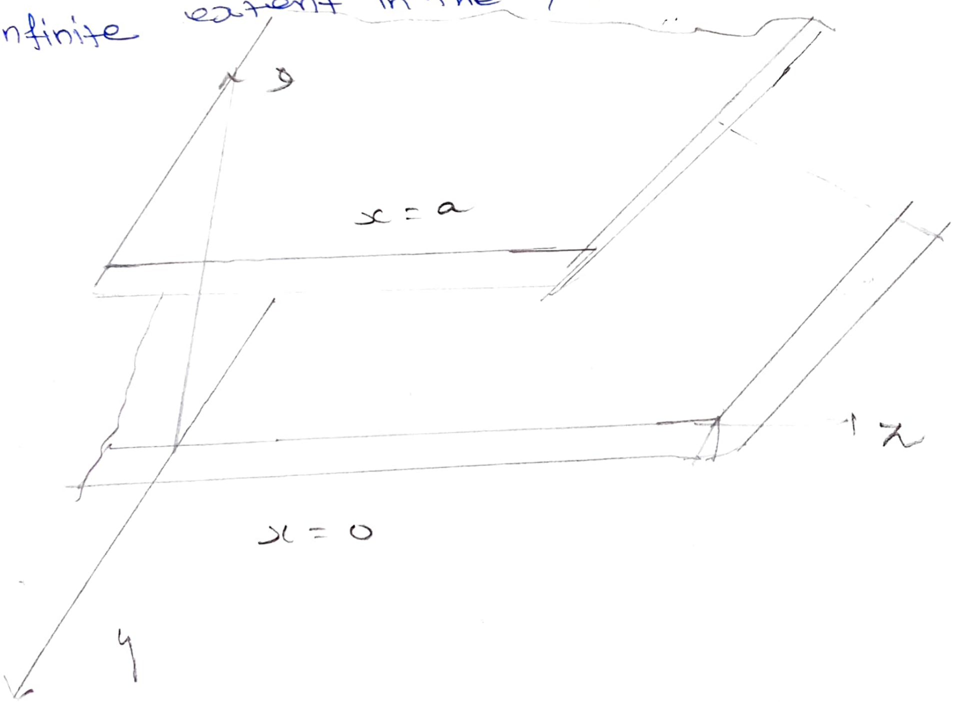
Guided waves: Aim: ~~To~~ study EM waves in wave guides

The EM waves that are guided along or over conducting or dielectric surfaces are called guided waves.

EX: EM waves along ordinary ||el wire
co-axial Txmn. lines
waves in waveguides
waves that are guided along the earth's surface from a radio txr to the rec.

Waves b/w Parallel Plates:

consider an EM wave propagating b/w a pair of parallel perfectly conducting planes of infinite extent in the y and z directions.



Maxwell's equations will be solved to determine the em field configurations in the rect. region.

Maxwell's equation for a non-conducting rect. region are given as

$\nabla \times H = j\omega \varepsilon E$
$\nabla \times E = -j\omega \mu H$

$$\nabla \times H = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \bar{a}_x \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] + \bar{a}_y \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right] + \bar{a}_z$$

$$\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right] \rightarrow (1)$$

$$\nabla \times H = j\omega\varepsilon \left[\bar{a}_x E_x + \bar{a}_y E_y + \bar{a}_z E_z \right] \rightarrow (2)$$

Equating eq. (1) & (2)

$$\left. \begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega\varepsilon E_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega\varepsilon E_z \end{aligned} \right\} \rightarrow (3)$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = \bar{a}_x \left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right]$$

$$+ \bar{a}_y \left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right]$$

$$+ a_y \left[-\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z} \right. \qquad + \bar{a}_z \left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right] \rightarrow (4)$$

$$= -j\omega\mu \left[\bar{a}_x H_x + \bar{a}_y H_y + \bar{a}_z H_z \right] \rightarrow (5)$$

Eq. (4) = (5)

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$\rightarrow$ (6)

The wave equation

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

$$\gamma^2 = [\sigma + j\omega\varepsilon][j\omega\mu]$$

Dielectric [Non-conducting medium]

$$\sigma = 0$$

$$\gamma^2 = j^2\omega^2\mu\varepsilon = -\omega^2\mu\varepsilon$$

$$\nabla^2 E = -\omega^2\mu\varepsilon E$$

$$\nabla^2 H = -\omega^2\mu\varepsilon H$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2\mu\varepsilon E$$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2\mu\varepsilon H$$

$\rightarrow$ (7)

It's assumed that the propagation is in the z-direction. $\rightarrow e^{-\gamma z}$

$\hookrightarrow e^{-j\beta z}$

$\gamma = \alpha + j\beta \quad (\alpha = 0)$

$\gamma = j\beta$

Let $H_y = H_y^0 e^{-\gamma z}$

$$\frac{\partial H_y}{\partial z} = H_y^0(-\gamma)e^{-\gamma z}$$

$$= -\gamma H_y$$

$H_x = H_x^0 e^{-\gamma z}$

$$\frac{\partial H_x}{\partial z} = -\gamma H_x^0 e^{-\gamma z} = -\gamma H_x$$

Let $E_y = E_y^0 e^{-\gamma z}$

$$\frac{\partial E_y}{\partial z} = -\gamma E_y^0 e^{-\gamma z} = -\gamma E_y$$

$E_x = E_x^0 e^{-\gamma z}$

$$\frac{\partial E_x}{\partial z} = -\gamma E_x^0 e^{-\gamma z}$$
$$= -\gamma E_x$$

There is no variation in the Y-direction ∴ derivative is zero.

sub. the values in eq. 3, 4, 6

Eq. 3 becomes,

$$\gamma H_y = j\omega\varepsilon E_x$$

$$\left.\begin{aligned} &\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \\ &-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \\ &\frac{\partial H_y}{\partial x} - 0 = j\omega\varepsilon E_z \rightarrow \frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z \end{aligned}\right\} \rightarrow (8)$$

$$\left.\begin{aligned} &\gamma E_y = -j\omega\mu H_x \\ &-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \\ &\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \end{aligned}\right\} \rightarrow (9)$$

$$\left.\begin{aligned} &\frac{\partial^2 E}{\partial x^2} + \gamma^2 E = -\omega^2\mu\varepsilon E \\ &\frac{\partial^2 H}{\partial x^2} + \gamma^2 H = -\omega^2\mu\varepsilon H \end{aligned}\right\} \rightarrow (10)$$

$$\frac{\partial^2 E}{\partial y^2} = 0 \qquad \frac{\partial^2 H}{\partial y^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} = \gamma^2 E$$

$$\frac{\partial^2 H}{\partial z^2} = \gamma^2 H$$

solve equ. (9) & (8) H_x, H_y, E_y & E_x → found

→ solve H_x:

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\gamma E_y = -j\omega\mu H_x$$

$$H_x = \frac{-\gamma E_y}{j\omega\mu} \rightarrow (11)$$

$$E_y = \frac{-1}{j\omega\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right] \rightarrow (12)$$

sub (12) in (11)

$$H_x = \frac{-\gamma}{j\omega\mu}\left[\frac{-1}{j\omega\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right]\right]$$

$$= \frac{-\gamma}{\omega^2\mu\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right]$$

$$H_x = -\frac{\gamma^2 H_x}{\omega^2\mu\varepsilon} + \frac{\gamma\,\partial H_z}{\omega^2\mu\varepsilon\,\partial x}$$

$$H_x + \frac{\gamma^2 H_x}{\omega^2\mu\varepsilon} = \frac{-\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x\left[1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right] = \frac{-\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x\left[\frac{\omega^2\mu\varepsilon + \gamma^2}{\omega^2\mu\varepsilon}\right] = \frac{-\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{\omega^2\mu\varepsilon} \times \left(\frac{\gamma^2 + \omega^2\mu\varepsilon}{\omega^2\mu\varepsilon}\right)^{-1} \frac{\partial H_z}{\partial x}$$

$$= \frac{-\gamma}{\gamma^2 + \omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (13)$$

$$(h^2 = \gamma^2 + \omega^2\mu\varepsilon)$$

To solve H_y

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\gamma H_y = j\omega\varepsilon E_x$$

$$H_y = \frac{j\omega\varepsilon E_x}{\gamma} \rightarrow (14)$$

$$E_x = \left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]\frac{1}{\gamma} \rightarrow (15)$$

sub (15) in (14)

$$H_y = \frac{j\omega\varepsilon}{\gamma^2}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

$$= \frac{-\omega^2\mu\varepsilon H_y}{\gamma^2} - \frac{j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y + \frac{\omega^2\mu\varepsilon H_y}{\gamma^2} = -\frac{j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y\left[1 + \frac{\omega^2\mu\varepsilon}{\gamma^2}\right] = \frac{-j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y\left[\gamma^2 + \omega^2\mu\varepsilon\right] = -j\omega\varepsilon\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (16)$$

To solve E_x:

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$H_y = \frac{j\omega\varepsilon}{\gamma}E_x \rightarrow (17)$$

$$E_x = \left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]\frac{1}{\gamma} \rightarrow (18)$$

sub. eq. (17) in (18)

$$E_x = \frac{1}{\gamma}\left[j\omega\mu\left[\frac{j\omega\varepsilon}{\gamma}E_x\right] - \frac{\partial E_z}{\partial x}\right]$$

$$E_x = \frac{1}{\gamma}\left[\frac{-\omega^2\mu\varepsilon E_x}{\gamma} - \frac{\partial E_z}{\partial x}\right]$$

$$E_x + \frac{\omega^2\mu\varepsilon E_x}{\gamma^2} = \left[-\frac{\partial E_z}{\partial x}\right]^1/\gamma$$

$$\gamma^2 E_x + \omega^2\mu\varepsilon E_x = \gamma\left[\frac{-\partial E_z}{\partial x}\right]$$

$$E_x\left[\gamma^2+\omega^2\mu\varepsilon\right] = -\gamma\left[\frac{\partial E_z}{\partial x}\right]$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (19)$$

To solve E_y

$$\gamma H_x + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y \rightarrow (21)$$

$$H_x = \frac{-\gamma E_y}{j\omega\mu} \rightarrow (20)$$

sub (20) in (21)

$$\frac{\gamma^2 E_y}{j\omega\mu} + j\omega\varepsilon E_y = \frac{-\partial H_z}{\partial x}$$

$$-\gamma^2 E_y - \omega^2\mu\varepsilon E_y = \left[\frac{-1}{j\omega\mu}\right]^{-1}\frac{\partial H_z}{\partial x}$$

$$= -j\omega\mu\frac{\partial H_z}{\partial x}$$

$$E_y\left[\gamma^2+\omega^2\mu\varepsilon\right] = j\omega\mu\frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (22)$$

The comp. of E & M.F strengths $[E_x, E_y, H_x, H_y]$ are expressed interms of E_z and H_z.

outcome:

Learned about wave propagation in parallel plates.