Half Range cosine Series and Half range Sine Series

The half range cosine series in the interval $(0, l)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

where $a_0 = \frac{2}{l} \int_0^l f(x) \, dx$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

The half range sine series is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where $b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

① Find half range cosine Series for

$f(x) = x(2-x)$ in $0 \leq x \leq 2$

Soln:
$\qquad l = 2$

Half range cosine Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right) \longrightarrow ①$$

To find a_0:

$$a_0 = \frac{2}{2} \int_0^2 x(2-x)\,dx$$

$$= \int_0^2 (2x - x^2)\,dx$$

$$= \left(\frac{2x^2}{2} - \frac{x^3}{3} \right)_0^2$$

$$\boxed{a_0 = \frac{4}{3}}$$

To find a_n :

$$a_n = \frac{2}{2} \int_0^2 (2x - x^2) \cos\left(\frac{n\pi x}{2}\right) dx$$

$$= \left[(2x - x^2) \sin\left(\frac{n\pi x}{2}\right) \frac{2}{n\pi} + \right.$$

$$\left. (2 - 2x) \frac{4}{\pi^2 n^2} \cos\left(\frac{n\pi x}{2}\right) - 2 \cdot \frac{8}{n^3 \pi^3} \sin\left(\frac{n\pi x}{2}\right) \right]_0^2$$

$$\boxed{a_n = \frac{-8}{n^2 \pi^2} \left[1 + (-1)^n \right]}$$

$$f(x) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{-8}{n^2 \pi^2} \left[1 + (-1)^n \right] \cos\left(\frac{n\pi x}{2}\right)$$

② Find half range Sine Series for

$$f(x) = x(\pi - x) \text{ in } (0, \pi)$$

C   Soln:

$$b_n = \frac{-4}{\pi n^3} \left[(-1)^n - 1 \right]$$

(3) Find half range Cosine Series for $f(x) = l - x$ in $(0, l)$. [RMS VALUE]

Soln:

$$a_0 = l \; , \quad a_n = \frac{2l}{n^2 \pi^2} [1 - (-1)^n]$$

(4) Find half range Cosine series for $f(x) = x \sin x$ in $(0, \pi)$.

Soln:

$$a_0 = 2 \; , \quad a_n = \frac{-2(-1)^n}{n^2 - 1} \; , \quad a_1 = \frac{-1}{4} .$$

(5) Find half range Fourier Sine Series for

$$f(x) = \begin{cases} x & , \; 0 \le x \le l/2 \\ l - x & , \; l/2 \le x \le l. \end{cases}$$

Soln:

$$b_n = \frac{4l}{n^2 \pi^2} \sin\left(\frac{n\pi}{2}\right)$$

(6) Find half range Sine Series for $f(x) = 1 - x$ in $(0, 1)$.

Soln:

$$b_n = \frac{-2}{n\pi}$$

(7) Find the half range Sine Series for $f(x) = x$ in $0 < x < l$. and

Soln:

$$b_n = \frac{-2l}{n\pi} (-1)^n$$

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ROOT MEAN SQUARE VALUE
[RMS VALUE]

The root mean square value or RMS value of $f(x)$ over the interval (a, b) is defined as,

$$RMS = \sqrt{\frac{\int_a^b (f(x))^2 \, dx}{b-a}}$$

① Find RMS value for $f(x) = x^2$ in $(0, 2\pi)$

Soln:

$$RMS = \sqrt{\frac{\int_0^{2\pi} x^4 \, dx}{2\pi}} = \frac{1}{\sqrt{2\pi}} \sqrt{\left(\frac{x^5}{5}\right)_0^{2\pi}}$$

$$= \frac{\sqrt{32\pi^5}}{\sqrt{2\pi}\sqrt{5}}$$

② Find RMS value for $f(x) = x$ in $(-1, 1)$

Soln:

$$RMS = \frac{1}{\sqrt{3}}$$