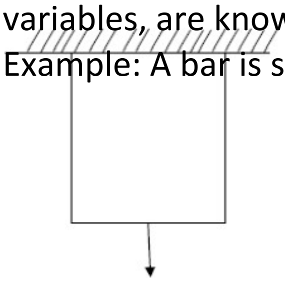
SNS COLLEGE OF TECHNOLOGY

COIMBATORE-35

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT I INTRODUCTION & UNIT II ONE DIMENSIONAL PROBLEMS

16ME401 Finite Element Analysis

1	**Why are polynomial types of interpolation functions preferred over trigonometric functions?**	***(May/June 2013)***
	Polynomial functions are preferred over trigonometric functions due to the following reasons: 1. It is easy to formulate and computerize the finite element equations 2. It is easy to perform differentiation or integration 3. The accuracy of the results can be improved by increasing the order of the polynomial.	
2.	**Distinguish Natural & Essential boundary condition**	**(May/June 2009)**
	There are two types of boundary conditions. They are: **1. Primary boundary condition (or) Essential boundary condition** The boundary condition, which in terms of field variable, is known as primary boundary condition. 2. Secondary boundary condition or natural boundary conditions The boundary conditions, which are in the differential form of field variables, are known as secondary boundary condition. Example: A bar is subjected to axial load shown in fig.  In this problem, displacement u at node 1 = 0, that is primary boundary condition. $AE \frac{du}{dx} = P$, that is secondary boundary condition.	
3.	**What do you mean by Boundary value problem?**	
	The solution of differential equation is obtained for physical problems, which satisfies some specified conditions known as boundary conditions. The differential equation together with these boundary conditions, subjected to a boundary value problem. Examples: Boundary value problem. $\frac{d^2y}{dx^2} - a(x)\frac{dy}{dx} - b(x)\,y = 0$ with boundary conditions, y(m) = S and y(n) = T.	
4.	**What do you mean by weak formulation? State its advantages.** A weak form is a weighted integral statement of a differential equation in which the differentiation is distributed among the dependent variable and the weight function and also includes the natural boundary conditions of the problem. • A much wider choice of trial functions can be used.	***(April/May 2015), (May/June 2013)***

	• The weak form can be developed for any higher order differential equation. • Natural boundary conditions are directly applied in the differential equation. • The trial solution satisfies the essential boundary conditions	
5.	**What is Rayleigh-Ritz method?**	**(NOV/DEC 2015)**
	Rayleigh-Ritz method is a integral approach method which is useful for solving complex structural problems, encountered in finite element analysis. This method is possible only if a suitable functional is available.	
6.	**What is meant by degrees of freedom?** When the force or reactions act at nodal point, node is subjected to deformation. The deformation includes displacement, rotations, and/or strains. These are collectively known as degrees of freedom.	
7.	**What is "Aspect ratio"?**	
	Aspect ratio is defined as the ratio of the largest dimension of the element to the smallest dimension. In many cases, as the aspect ratio increases, the inaccuracy of the solution increases. The conclusion of many researches is that the aspect ratio should be close to unity as possible.	
8.	**What are 'h' and 'p' versions of finite element method?**	
	h' versions and 'p' versions are used to improve the accuracy of the finite element method. In 'h' versions, the order of polynomial approximation for all elements is kept constant and the numbers of elements are increased. In 'p' version, the numbers of elements are maintained constant and the order of polynomial approximation of element is increased.	
9.	**What is Discretization?**	**(NOV/DEC 2015)**
	The art of subdividing a structure into a convenient number of smaller components is known as Discretization.	
10.	**During Discretization, mention the places where it is necessary to place a node?**	
	The following places are necessary to place a node during Discretization process. (i) Concentrated load acting point. (ii) Cross-section changing point. (iii) Different material inter junction point. (iv) Sudden change in load point.	
11.	**What is natural co-ordinate?**	***(Nov/Dec 2014), (April/May 2011)***
	A natural co-ordinate system is used to define any point inside the element by a set of dimensionless numbers, whose magnitude never exceeds unity, this system is useful in assembling of stiffness matrices.	
12.	**Explain force method and stiffness method?** In force method, internal forces are considered as the unknowns of the problem. In displacement or stiffness method, displacements of the	

	nodes are considered as the unknowns of the problem. Among them two approaches, displacement method is desirable.	
13.	**Define shape function. State its characteristics**	*(May/June 2014), (Nov/Dec 2014), (Nov/Dec 2012)*
	In finite element method, field variables within an element are generally expressed by the following approximate relation: $u(x, y) = N_1(x, y)u_1 + N_2(x, y)u_2 + N_3(x, y)u_3$ Where u_1, u_2, u_3 are the values of the field variable at the nodes and N_1, N_2, N_3 are interpolation function. N_1, N_2, N_3 is called shape functions because they are used to express the geometry or shape of the element. The characteristics of the shape functions are follows: 1. The shape function has unit value at one nodal point and zero value at the other nodes. 2.The sum of the shape function is equal to one.	
14.	**How do you calculate the size of the global stiffness matrix?**	
	Global the matrix size=Number of nodes × $\{Degress\ of\ \text{freedom}\}$	
15.	**Give the general expression for element stiffness matrix.**	**(Nov/Dec 2015)**
	stiffness matrix, $[K] = \int_v [B]^T[D][B]dv$ $[B] \Rightarrow$ *strain displacement matrix* [Row matrix] $[D] \Rightarrow$ *stress, strain relationship* matrix [*Row matrix*]	
16.	**Write down the expression of stiffness matrix for one dimensional bar element.**	
	For 1D linear bar element $[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ A-Area of the element mm^2 E-Young's Modulus of the element N/mm^2 L-length of the element	
17.	**State the properties of a stiffness matrix.**	**[AU, Jan 2006]**
	The properties of a stiffness matrix [K] are: 1. It is symmetric matrix. 2. The sum of elements in any column must be equal to zero. 3. It is an unstable element. So, the determinant is equal to zero	
18.	**Write down the general finite element equation**	
	General finite element equation is, $\{F\} = [K]\{u\}$ where, $\{F\} \rightarrow$ Force vector [Column matrix]. $[K] \rightarrow$ Stiffness matrix [Row matrix]. $\{u\} \rightarrow$ Degrees of freedom [Column matrix].	
19.	**Write down the finite element equation for one dimensional two noded bar element.**	
	The finite element equation for one dimensional two noded bar element is, $\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$	

20.	**Define total potential energy.** The total potential energy 1t of an elastic body is defined as the sum of total strain energy U andthe potential energy ofthe external forces, (W). Total potential energy, π = Strain energy (U) + {Potential energy of the external forces (W)}	

1- Solve the following system of equation using gauss elimination method.

$$x_1 - x_2 + x_3 = 1, \quad -3x_1 + 2x_2 - 3x_3 = -6, \; 2x_1 - 5x_2 + 4x_3 = 5$$

In form matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -3 \\ 2 & -5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & -3 & 2 & 3 \end{bmatrix}$$

$\Rightarrow R_2 = [R_2 + 3R_1]$

$\Rightarrow R_3 = [R_3 - 2R_1]$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 2 & 12 \end{bmatrix}$$

$\Rightarrow R_3 = [R_3 - 3R_2]$

$$2x_3 = 12$$
$$-x_2 = -3$$
$$x_1 - x_2 + x_3 = 1$$

Re *sult*:

$x_3 = 6, \; x_2 = 3, \; x_1 = -2$

It is a method which is easily adapted to the computer and is based on triangularization of the coefficient matrix and evaluation of the unknowns by back- substitution starting from the last equation

2- Describe the step by step procedure of solving FEA.

GeneralStepstobefollowedwhilesolvingastructuralproblembyusingFEM:
1.Discretize and select the element type
2.Choose a displacement function
3.Define the strain/displacement and stress/ strain relationships
4.Derive the element stiffness matrix and equations by using direct or variational or Galerkin's approach
5.Assemble the element equations to obtain the global equations and introduce boundary conditions
6.Solve for the unknown degrees of freedom or generalized displacements
7.Solve for the element strains and stresses
8.Interpret the results

Step 1. Discretize and Select element type

- Dividing the body in to an equivalent system of finite elements with associated nodes
- Choose the most appropriate element type
- Decide what number, size and the arrangement of the elements
- The elements must be made small enough to give us able results and yet large enough to reduce computation effort

Step 2. Selection of the displacement function

- Choose displacement function within the element using nodal values of the element
- *Linear, quadratic, cubic polynomials can be used*
- The same displacement function can be used repeatedly for each element

Step 3. Define the strain/displacement and stress/strain relationships

- Strain/displacement and stress/strain relationships are necessary for deriving the equations for each element
- In case of 1-D, deformation, say in x-direction is given by, $\varepsilon = \frac{du}{dx}$
- Stress / strain law is , Hooke's law given by, $\sigma_x = E\varepsilon_x$

Step 4. Derive element stiffness matrix and equations

Following methods can be used

- Direct equilibrium method
- Work or energy methods
- Method of weighted residuals such as Galerkin's method

Any one of the above methods will produce the equations to describe the behavior of an element

The equations are written conveniently in matrix form as, $\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

Step 5. Assemble the element equations to obtain the global or total equations and introduce boundary conditions

- The element equations generated in the step 4 can be added together using the method of superposition
- The final assembled or global equations will be of the matrix form $[K]\{u\}=\{F\}$
- Now introduce the boundary conditions or supports or constraints
- Invoking boundary conditions results in a modification of the global equation

Step 6. Solve for the unknown degrees of After introducing boundary conditions, we get a set of simultaneous algebraic equations and these equations can be written in the expanded form as

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \\ \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & ------- & K_{1n} \\ K_{21} & K_{22} & K_{23} & ------- & K_{2n} \\ K_{31} & K_{32} & K_{33} & ------- & K_{3n} \\ \\ \\ K_{n1} & ------------- & & & K_{nn} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \\ \\ d_n \end{Bmatrix}$$

The above equations can be solved for unknown degrees of freedom by using an elimination method such as Gauss 's method or an iteration method such as the Gauss-Seidel method

Step7. Solve for the element strain and stress

Secondary quantities such as strain and stress , moment or shear force can now be obtained

Step 8. Interpret the results

- The final goal is to interpret and analyze the results for use in design / analysis process.
- Determine the locations where large deformations and large stresses occur in the structure
- Now make design and analysis decisions

3- List the advantages, disadvantages and applications of FEM.

Applications of FEM:

- ***Equilibrium problems or time independent problems**.*

e. g. i) To find displacement distribution and stress distribution for a mechanical or thermal loading in solid mechanics. ii) To find pressure, velocity, temperature, and density distributions of equilibrium problems in fluid mechanics.

- ***Eigenvalue problems of solid and fluid mechanics**.*

e. g. i) Determination of natural frequencies and modes of vibration of solids and fluids. ii) Stability of structures and the stability of laminar flows.

***Time-dependent or propagation problems of continuum mechanics**. e.g. This category is composed of the problems that results when the time dimension is added to the problems of the first two categories.*

Engineering applications of the FEM:

- Civil Engineering structures
- Air-craft structures
- Heat transfer
- Geomechanics
- Hydraulic and water resource engineering and hydrodynamics
- Nuclear engineering
- Biomedical Engineering
- Mechanical Design- stress concentration problems, stress analysis of pistons, composite materials, linkages, gears, stability of linkages, gears and machine tools. Cracks and fracture problems under dynamic loads etc

Advantages of Finite Element Method

- *Model irregular shaped bodies quite easily*
- *Can handle general loading/ boundary conditions*
- *Model bodies composed of composite and multiphase materials because the element equations are evaluated individually*
- *Model is easily refined for improved accuracy by varying element size and type*
- *Time dependent and dynamic effects can be included*
- *Can handle a variety nonlinear effects including material behaviour, large deformation, boundary conditions etc.*

Disadvantages:

- *Needs computer programmes and computer facilities*
- *The computations involved are too numerous for hand calculations even when solving very small problems*
- *Computers with large memories are needed to solve large complicated problems*

Computer Programmes for the FEM

1. *Algor*
2. *ANSYS – Engineering Analysis System*
3. *COSMOS/M*
4. *STARDYNE*
5. *IMAGES-3D*
6. *MSC/NASTRAN- NASA Structural Analysis*
7. *SAP90- Structural Analysis Programme*
8. *GT- STRUDL – Structural Design Language*
9. *SAFE- Structural Analysis by Finite Elements*
10. *NISA- Non linear Incremental Structural Analysis etc.*