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## Internal Assessment Examination -I Academic Year 2022-2023(Even) <br> Third Semester <br> 19MAT203 - PROBABILITY AND RANDOM PROCESSES (REGULATION 2019)

TIME: 1 1/2 HOURS
MAXIMUM MARKS: 50
ANSWER ALL QUESTIONS
PART A - ( $5 \times 2=10$ Marks $)$

|  |  |  |  |  |  | CO | BL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | A.R.V. X has the probability <br> Find k and the mean value of | $\begin{array}{\|l\|} \hline \text { anctio } \\ \hline-2 \\ \hline 0.4 \\ \hline \end{array}$ | -1 k | $\begin{array}{\|l\|} \hline 0 \\ \hline 0.2 \end{array}$ | $\begin{array}{l\|} \hline 1 \\ \hline 0.3 \\ \hline \end{array}$ | CO1 | Rem | 2 |
| 2. | A Continuous R.V X that can assume any value between $x=2$ and $x=5$ had the p.d.f $f(x)=k(1+x)$. Find $P(x<4)$. |  |  |  |  | CO1 | App | 2 |
| 3. | If a R.V X has the $\mathrm{MGF}_{\mathrm{X}}(\mathrm{t})=\frac{3}{3-t}$, Obtain the mean and variance of X . |  |  |  |  | CO1 | Und | 2 |
| 4. | The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda=\frac{1}{3}$. What is the probability that the repair time exceeds 3 hours? |  |  |  |  | CO1 | Rem | 2 |
| 5. | The joint p.d.f of a bivariate random variable ( $\mathrm{X}, \mathrm{Y}$ ) is given by$f(x, y)=\left\{\begin{array}{l} k x y, 0<x<1,0<y<1 \\ 0, \\ \text { otherwise } \end{array}\right. \text {, find K. }$ |  |  |  |  | CO2 | App | 2 |

PART B - (13+13+14=40 Marks)

| 6.1 (a) | (i) | A random variable $X$ has the following probability distribution. $\begin{array}{ccccccccc} X & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ P(X) & 0 & k & 2 k & 2 k & 3 k & k^{2} & 2 k^{2} & 7 k^{2}+k \end{array}$ <br> Find <br> (1) The value of $k$ <br> (2) Evaluate $\mathrm{P}(\mathrm{X}<6), \mathrm{P}(0<\mathrm{X}<5)$ <br> (3) The smallest value of a for which $P(X \leq a)>\frac{1}{2}$. <br> (4) The Cumulative distribution function. | CO1 | App | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (ii) | A manufacturer of pins knows that $2 \%$ of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective what is the probability that a box will fail to meet the guaranteed quality? $\left(e^{-2}=\right.$ 0.13534) | CO1 | Ana | 6 |


|  | (b) | (i) | A continuous random variable $f(x)=\left\{\begin{array}{cc}2 x, & 0<x<1 \\ 0, & \text { Otherwise }\end{array}\right.$ is a pdf and find <br> i) $P\left(X<\frac{1}{2}\right)$ <br> ii) $P\left(\frac{1}{4}<X<\frac{1}{2}\right)$ <br> iii) $P\left(X>\frac{3}{4} / X>\frac{1}{2}\right)$. |  |  |  |  |  |  | CO1 | App | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | (ii) | Derive the MGF of Poisson distribution and hence find its mean and variance. |  |  |  |  |  |  | CO1 | Und | 7 |
| 7. | (a) | (i) | A random variable has the p.d.f given by $f(x)=\left\{\begin{array}{cc} 2 e^{-2 x} & x \geq 0 \\ 0 & x<0 \end{array}\right.$ <br> Find a) The moment generating function <br> b) First two moments about the origin. |  |  |  |  |  |  | CO1 | App | 7 |
|  |  | (ii) | The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 70 with a S.D. of Rs.5. Estimate the number of workers whose weekly wages will be (i) between Rs. 69 and Rs. 72 (ii) less than Rs. 69 (iii) more than Rs. 72. |  |  |  |  |  |  | CO1 | Ana | 6 |
| (OR) |  |  |  |  |  |  |  |  |  |  |  |  |
|  | (b) | (i) | The two dimentional random variable (X,Y) has joint probability mass function $f(x, y)=\frac{x+2 y}{27}, x=0,1,2 ; y=$ $0,1,2$. Find the conditional distribution of Y for $\mathrm{X}=\mathrm{x}$. Also find conditional distribution of $Y$ given $X=x$. |  |  |  |  |  |  | CO2 | App | 6 |
|  |  | (ii) | The joint probability function ( $\mathrm{X}, \mathrm{Y}$ ) is given by $P(x, y)=k(2 x+3 y), x=0,1,2 ; y=1,2,3$ <br> i) Find the marginal distributions. <br> ii) Find the probability distributions of $(\mathrm{X}+\mathrm{Y})$ <br> iii) Find all conditional probability distributions. |  |  |  |  |  |  | CO2 | App | 7 |
| 8. | (a) |  | Derive the MGF of Exponential distribution and hence find its mean and variance |  |  |  |  |  |  | CO1 | App | 14 |
|  |  |  | (OR) |  |  |  |  |  |  |  |  |  |
|  | (b) |  | From the follo Find <br> i) $P(X \leq 1)$ iv) $P(X \leq 1 / Y$ <br> vii) Marginal d viii) Condition ix) Estimate X | $\begin{aligned} & \text { ving t } \\ & \text { ii) } P \\ & \text { } P 3 \text { 3 } \\ & \text { stribu } \\ & 1 \text { dist } \\ & \hline \text { Y a } \\ & \hline 1 \\ & \hline 0 \\ & \hline \frac{1}{16} \\ & \hline \frac{1}{32} \\ & \hline \end{aligned}$ | ble for <br> ve $P($ <br> ion fio <br> eutio <br> 2 <br> 0 <br> $\frac{1}{16}$ <br> $\frac{1}{32}$ | bivar $\leq 3$ inion of $X$ 3 $\frac{1}{32}$ $\frac{1}{8}$ $\frac{1}{64}$ | $\begin{aligned} & \text { ate di } \\ & P\left(\begin{array}{l} P \\ \leq 1 \\ \text { of X } \\ \text { iven } \\ \text { t. } \\ \hline 4 \\ \hline 4 \\ \hline \frac{2}{32} \\ \hline \frac{1}{8} \\ \hline \frac{1}{64} \\ \hline \end{array}\right. \end{aligned}$ | $1, Y$ i) $P(X$ 2 5 $\frac{2}{32}$ $\frac{1}{8}$ 0 | $\begin{aligned} & (\mathrm{X}, \mathrm{Y}) . \\ & Y \leq 4) \\ & \hline 6 \\ & \hline \frac{3}{32} \\ & \hline \frac{1}{8} \\ & \hline \frac{2}{64} \\ & \hline \end{aligned}$ | CO 2 | Ana | 14 |

Rem/und: Remember/Understand App:Apply Ana:Analyze Eva: Evaluate

