-iron tube also undergo the same normal compressive strain.

$\varepsilon_{Cu} = \varepsilon_{CI}$

Normal Compressive strain =

$$\frac{\text{decrease in length}}{\text{initial length}} = \frac{0.0795775}{2000} = 3.978875\times10^{-5} = 39.78875\times10^{-6}\varepsilon = 39.78875\ \mu\varepsilon$$

Normal stress (compressive) in copper rod =

$$\sigma_{copper} = \frac{P_{copper}}{A_{copper}} = \frac{3,750N}{1,256.63706mm^2} = 2.984155\frac{\text{N}}{\text{mm}^2} = 2.984155 \text{ MPa}(C)$$

Normal stress (compressive) in cast-iron tube =

$$\sigma_{\text{CI tube}} = \frac{P_{\text{CI tube}}}{A_{\text{CI tube}}} = \frac{26,250 \text{ N}}{3,769.91118 \text{ mm}^2} = 6.963028\frac{\text{N}}{\text{mm}^2} = 6.963028 \text{ MPa}(C)$$