Problem (1)

Evaluate by Green's theorem $\int (xy + x^2)\,dx + (x^2+y^2)\,dy$ where c is the square formed by $x=-1, x=1, y=-1$ $y=1$.

Sol

By Green's theorem

$$\int_c M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

Here $M = xy + x^2$

$N = x^2 + y^2$

$$\frac{\partial M}{\partial y} = x \quad , \quad \frac{\partial N}{\partial x} = 2x$$

$$\int_c (xy + x^2)\,dx + (x^2+y^2)\,dy = \iint_R (2x - x)\,dx\,dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} (2x - x)\,dx\,dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} x\,dx\,dy$$

$$= \int_{-1}^{1} \left[\frac{x^2}{2}\right]_{-1}^{1} dy$$

$$= \int_{-1}^{1} \left(\frac{1}{2} - \frac{1}{2}\right) dy$$

$$= 0$$

② Evaluate by Green's theorem in XY plane for $\int_c (3x^2-8y^2)\,dx + (4y-6xy)\,dy$ where c is the boundary of the region defined by $x=y^2,\ y=x^2$

Sol

By Green's theorem.

$$\int_c M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$M = 3x^2-8y^2 \quad , \quad N = 4y-6xy$$

$$\frac{\partial M}{\partial y} = -16y \qquad \frac{\partial N}{\partial x} = -6y$$

$$\int_c (3x^2-8y^2)\,dx + (4y-6xy)\,dy = \int_0^1\int_{y^2}^{\sqrt{y}} (-6y+16y)\,dx\,dy$$

$$= \int_0^1\int_{y^2}^{\sqrt{y}} (10y)\,dx\,dy$$

$$= \int_0^1 \left[10yx\right]_{y^2}^{\sqrt{y}} dy$$

$$= \int_0^1 (10y\sqrt{y} - 10y^3)\,dy$$

$$= \left[\frac{10\,y^{5/2}}{5/2} - \frac{10y^4}{4}\right]_0^1$$

$$= \frac{2\times 10\,(1)^{5/2}}{5} - \frac{10\,(1)^4}{4}$$

$$= \frac{20}{5} - \frac{5}{2}$$

$$= \frac{40-25}{10} = \frac{15}{10}$$

$$= \frac{3}{2}$$

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