SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF MECHANICAL ENGINEERING

Finite Element Analysis

IV Year VII Sem

Unit I Introduction

Topic – Rayleigh Ritz method Example Problem-1

SNS *Design Thinkers*

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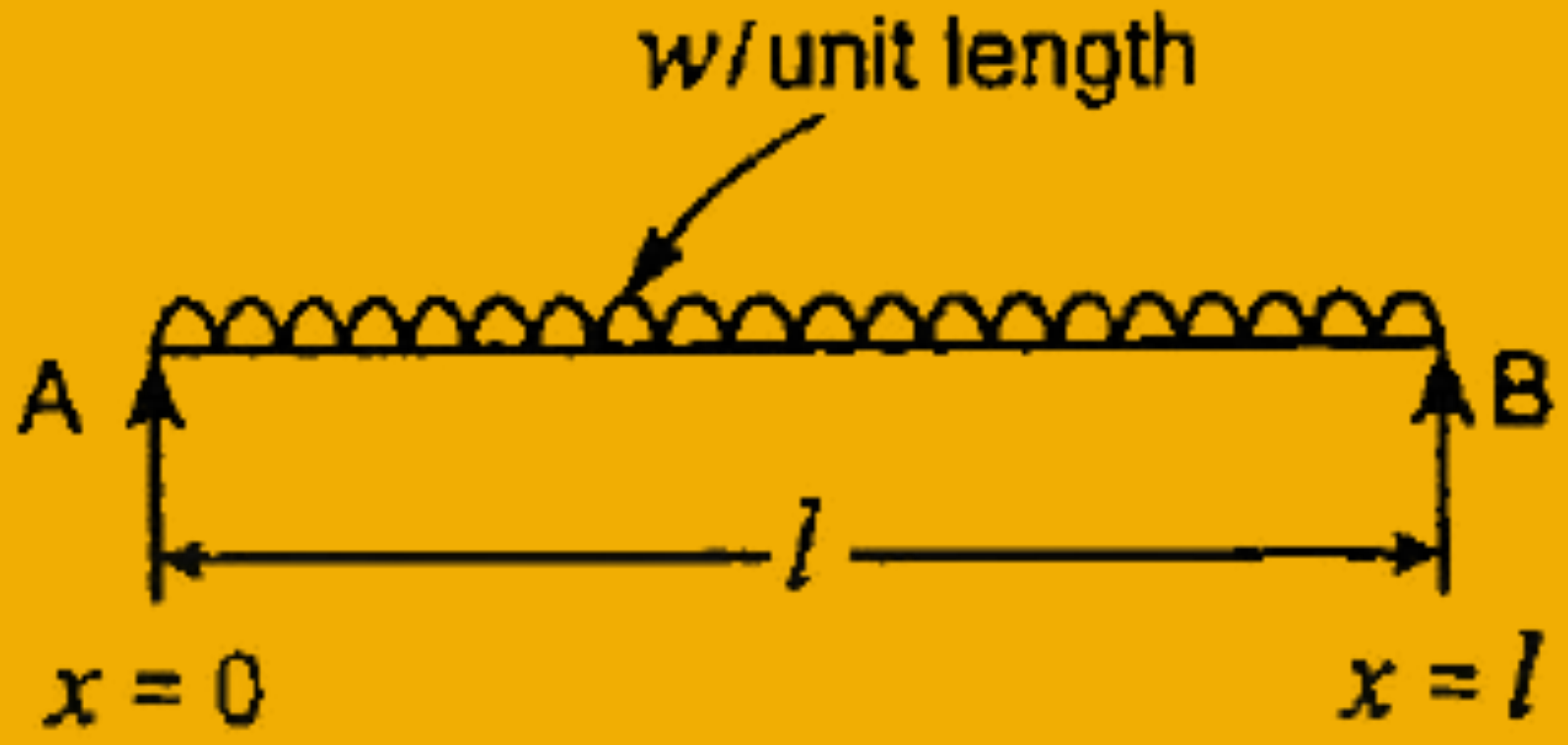
Rayleigh Ritz method Example Problem-1

A simply supported beam subjected to uniformly distributed load over entire span. Determine the bending moment and deflection at midspan by using Rayleigh-Ritz method and compare with exact solution.



To find:

1.Delection and Bending moment at midspan

2. Compare with exact solution

Rayleigh-Ritz Method-Steps

Ritz Steps

Step1

Setting an approximation function

Either polynomial

$$y = a_0 + a_1 x + a_2 x^2 + \cdots$$

Or trigonometric

$$y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l} + \cdots$$

Step:2

Strain Energy (U)

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx$$

Step:3

Work done (H)

$$H = \int_0^l wy\,dx$$

Step:4

Total Energy

$$\pi = U - H$$

Step:5

Finding Rayleigh-Ritz parameter

$$\frac{\partial \pi}{\partial a_1} = 0\ ; \quad \frac{\partial \pi}{\partial a_2} = 0$$

Step:6

Determine the deflection, Bending moment stresses

Rayleigh Ritz method Example Problem-1

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Solution: We know that, for simply supported beam, the Fourier series,

$$y = \sum_{n=1,3}^{\infty} a \sin \frac{n\pi x}{l} \text{ is the approximating function}$$

To make this series more simple let us consider only two terms.

$$\text{Deflection, } y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l} \quad (1)$$

where, a_1, a_2 are Ritz parameters.

$$\text{Deflection, } y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}$$

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we know that,

Total potential energy of the beam, π=U-H (2)

Where, U - Strain energy

H-Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \quad (3)$$

$$y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l}$$

$$\frac{dy}{dx} = a_1 \cos\frac{\pi x}{l} \times \left(\frac{\pi}{l}\right) + a_2 \cos\frac{3\pi x}{l}\left(\frac{3\pi}{l}\right)$$

$$\frac{dy}{dx} = \frac{a_1\pi}{l}\cos\frac{\pi x}{l} + \frac{a_2 3\pi}{l}\cos\frac{3\pi x}{l}$$

Rayleigh Ritz method Example Problem-1

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$$\frac{dy}{dx} = a_1 \cos\frac{\pi x}{l} \times \left(\frac{\pi}{l}\right) + a_2 \cos\frac{3\pi x}{l}\left(\frac{3\pi}{l}\right)$$

$$\frac{dy}{dx} = \frac{a_1 \pi}{l}\cos\frac{\pi x}{l} + \frac{a_2 3\pi}{l}\cos\frac{3\pi x}{l}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-a_1 \pi}{l}\sin\frac{\pi x}{l} \times \frac{\pi}{l} - a_2\frac{3\pi}{l}\sin\frac{3\pi x}{l} \times \frac{3\pi}{l}$$

$$= \frac{-\pi^2 a_1}{l^2}\sin\frac{\pi x}{l} - a_2\frac{9\pi^2}{l^2}\sin\frac{3\pi x}{l}$$

$$\frac{d^2y}{dx^2} = \left[-\frac{a_1 \pi^2}{l^2}\sin\frac{\pi x}{l} - \frac{a_2 9\pi^2}{l^2}\sin\frac{3\pi x}{l}\right] \qquad (4)$$

we know that,

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$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad (3)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-a_1\pi}{l}\sin\frac{\pi x}{l}\times\frac{\pi}{l} - a_2\frac{3\pi}{l}\sin\frac{3\pi x}{l}\times\frac{3\pi}{l}$$

$$= \frac{-\pi^2 a_1}{l^2}\sin\frac{\pi x}{l} - a_2\frac{9\pi^2}{l^2}\sin\frac{3\pi x}{l}$$

$$\frac{d^2y}{dx^2} = \left[-\frac{a_1\pi^2}{l^2}\sin\frac{\pi x}{l} - \frac{a_2 9\pi^2}{l^2}\sin\frac{3\pi x}{l}\right] \qquad (4)$$

Substituting $\frac{d^2y}{dx^2}$ value in equation (3),

Rayleigh Ritz method Example Problem-1

we know that,

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The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2} \int_0^l \left(\frac{d^2y}{dx^2} \right)^2 dx \qquad (3)$$

$$\frac{d^2y}{dx^2} = \left[-\frac{a_1 \pi^2}{l^2} \sin \frac{\pi x}{l} - \frac{a_2 9 \pi^2}{l^2} \sin \frac{3\pi x}{l} \right]$$

Substituting $\frac{d^2y}{dx^2}$ value in equation (3),

$$U = \frac{EI}{2} \int_0^l \left[-\frac{a_1 \pi^2}{l^2} \sin \frac{\pi x}{l} - \frac{a_2 9\pi^2}{l^2} \sin \frac{3\pi x}{l} \right]^2 dx$$

$$= \frac{EI}{2} \int_0^l \left[\frac{a_1 \pi^2}{l^2} \sin \frac{\pi x}{l} + \frac{a_2 9\pi^2}{l^2} \sin \frac{3\pi x}{l} \right]^2 dx$$

$$= \frac{EI}{2} \times \frac{\pi^4}{l^4} \int_0^l \left[a_1 \sin \frac{\pi x}{l} + 9\, a_2 \sin \frac{3\pi x}{l} \right]^2 dx$$

we know that,

Total potential energy of the beam, π=U-H (2)

Where, U - Strain energy

H-Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2} \int_0^l \left(\frac{d^2y}{dx^2} \right)^2 dx \qquad (3)$$

$$= \frac{EI}{2} \times \frac{\pi^4}{l^4} \int_0^l \left[a_1 \sin \frac{\pi x}{l} + 9\, a_2 \sin \frac{3\pi x}{l} \right]^2 dx$$

$$U = \frac{EI}{2} \times \frac{\pi^4}{l^4} \int_0^l \left[a_1^2 \sin^2 \frac{\pi x}{l} + 81\, a_2^2 \sin^2 \frac{3\pi x}{l} + 2\, a_1 \sin \frac{\pi x}{l}\, 9\, a_2 \sin \frac{3\pi x}{l} \right] dx$$

$$[\because (a+b)^2 = a^2 + b^2 + 2\,ab]$$

$$U = \frac{EI}{2} \frac{\pi^4}{l^4} \int_0^l \left[a_1^2 \sin^2 \frac{\pi x}{l} + 81\, a_2^2 \sin^2 \frac{3\pi x}{l} + 18\, a_1\, a_2 \sin \frac{\pi x}{l} \sin \frac{3\pi x}{l} \right] dx \qquad (5)$$

we know that,

Total potential energy of the beam, π=U-H (2)

Where, U - Strain energy

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The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \quad (3)$$

$$U = \frac{EI}{2}\frac{\pi^4}{l^4}\int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l} + 81\, a_2^2 \sin^2\frac{3\pi x}{l} + 18\, a_1 a_2 \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l}\right] dx \quad (5)$$

$$\int_0^l a_1^2 \sin^2\frac{\pi x}{l}\, dx = a_1^2 \int_0^l \frac{1}{2}\left(1 - \cos\frac{2\pi x}{l}\right) dx$$

$$\left[\because \sin^2 x = \frac{1-\cos 2x}{2}\right]$$

$$= \frac{a_1^2}{2}\int_0^l \left(1 - \cos\frac{2\pi x}{l}\right) dx$$

$$= \frac{a_1^2}{2}\left[\int_0^l dx - \int_0^l \cos\frac{2\pi x}{l}\, dx\right]$$

we know that,

Total potential energy of the beam, π=U-H

(2)

Where, U - Strain energy

H-Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad (3)$$

$$= \frac{a_1^2}{2}\left[\int_0^l dx - \int_0^l \cos\frac{2\pi x}{l}\,dx\right]$$

$$= \frac{a_1^2}{2}\left[(x)_0^l - \left(\frac{\sin\frac{2\pi x}{l}}{\frac{2\pi}{l}}\right)_0^l\right]$$

$$= \frac{a_1^2}{2}\left[l - 0 - \frac{l}{2\pi}\left(\sin\frac{2\pi l}{l} - \sin 0\right)\right]$$

$$= \frac{a_1^2}{2}\left[l - \frac{l}{2\pi}(0-0)\right] = \frac{a_1^2 l}{2} \qquad [\because \sin 2\pi = 0;\ \sin 0$$

$$\int_0^l a_1^2 \sin^2\frac{\pi x}{l}\,dx = \frac{a_1^2 l}{2} \qquad (6)$$

we know that,

Total potential energy of the beam, π=U-H (2)

Where, U - Strain energy

H-Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad (3)$$

$$U = \frac{EI}{2}\frac{\pi^4}{l^4}\int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l} + 81\, a_2^2 \sin^2\frac{3\pi x}{l} + 18\, a_1\, a_2 \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l}\right] dx \qquad (5)$$

Similarly

,

$$\left[\because \sin^2 x = \frac{1-\cos 2x}{2}\right]$$

$$\int_0^l 81\, a_2^2 \sin^2\frac{3\pi x}{l} = 81\, a_2^2 \int_0^l \frac{1}{2}\left(1-\cos\frac{6\pi x}{l}\right) dx$$

$$= \frac{81\, a_2^2}{2}\left[\int_0^l dx - \int_0^l \cos\frac{6\pi x}{l}\, dx\right]$$

$$= \frac{81\, a_2^2}{2}\left[(x)_0^l - \left(\frac{\sin\frac{6\pi x}{l}}{\frac{6\pi}{l}}\right)_0^l\right]$$

we know that,

Total potential energy of the beam, π=U-H (2)

Where, U - Strain energy

H-Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad (3)$$

$$U = \frac{EI}{2}\frac{\pi^4}{l^4}\int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l} + 81\, a_2^2 \sin^2\frac{3\pi x}{l} + 18\, a_1 a_2 \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l}\right] dx \qquad (5)$$

$$= \frac{81\, a_2^2}{2}\left[(x)_0^l - \left(\frac{\sin\frac{6\pi x}{l}}{\frac{6\pi}{l}}\right)_0^l \right]$$

$$= \frac{81\, a_2^2}{2}\left[l - 0 - \frac{l}{6\pi}\left(\sin\frac{6\pi l}{l} - \sin 0\right)\right]$$

$$= \frac{81\, a_2^2}{2}\left[l - \frac{l}{6\pi}(\sin 6\pi - \sin 0)\right]$$

$$= \frac{81\, a_2^2}{2}\left[l - 0 \right] \qquad [\because \sin 6\pi = 0;\ \sin 0 = 0]$$

$$\int_0^l 81\, a_2^2 \sin^2\frac{3\pi x}{l}\, dx = \frac{81\, a_2^2\, l}{2} \qquad (7)$$

we know that,

Total potential energy of the beam, π=U-H (2)

Where, U - Strain energy

H-Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad (3)$$

$$\left[\because \sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}\right]$$

$$U = \frac{EI}{2}\frac{\pi^4}{l^4}\int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l} + 81\, a_2^2 \sin^2\frac{3\pi x}{l} + 18\, a_1 a_2 \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l}\right] dx \qquad (5)$$

$$\int_0^l 18\, a_1 a_2 \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l} = 18\, a_1 a_2 \int_0^l \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l}$$

$$= 18\, a_1 a_2 \int_0^l \sin\frac{3\pi x}{l}\sin\frac{\pi x}{l}$$

$$= 18\, a_1 a_2 \int_0^l \frac{1}{2}\left(\cos\frac{2\pi x}{l} - \cos\frac{4\pi x}{l}\right) dx$$

18a1a2

$$= \frac{18 a_1 a_2}{2}\left[\int_0^l \cos\frac{2\pi x}{l}dx - \int_0^l \cos\frac{4\pi x}{l}dx\right]^0$$

$$= \frac{18\, a_1 a_2}{2}\left[\left(\frac{\sin\frac{2\pi x}{l}}{\frac{2\pi}{l}}\right)_0^l - \left(\frac{\sin\frac{4\pi x}{l}}{\frac{4\pi}{l}}\right)_0^l\right]$$

$$= 9\, a_1 a_2 \,[0-0] = 0 \quad [\because \sin 2\pi = 0; \sin 4\pi = 0; \sin 0 = 0]$$

$$\int_0^l 18\, a_1 a_2 \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l} = 0 \qquad (8)$$

we know that,

Total potential energy of the beam, π=U-H (2)

Where, U - Strain energy

H-Work done by external force

The strain energy, U, of the beam due to bending is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad (3)$$

$$U = \frac{EI}{2}\frac{\pi^4}{l^4}\int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l} + 81\, a_2^2 \sin^2\frac{3\pi x}{l} + 18\, a_1 a_2 \sin\frac{\pi x}{l} \sin\frac{3\pi x}{l} \right] dx \qquad (5)$$

Substitute (6), (7) and (8) in equation (5),

$$U = \frac{EI}{2}\frac{\pi^4}{l^4}\left[\frac{a_1^2 l}{2} + \frac{81\, a_2^2 l}{2} + 0 \right]$$

$$U = \frac{EI\,\pi^4 l}{4\, l^4}\left[a_1^2 + 81\, a_2^2 \right]$$

Strain energy,

U

$$= \frac{EI\,\pi^4}{4\, l^3}\left[a_1^2 + 81\, a_2^2 \right] \qquad (9)$$

We know that,

Work done by external force,

$$H = \int_0^l \omega\, y\, dx = \int_0^l \omega \left(a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l} \right) dx$$

$$= \omega \int_0^l \left(a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l} \right) dx$$

$$= \omega \left[a_1 \int_0^l \sin\frac{\pi x}{l}\, dx + a_2 \int_0^l \sin\frac{3\pi x}{l}\, dx \right]$$

$$= \omega \left[a_1 \left(\frac{-\cos\frac{\pi x}{l}}{\frac{\pi}{l}} \right)_0^l + a_2 \left(\frac{-\cos\frac{3\pi x}{l}}{\frac{3\pi}{l}} \right)_0^l \right]$$

We know that,

Work done by external force,

$$H = \int_0^l \omega\, y\, dx = \int_0^l \omega \left(a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l} \right) dx$$

$$= \omega \left[\frac{-a_1 l}{\pi} \left(\cos\frac{\pi x}{l} \right)_0^l - \frac{a_2 l}{3\pi} \left(\cos\frac{3\pi x}{l} \right)_0^l \right]$$

$$= \omega \left[\frac{-a_1 l}{\pi} [(-1) - 1] - \frac{a_2 l}{3\pi} (-1 - 1) \right]$$

$$= \omega \left[\frac{2a_1 l}{\pi} + \frac{2a_2 l}{3\pi} \right] \qquad [\because \cos 0 = 1; \; \cos\pi = -1; \; \cos 3\pi = -1]$$

$$= \frac{2\omega l}{\pi} \left[a_1 + \frac{a_2}{3} \right]$$

$$H = \frac{2\omega l}{\pi} \left(a_1 + \frac{a_2}{3} \right) \qquad (10)$$

We know that,

Work done by external force,

$$H = \int_0^l \omega\, y\, dx = \int_0^l \omega \left(a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l} \right) dx$$

we know that,

Total potential energy of the beam, π=U-H (2)

Substitute (9) and (10) values in equation (2).

$$(2) \Rightarrow \pi = U - H$$

$$\Rightarrow \pi = \frac{E I \pi^4}{4 l^3} (a_1^2 + 81 a_2^2) - \frac{2 \omega l}{\pi} \left(a_1 + \frac{a_2}{3} \right) \quad (11)$$

For stationary value of π, the following conditions must be satisfied.

$$\frac{\partial \pi}{\partial a_1} = 0 \text{ and } \frac{\partial \pi}{\partial a_2} = 0$$

$$\frac{\partial \pi}{\partial a_1} = \frac{E I \pi^4}{4 l^3} (2 a_1) - \frac{2\omega l}{\pi} = 0$$

$$\frac{EI \pi^4}{4 l^3} \times 2 a_1 = \frac{2\omega l}{\pi} \qquad a_1 = \frac{4\omega l^4}{EI\pi^5}$$

Rayleigh Ritz method Example Problem-1

We know that,

Work done by external force,

$$H = \int_0^l \omega\, y\, dx = \int_0^l \omega \left(a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l} \right) dx$$

we know that,

Total potential energy of the beam, π=U-H (2)

$$\pi = \frac{E I \pi^4}{4 l^3} (a_1^2 + 81 a_2^2) - \frac{2 \omega l}{\pi} \left(a_1 + \frac{a_2}{3} \right)$$

Similarly,

$$\frac{\partial \pi}{\partial a_2} = \frac{EI \pi^4}{4 l^3} (162 a_2) - \frac{2\omega l}{\pi} \left(\frac{1}{3} \right) = 0$$

$$\Rightarrow \frac{EI \pi^4}{4 l^3} (162 a_2) = \frac{2\omega l}{\pi} \left(\frac{1}{3} \right)$$

$$\Rightarrow a_2 = \frac{2\omega l}{3\pi} \times \frac{4 l^3}{162 EI \pi^4} = \frac{4 \omega l^4}{243 EI \pi^5}$$

$$\boxed{a_2 = \frac{4\omega l^4}{243 EI \pi^5}}$$

We know that,

Deflection, $y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l}$

where, a_1, a_2 are Ritz parameters.

Substituting a1 and a2, values,

$$y = \frac{4\omega l^4}{EI\,\pi^5}\sin\frac{\pi x}{l} + \frac{4\omega l^4}{243\,EI\,\pi^5}\sin\frac{3\pi x}{l} \quad (12)$$

We know that, maximum deflection occurs at x=l/2

Substitute x=l/2 in equation (12),

$$y_{max} = \frac{4\omega l^4}{EI\,\pi^5}\sin\frac{\pi\times\frac{l}{2}}{l} + \frac{4\omega l^4}{243\,EI\,\pi^5}\sin\frac{3\pi\frac{l}{2}}{l}$$

$$y_{max} = \frac{4\omega l^4}{EI\,\pi^5}\sin\frac{\pi}{2} + \frac{4\omega l^4}{243\,EI\,\pi^5}\sin\frac{3\pi}{2}$$

$$\left[\because \sin\frac{\pi}{2} = 1;\ \sin\frac{3\pi}{2} = -1\right]$$

$$y_{max} = \frac{4\omega l^4}{EI\,\pi^5} - \frac{4\omega l^4}{243\,EI\,\pi^5}$$

$$= \frac{4\omega l^4}{EI\,\pi^5}\left[1 - \frac{1}{243}\right] \qquad \left[\because \sin\frac{\pi}{2} = 1;\ \sin\frac{3\pi}{2} = -1\right]$$

$$= \frac{4\omega l^4}{EI\,\pi^5}\,(0.9958) = \frac{3.98\,\omega\, l^4}{EI\,\pi^5}$$

$$y_{max} = 0.0130\,\frac{\omega\, l^4}{EI} \quad (13)$$

We know that, simply supported beam subjected to uniformly distributed load, maximum deflection is,

$$y_{max} = \frac{5}{384}\,\frac{\omega\, l^4}{EI}$$

$$y_{max} = 0.0130\,\frac{\omega\, l^4}{EI} \quad (14)$$

From equations (13) and (14), we know that, exact solution and solution obtained by using Rayleigh-Ritz method are same.

Bending Moment at Mid span

We know that,

Bending moment,

$$M = EI \frac{d^2y}{dx^2} \quad (15)$$

From equation (4), we know

$$\frac{d^2y}{dx^2} = -\left[\frac{a_1 \pi^2}{l^2} \sin\frac{\pi x}{l} + \frac{a_2 9 \pi^2}{l^2} \sin\frac{3\pi x}{l}\right]$$

Substituting a1 and a2, values,

$$\frac{d^2y}{dx^2} = -\left[\frac{4\omega l^4}{EI \pi^5} \times \frac{\pi^2}{l^2} \sin\frac{\pi x}{l} + \frac{4\omega l^4}{243 EI \pi^5} \times \frac{9\pi^2}{l^2} \sin\frac{3\pi x}{l}\right]$$

Maximum bending occurs at x=l/2

$$\frac{d^2y}{dx^2} = -\left[\frac{4\omega l^4}{EI \pi^5} \times \frac{\pi^2}{l^2} \sin\frac{\pi \frac{l}{2}}{l} + \frac{4\omega l^4}{243 EI \pi^5} \times \frac{9\pi^2}{l^2} \sin\frac{3\pi \frac{l}{2}}{l}\right]$$

$$= -\left[\frac{4\omega l^4}{EI \pi^5} \times \frac{\pi^2}{l^2} \sin\frac{\pi}{2} + \frac{4\omega l^4}{243 EI \pi^5} \times \frac{9\pi^2}{l^2} \sin\frac{3\pi}{2}\right]$$

$$= -\left[\frac{4\omega l^4}{EI \pi^5} \frac{\pi^2}{l^2} (1) + \frac{4\omega l^4}{243 EI \pi^5} \times \frac{9 \pi^2}{l^2} (-1)\right]$$

$$\left[\because \sin\frac{\pi}{2} = 1; \ \sin\frac{3\pi}{2} = -1\right]$$

Bending Moment at Mid span

We know that,

Bending moment,

$$M = EI \frac{d^2y}{dx^2} \quad (15)$$

From equation (4), we know

$$\frac{d^2y}{dx^2} = -\left[\frac{a_1 \pi^2}{l^2} \sin\frac{\pi x}{l} + \frac{a_2 9\pi^2}{l^2} \sin\frac{3\pi x}{l}\right]$$

Substituting a1 and a2, values,

$$= -\left[\frac{4\omega l^2 \pi^2}{EI \pi^5} - \frac{36 \omega l^2 \pi^2}{243 EI \pi^5}\right]$$

$$= -\frac{4\omega l^2}{EI \pi^3} + \frac{36 \omega l^2}{243 EI \pi^3}$$

$$= -\frac{4\omega l^2}{EI \pi^3} + \frac{0.148 \omega l^2}{EI \pi^3} = -3.852 \frac{\omega l^2}{EI \pi^3}$$

$$\frac{d^2y}{dx^2} = -0.124 \frac{\omega l^2}{EI}$$

Rayleigh Ritz method Example Problem-1

Bending moment,

$$M = EI \frac{d^2y}{dx^2} \quad (15)$$

{Negative sign indicates downward load] We know that, for simply supported beam subjected to uniformly distributed load, maximum bending moment is,

$$M_{centre} = \frac{\omega l^2}{8}$$

$$\boxed{M_{centre} = 0.125\, \omega l^2}$$

Substituting $\frac{d^2y}{dx^2}$ value in bending moment equation,

$$(15) \Rightarrow \quad M_{centre} = EI \times -(0.124) \frac{\omega l^2}{EI}$$

$$\Rightarrow \quad \boxed{M_{centre} = -0.124\, \omega l^2} \quad (16)$$

$$M_{centre} = \frac{\omega l^2}{8}$$

$$\boxed{M_{centre} = 0.125\, \omega l^2} \quad (17)$$

From equation (16) and (17), we know that, exact solution and solution obtained by using8 Rayleigh-Ritz method are almost same. In order to get accurate result, more terms in Fourier series should be taken.

Thank you