19MAT201

TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT - I - FOURIER SERIES

BASIC FORMULAS:

(1) $\frac{d}{dx}(x^n) = nx^{n-1}$

(2) $\frac{d}{dx}(e^{ax}) = ae^{ax}$

(3) $\frac{d}{dx}(\sin ax) = a\cos ax$

(4) $\frac{d}{dx}(\cos ax) = -a\sin ax$

(5) $\int \sin ax\, dx = -\frac{\cos ax}{a} + c$

(6) $\int \cos ax\, dx = \frac{\sin ax}{a} + c$

(7) $\int e^{ax}\, dx = \frac{e^{ax}}{a} + c$

(8) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$

(9) $\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

(10) $\int \frac{dx}{x} = \log x + c$.

Bernoulli's formula:

* $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \cdots\cdots$

* $\int e^{ax}\sin bx\,dx = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

* $\int e^{ax}\cos bx\,dx = \dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

* $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

* $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

* $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

* $\cos(A+B) - \cos(A-B) = -2\sin A\sin B$

* $\sin 2x = 2\sin x\cos x$

* $\sin^2 x = \dfrac{1-\cos 2x}{2}$

* $\cos^2 x = \dfrac{1+\cos 2x}{2}$

* $\sin n\pi = 0$ ~~for all values of n~~, if n is an integer
  $\sin\pi = \sin 2\pi = \sin 3\pi = \cdots\cdots = \sin n\pi = 0$

* $\cos n\pi = (-1)^n$, n is an integer
  $\cos\pi = \cos 3\pi = \cos 5\pi = \cos 7\pi = \cdots = (-1)$
  $\cos 2\pi = \cos 4\pi = \cos 6\pi = \cdots\cdots = 1$

* $\sin(-\theta) = -\sin\theta$
  $\cos(-\theta) = \cos\theta$

Problems based on Bernoulli's formula :

(1) Evaluate $\int x^2 \sin x \, dx$

Soln: $I = \int uv \, dx = uv_1 - u'v_2 + u''v_3 - \cdots$

$u = x^2 \qquad v = \sin x$

$u' = 2x \qquad v_1 = -\cos x$

$u'' = 2 \qquad v_2 = -\sin x$

$u''' = 0 \qquad v_3 = \cos x$

$\therefore I = [-x^2 \cos x + 2x \sin x + 2\cos x] + c$

(2) Evaluate $\int_0^{2\pi} x^2 \cos nx \, dx$

Soln:

$u = x^2 \quad , \quad v = \cos nx$

$u' = 2x \quad ; \quad v_1 = \frac{\sin nx}{n}$

$u'' = 2 \quad ; \quad v_2 = \frac{-\cos nx}{n^2}$

$u''' = 0 \quad ; \quad v_3 = \frac{-\sin nx}{n^3}$

$$\therefore I = \left[x^2 \frac{\sin nx}{n} + 2x \frac{\cos nx}{n^2} - 2\frac{\sin nx}{n^3} \right]_0^{2\pi}$$

$$= 0 + 4\pi\left(\frac{1}{n^2}\right) - 0 - 0$$

$$\boxed{I = \frac{4\pi}{n^2}}$$

(3) Evaluate $\int_0^{2\pi} x \sin nx \, dx$

Soln:

$u = x \qquad V = \sin nx$

$u' = 1 \qquad V_1 = \frac{-\cos nx}{n}$

$u'' = 0 \qquad V_2 = \frac{-\sin nx}{n^2}$

$$I = \left[-x \frac{\cos nx}{n} + \frac{\sin nx}{n^2} \right]_0^{2\pi}$$

$$\boxed{I = \frac{-2\pi}{n}}$$

(4) Evaluate $\int_{-\pi}^{\pi} x^2 \sin nx \, dx$

Soln:

$u = x^2 \qquad V = \sin nx$

$u' = 2x \qquad V_1 = \frac{-\cos nx}{n}$

$u'' = 2 \qquad V_2 = \frac{-\sin nx}{n^2}$

$u''' = 0 \qquad V_3 = \frac{\cos nx}{n^3}$

$$I = \left[-x^2 \frac{\cos nx}{n} + 2x \frac{\sin nx}{n^2} + 2 \frac{\cos nx}{n^3} \right]_{-\pi}^{\pi}$$

$$= -\pi^2 \frac{(-1)^n}{n} + 0 + 2 \frac{\cos n\pi}{n^3} + \frac{\pi^2 (-1)^n}{n} - 0 - \frac{2\cos n\pi}{n^3}$$

$$\boxed{I = 0}$$

$(\cos(-\theta) = \cos\theta)$

(5) Evaluate $\int_0^{2\pi} (x+x^2)\cos nx\, dx$

Soln:

$u = x+x^2 \qquad v = \cos nx$

$u' = 1+2x \qquad v_1 = \frac{\sin nx}{n}$

$u'' = 2 \qquad v_2 = \frac{-\cos nx}{n^2}$

$u''' = 0 \qquad v_3 = \frac{-\sin nx}{n^3}$

$$I = \left[(x+x^2)\frac{\sin nx}{n} + (1+2x)\frac{\cos nx}{n^2} - 2\frac{\sin nx}{n^3}\right]_0^{2\pi}$$

$$= 0 + (1+2\pi)\frac{1}{n^2} - \frac{1}{n^2}$$

$$\boxed{I = \frac{2\pi}{n^2}}$$

(6) Evaluate $\int_{-\pi}^{\pi} x\cos nx\, dx$

Soln:

$u = x \qquad v = \cos nx$

$u' = 1 \qquad v_1 = \frac{\sin nx}{n}$

$u'' = 0 \qquad v_2 = \frac{-\cos nx}{n^2}$

$$I = \left[x\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right]_{-\pi}^{\pi}$$

$$= 0 + \frac{(-1)^n}{n^2} - 0 - \frac{(-1)^n}{n^2}$$

$$\boxed{I = 0}$$

UNIT - I

FOURIER SERIES

Periodic function:

A function $f(x)$ which satisfies the relation $f(x+T) = f(x)$ for all x and some T is called a periodic function. The smallest positive number T for which the relation holds is called the period of $f(x)$.

Example:

* $\sin x$, $\cos x$ are periodic function with period 2π.
* $\sin nx$ and $\cos nx$ are periodic function with period $\frac{2\pi}{n}$.
* $\tan x$ is a periodic function with period π.

Dirichlet's Conditions:

* $f(x)$ is periodic, single valued and finite.
* $f(x)$ has finite number of finite discontinuities in any one period.
* $f(x)$ has a finite number of maxima ~~and~~ or minima.