Calculate the deflection under the load in the statically indeterminate beam in figure.1., and predict the shear force and bending moment distributions.

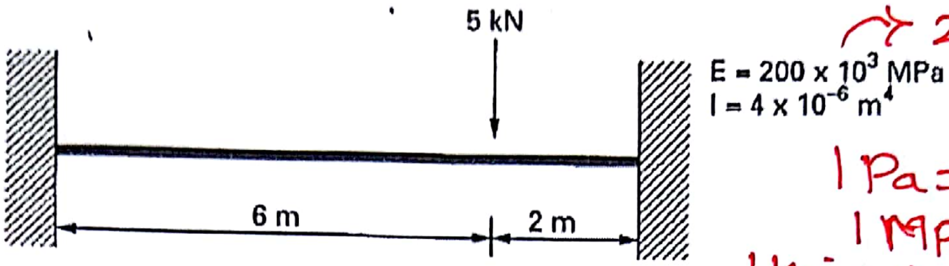


Figure .1.

$\rightarrow 200\times10^3\times10^6\ N/m^2$

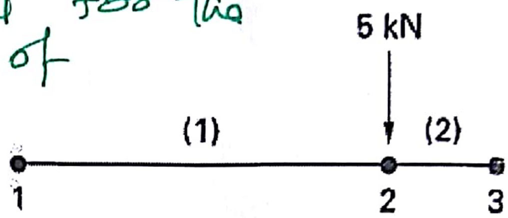
$1\ Pa = 10^{-6}\ N/mm^2$

$1\ MPa = 1\ N/mm^2$

$1\ ksi = 6.895\ N/mm^2$

$1\ bar = 0.1\ N/mm^2$

The finite element model for the beam need only consist of two elements.



Stiffness matrix for Element (1)

$$K^{(1)} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$= \frac{200\times10^3\times10^6\times4\times10^{-6}}{(6)^3}\begin{bmatrix} 12 & 6(6) & -12 & 6(6) \\ 6(6) & 4(6)^2 & -6(6) & 2(6)^2 \\ -12 & -6(6) & 12 & -6(6) \\ 6(6) & 2(6)^2 & -6(6) & 4(6)^2 \end{bmatrix}$$

$$= \frac{0.8\times10^{+6}}{216}\begin{bmatrix} 12 & 36 & -12 & 36 \\ 36 & 144 & -36 & 72 \\ -12 & -36 & 12 & -36 \\ 36 & 72 & -36 & 144 \end{bmatrix}$$

$$= 10^{-6}\begin{bmatrix} 0.044 & 0.133 & -0.044 & 0.133 \\ 0.133 & 0.533 & -0.133 & 0.266 \\ -0.044 & -0.133 & 0.044 & -0.133 \\ 0.133 & 0.266 & -0.133 & 0.533 \end{bmatrix}$$

Stiffness matrix for element (2)

$$= \frac{0.8\times10^{+6}}{2^3}\begin{bmatrix} 12 & 6(2) & -12 & 6(2) \\ 6(2) & 4(2)^2 & -6(2) & 2(2)^2 \\ -12 & -6(2) & 12 & -6(2) \\ 6(2) & 2(2)^2 & -6(2) & 4(2)^2 \end{bmatrix}$$

$$= 10^{+6} \times \left[\tfrac{1}{10}\right]\begin{bmatrix} 12 & 12 & -12 & 12 \\ 12 & 16 & -12 & 8 \\ -12 & -12 & 12 & -12 \\ 12 & 8 & -12 & 16 \end{bmatrix}$$

$$= 10^{+6}\begin{bmatrix} 1.2 & 1.2 & -1.2 & 1.2 \\ 1.2 & 1.6 & -1.2 & 0.8 \\ -1.2 & -1.2 & +1.2 & -1.2 \\ 1.2 & 0.8 & -1.2 & 1.6 \end{bmatrix}$$

Including the load of 5kN applied at node 2, the final set of system equation is then

$[K]\{U\} = \{F\}$

$$10^{+6}\begin{bmatrix} 0.044 & 0.133 & -0.044 & 0.133 & 0.0 & 0.0 \\ 0.133 & 0.533 & -0.133 & 0.266 & 0.0 & 0.0 \\ -0.044 & -0.133 & 1.244 & 1.067 & -1.2 & 1.2 \\ 0.133 & 0.266 & 1.067 & 2.133 & -1.2 & 0.8 \\ 0.0 & 0.0 & -1.2 & -1.2 & 1.2 & -1.2 \\ 0.0 & 0.6 & 1.2 & 0.8 & -1.2 & \underset{1.6}{0.8} \end{bmatrix} \times \begin{bmatrix} \cancel{v_1} \to 0 \\ \cancel{\theta_1} \to 0 \\ v_2 \\ \theta_2 \\ \cancel{v_3} \to 0 \\ \cancel{\theta_3} \to 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -5\times10^3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

However, before the equation can be solved, the constant conditions must be applied, namely $v_1 = \theta_1 = v_3 = \theta_3 = 0$

These result in reduced set of equation

$$10^{+6} \times \begin{bmatrix} 1.244 & 1.067 \\ 1.067 & 2.133 \end{bmatrix} \times \begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \begin{Bmatrix} -5\times10^3 \\ 0 \end{Bmatrix}$$

$$1.244\times10^{+6}\, v_2 + 1.067\times10^{+6}\,\theta_2 = -5\times10^3 \quad \text{— (1)}$$

$$1.067\times10^{+6}\, v_2 + 2.133\times10^{+6}\,\theta_2 = 0 \quad \text{— (2)}$$

Solve

$v_2 = -0.007037$ m $\qquad \theta_2 = +0.00352$ rad

$$
\begin{Bmatrix} F_1 \\ m_1 \\ F_2 \\ m_2 \\ F_3 \\ m_3 \end{Bmatrix} =
\begin{array}{c|cccccc|}
 & v_1 & \theta_1 & v_2 & \theta_2 & v_3 & \theta_3 \\
v_1 & 44{,}444.4 & 133{,}333.2 & -44{,}444.4 & 133{,}333.2 & 0 & 0 \\
\theta_1 & 133{,}333.2 & 533{,}332.8 & -133{,}333.2 & 266{,}666.4 & 0 & 0 \\
v_2 & -44{,}444.4 & -133{,}333.2 & \boxed{\begin{matrix}44{,}444.4\\+1200{,}000\end{matrix}} & \boxed{\begin{matrix}-133{,}333.2\\1200{,}000\end{matrix}} & -1200{,}000 & 1200{,}000 \\
\theta_2 & 133{,}333.2 & 266{,}666.4 & \boxed{\begin{matrix}-133{,}333.2\\1200{,}000\end{matrix}} & \boxed{\begin{matrix}533{,}332.8\\1600{,}000\end{matrix}} & -1200{,}000 & 800{,}000 \\
v_3 & 0 & 0 & -1200{,}000 & -1200{,}000 & 1200{,}000 & -1200{,}000 \\
\theta_3 & 0 & 0 & 1200{,}010 & 800{,}000 & -1200{,}000 & 1600{,}000
\end{array}
\begin{Bmatrix} v_1 = 0 \\ \theta_1 = 0 \\ v_2 = -0.007031 \\ \theta_2 = 3.51563\times10^{-3} \\ v_3 = 0 \\ \theta_3 = 0 \end{Bmatrix}
$$

$$-5000 = 1244{,}444.4\, V_2 + 1066{,}666.8\, \theta_2$$

$$0 = 1066{,}666.8\, V_2 + 2133{,}332.8\, \theta_2$$

$\cancel{V_2 = +0.00703\ m}$

$\cancel{\theta_2 = -0.00352\ rad}$

$$\boxed{\begin{aligned} V_2 &= -0.007031\ m \\ \theta_2 &= 3.5156\times10^{-3}\ rad \end{aligned}}$$

~~F_1 = 781.776996 N~~

~~m_1 = -1875.99824 N-m~~

$F_1 = 779.9\ N$

$m_1 = 1874.966.$

$F_3 = 4218.444\ N.$

$m_3 = -5624.696\ N\text{-}m.$

$M_A = \frac{Pab^2}{l^2} = 1875$

$M_B = \frac{Pa^2b}{l^2} = 5625$