Fluid Mechanics and Machineries-Flow characteristics: concepts of control volume Application of control volume to continuity equation

(11)

KINEMATICS OF FLOW

Kinematics is defined as that branch of science which deals with motion of particles without considering the force causing the motion.

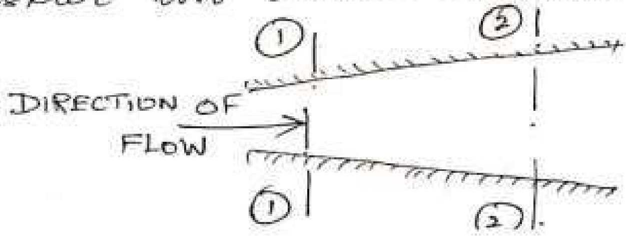
TYPES OF FLUID FLOW & FLOW CHARACTERISTICS :-

(i) Steady and Unsteady flows
(ii) Uniform and non-Uniform flows
(iii) Laminar and turbulent flows
(iv) Compressible and incompressible flows
(v) Rotational and irrotational flows
(vi) One, two and three-dimensional flows.

CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

Consider two cross-section of a pipe



Steady and Unsteady Flows:

Steady flow is defined as that type of flow in which the fluid characteristics like velocity Pressure density etc at a point do not change with time. Thus for steady flow, mathematically we have

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

(ρ = Density)

∂t = change in time.

∂v = change in velocity

∂p = Change in Pressure

$\partial \rho$ = change in density

∂ → partial at a pt.

Where (x_0, y_0, z_0) is a fixed point in fluid field

Unsteady flow is that type of flow, in which the velocity, Pressure or density at a point changes with respect to time, Thus, mathematically, for unsteady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0 z_0} \neq 0 \quad \left(\frac{\partial p}{\partial t}\right)_{x_0 y_0 z_0} \neq 0 \text{ etc}$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0 y_0 z_0} \neq 0$$

Let V_1 = Average Velocity at Cross - Section 1-1

ρ_1 = density at Section 1-1

A_1 = Area of pipe at Section 1-1

and V_2, ρ_2, A_2 are Corresponding values at Section 2-2

Then rate of flow at Section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at Section 2-2 = $\rho_2 A_2 V_2$

According to Law of Conservation of Mass

Rate of flow at section 1-1 = Rate of flow at Section 2-2

or
$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2} \quad \text{—(A)}$$

Equation (A) is applicable to the Compressible as well as incompressible fluids and is Called Continuity equation.

when the fluid is incompressible then $\rho_1 = \rho_2$ and Continuity equation reduces to

$$\boxed{A_1 V_1 = A_2 V_2}$$

UNIFORM and Non-Uniform Flows :

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e) Length of direction of the flow, Mathematically, for uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t=c} = 0$$

where

∂v = Change of Velocity

∂s = Length of flow in the direction s

Non-Uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically for non uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t=c} \neq 0$$

Steady and Unsteady Flows.

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, Pressure, density, etc. at a point do not change with time.

Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where $(x_0\ y_0\ z_0)$ is a fixed point in fluid field

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

unsteady flow is that type of flow, in which the velocity, Pressure or density at a point changes with respect to time.

Thus mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0\ y_0\ z_0} \neq 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0\ y_0\ z_0} \neq 0 \text{ etc}$$

Laminar and Turbulent Flows

Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible.

For high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynolds Number

where D = Diameter of pipe

V = Mean Velocity of flow in pipe

ν = Kinematic viscosity of fluid

when Re < 2000 flow is Laminar & > 4000 is Turbulent

Re < 2000 - 400 > may be Laminar or Turbulent flow

Compressible and Incompressible flows

Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow.

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically for compressible flow

$$\rho = \text{Constant}.$$

Rotational and Irrotational Flows:

Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis, And if the fluid particles while flowing along stream-lines, do not rotate about their own axis then that type of flow is called irrotational flow.

One, Two and Three - Dimensional Flows.

One - Dimensional Flow:

Is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space co-ordinate only.

The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x) \quad v = 0 \quad \text{and} \quad w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus mathematically for two-dimensional flow

$$u = f_1(x, y) \quad v = f_2(x, y) \quad \text{and} \quad w = 0$$

Three dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow

$$u = f_1(x, y, z)$$

$$v = f_2(x, y, z)$$

$$w = f_3(x, y, z)$$