The Drag Polar.

* For every aerodynamic body, there is a relation between C_D and C_L that can be expressed as an equation or plotted on a graph.

* Both the equation and the graph are called drag polar.
* Virtually all the aerodynamic information about an airplane necessary for a performance analysis is wrapped up in the drag polar.

We can write the total drag for an airplane as the following sum:

Total drag = Parasite drag + wave drag + induced drag

$$\therefore C_D = C_{D_e} + C_{D,w} + C_{D_i} \longrightarrow (1)$$

We know $$C_{D_i} = \frac{C_L^2}{\pi e AR}. \longrightarrow (2)$$

C_{De} = Sum of value @ zero lift drag $C_{D,e,0}$ and increment in parasite drag $\Delta C_{D,e}$ due to lift

$\therefore \; C_{D,e} = C_{D,e0} + \Delta C_{De}$

$$C_{D,e} = C_{D,e,0} + K_1 C_L^2 \rightarrow (3)$$

$\therefore C_{De} \propto C_L^2$

$C_{De} = K_1 C_L^2$

Where K_1 is suitable proportionality constant.

Next, for wave drag co-efficient $C_{D,w}$, in a similar fashion, that is $C_{D,w}$ is the sum of zero-lift wave drag coeffient $C_{D,w,0}$ and the change $\Delta C_{D,w}$ due to lift.

$\therefore$ Recalling our discussion of supersonic drag.

$$C_{d,w} = \frac{4\alpha^2}{\sqrt{M_\alpha^2 - 1}} = \frac{4}{\sqrt{M_\alpha^2 - 1}} \left(\frac{C_l \sqrt{M_\alpha^2 - 1}}{4} \right)^2$$

$$C_{d,w} = \frac{C_l^2 \sqrt{M_\alpha^2 - 1}}{4} \rightarrow (4)$$

$\therefore C_{dw}$ is simply the wave drag coefficient due to lift

$\therefore C_{dw} \propto C_l^2$, we are comfortable with the the assumption that $\Delta C_{D,w}$ varies as C_L^2

$$\therefore C_{Dw} = C_{D,w,0} + \Delta C_{D,w}$$

$$= C_{Dw,0} + K_2 C_L^2 \rightarrow (5)$$

$$\therefore C_D = C_{D,e,0} + C_{D,w,0} + K_1 C_L^2 + K_2 C_L^2 + \frac{C_L^2}{\pi e AR} \rightarrow (6)$$

$$\therefore k_3 = \frac{1}{\pi e AR}$$

$$C_D = C_{D,e,0} + C_{D,w,0} + (k_1 + k_2 + k_3) C_L^2 \longrightarrow (7)$$

The Sum of First two terms is simply the zero-lift drag Coefficient $C_{D,0}$.

$$\therefore C_{D,e,0} + C_{D,w,0} = C_{D,0}$$

$$\therefore k_1 + k_2 + k_3 = k$$

$$\therefore C_D = C_{D,0} + k C_L^2 \longrightarrow (8)$$

This above eqn (8) is drag polar for the airplane

$\therefore C_D$ = Total drag coefficient.

$C_{D,0}$ = Zero lift parasite drag Coefficient.

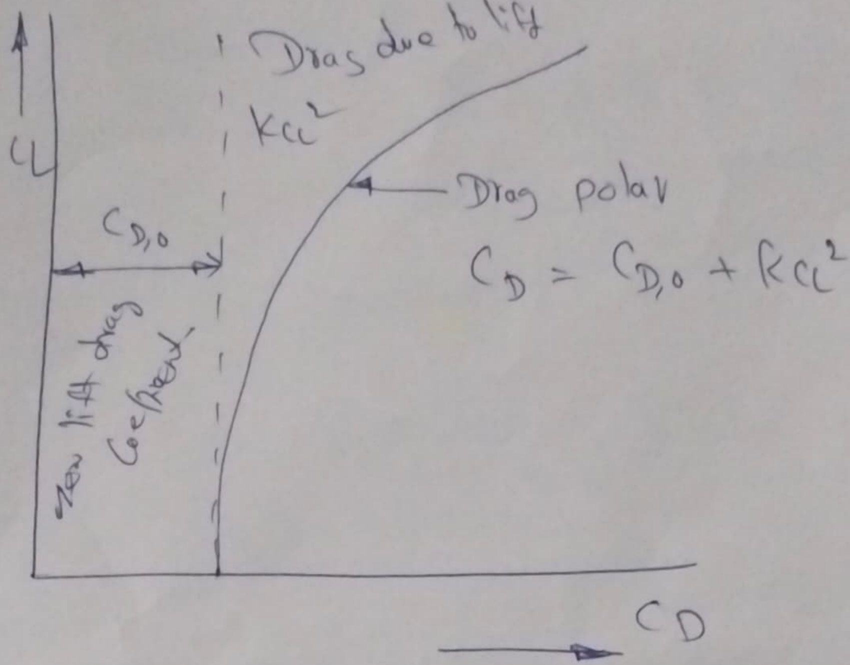
$k C_L^2$ = drag due to lift

This equation is valid for both subsonic and supersonic flight

@ Supersonic Speed $C_{D,0}$ Contains zero wave drag.

for friction, form drags, the effect of wave drag due to lift contained in the value used for k.

$\therefore$ A graph of C_L versus C_D is sketched



This is simply a plot of equation C_D. hence the curve itself is also called ~~plot~~ drag polar.

Another feature of the drag polar diagram, very