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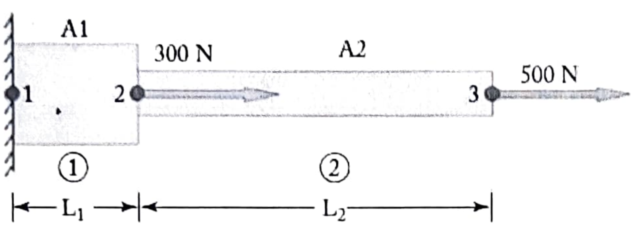
DEPARTMENT OF MECHANICAL ENGINEERING

16ME401 Finite Element Analysis

UNIT II One Dimensional Problems

A compound axial member is subjected to the loads shown in Fig. Given,



Youngs modulus $E_1 = 50MN/m^2, E_2 = 100MN/m^2$,

$E_1 = 50 \times 10^6 \, N/m^2$

$E_2 = 100 \times 10^6 \, N/m^2$

Length $L_1 = 0.5m, L_2 = 1m$, *Area* $A_1 = 20cm^2, A_2 = 10cm^2$

$A_1 = 0.002 \, m^2$

$A_2 = 0.001 \, m^2$

Figure compound axial member subjected to loads.

Calculate the following: (i) Nodal displacement (ii) Element strain (iii) Element stresses (iv) Support reaction, (v) Compare with exact solution, Using two bar elements model.

FEA Model

Solution:

Number of elements = 2; and number of nodes = 3.

The stiffness matrix of each element is computed from

$$[K]^{(e)} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

A - Area of the Element
E - Youngs modulus of the Element
L - Length of the Element

For element (1), the stiffness matrix is

$$[K]^{(1)} = \frac{A_1 E_1}{L_1} \Rightarrow$$

$$[K]^{(1)} = \frac{0.002 \times (50 \times 10^6)}{0.5}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(1)} = 2 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

$$[K]^{(1)} = 10^5 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

Node (displacement functions)

①

For element (2), the stiffness matrix is

$$[K]^{(2)} = \frac{A_2 E_2}{L_2} \Rightarrow$$

$$[K]^{(2)} = \frac{0.001 \times (100 \times 10^6)}{1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(2)} = 1 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

$$[K]^{(2)} = 10^5 \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{cc} 2 & 3 \\ \left[\begin{matrix} 1 & -1 \\ -1 & 1 \end{matrix}\right. & \left.\begin{matrix} \\ \end{matrix}\right] \end{array} \begin{array}{c} \\ 2 \\ 3 \end{array} \Leftarrow$$

The final global matrices of stiffness, displacement, and load matrix, are obtained by combining the element matrices as

$$[K]^G = 10^5 \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}; \{u\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}; \{F\} = \begin{Bmatrix} 0 \\ 300 \\ 500 \end{Bmatrix}.$$

$u_1 = 0$

Applying the fixed boundary conditions at node 1 and applying the external forces at nodes 2 and 3, we have

$$10^5 \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 300 \\ 500 \end{Bmatrix}.$$

Using the matrix partitioning to solve for u2 and u3, we have

$$10^5 \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 300 \\ 500 \end{Bmatrix}$$

$\Rightarrow 10^5(3u_2 - u_3) = 300 \to (1)$

$10^5(-u_2 + u_3) = 500 \to (2)$

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0.004 \\ 0.009 \end{Bmatrix} m.$$

Solve eqn (1) → (2) by gaussian elimination Approach.

(i) Nodal displacement

$u_2 = 0.004\,m \quad u_3 = 0.009\,m$

(ii) Nodal Element strain

Element (1) $\varepsilon^{(1)} = \frac{u_2 - u_1}{L_1} = \frac{0.004 - 0}{0.5} = 0.008$

Element (2) $\varepsilon^{(2)} = \frac{u_3 - u_2}{L_2} = \frac{0.009 - 0.004}{1} = 0.005$

(iii) Element stresses [Found by Hooke's law]

$\sigma^{(1)} = E_1 \times \varepsilon^{(1)} = 50 \times 10^6 \times 0.008 = 4 \times 10^5 \, N/m^2$

$\sigma^{(2)} = E_2 \times \varepsilon^{(2)} = 100 \times 10^6 \times 0.005 = 5 \times 10^5 \, N/m^2$

$\Rightarrow$ Theoretical stress

$\sigma^{(1)} = P_1/A_1 = \frac{800}{0.002} = 4 \times 10^5 \, N/m^2$

$\sigma^{(2)} = P_2/A_2 = 500/0.001 = 5 \times 10^5 \, N/m^2$

System of Equation.

$$10^5 \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.004 \\ 0.009 \end{bmatrix} = \begin{bmatrix} R_1 \\ 300 \\ 500 \end{bmatrix}$$

Using the first row of the global matrix to find R_1, we have

(iv) Support reaction $[R_1]$

$$R_1 = -2 \times 10^5 \times 0.004 = -800 \text{ N}.$$

Check for Equilibrium: Action Forces = - Reaction Forces

$$F_2 + F_3 = 300 + 500 = 800 \text{ N} \qquad R_1 = -800 \text{ N}.$$

Result;

(i) Nodal displacement

$u_1 = 0$ $u_2 = 0.004 m$

$u_3 = 0.009 m$

(ii) Element strain

$\varepsilon^{(1)} = 0.008$.

$\varepsilon^{(2)} = 0.005$

(iii) Element stresses

$\sigma^{(1)} = 4 \times 10^5 \, N/m^2$

$\sigma^{(2)} = 5 \times 10^5 \, N/m^2$

(iv) Support reaction

$R_1 = -800N$

(v) Compare with exact solution

– Verified / Same result obtained.