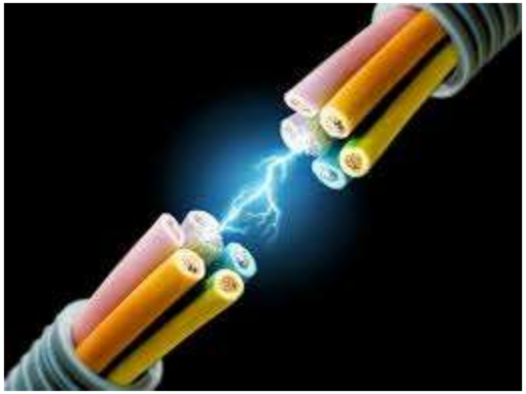
19EEB102 / ELECTRIC CIRCUIT ANALYSIS
I YEAR / II SEMESTER
UNIT-V: TRANSIENT CIRCUITS

RC , RLC TRANSIENT FOR DC INPUT

TOPIC OUTLINE

- RC Transient
- RLC Transient



Follow these basic steps to analyze a circuit using Laplace techniques:

1. Develop the differential equation in the time-domain using Kirchhoff's laws and element equations.

2. Apply the Laplace transformation of the differential equation to put the equation in the *s*-domain.

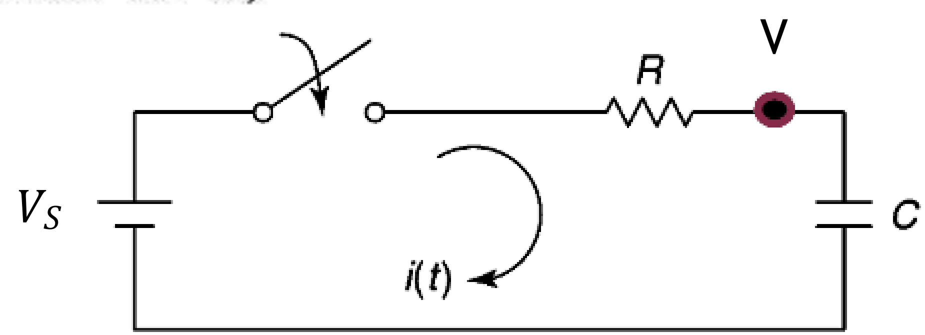
3. Algebraically solve for the solution, or response transform.

4. Apply the inverse Laplace transformation to produce the solution to the original differential equation described in the time-domain.

Transient Analysis of series RC Circuit (Differential Approach)

Consider a series *RC* circuit as shown in Fig. 10.124. The switch is closed at time $t = 0$. The capacitor is initially uncharged. Applying **KCL** to the circuit for $t > 0$,

$$\frac{V - V_s}{R} = -C\frac{dv}{dt}$$

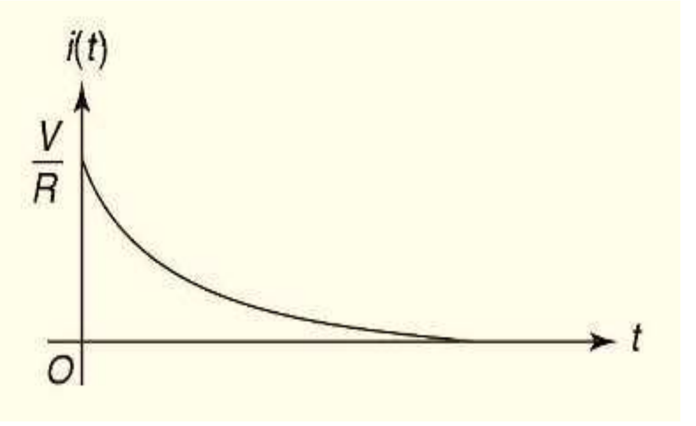


$$\frac{-1}{RC}.dt = \frac{1}{V - V_s}dv$$

Integrating both sides =>

$$\frac{-1}{RC}\int_0^t (1)dt = \int_0^{V(t)} \frac{1}{V - V_s}dv$$

$$\frac{-1}{RC}\left[t-0\right]=\ln\left(V-V_s\right)\Big|_0^{V(t)}$$

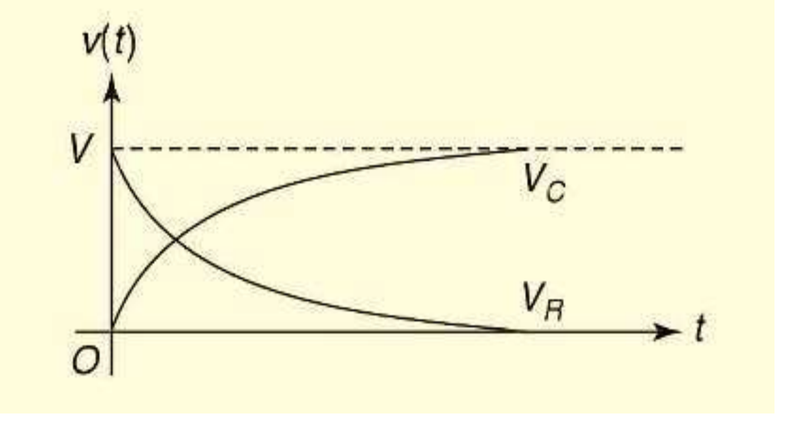


$$\frac{-t}{RC}=\ln[V(t)-V_s]-\ln[-V_s]$$

$$=\ln[\frac{V(t)-V_s}{-V_s}]=\ln[\frac{V_s-V(t)}{V_s}]$$

$$\frac{V_s-V(t)}{V_s}=e^{-t/RC}$$

$$V_s-V(t)=V_s e^{-t/RC}$$



$$V(t)=V_s[1-e^{-t/RC}]$$

$$i=\frac{V_s}{R}e^{-\frac{1}{RC}t}\quad \text{for } t>0$$

Transient Analysis of series RC Circuit (Laplace Approach)

Consider a series *RC* circuit as shown in Fig. 10.124. The switch is closed at time $t = 0$. The capacitor is initially uncharged. Applying KCL to the circuit for $t > 0$,

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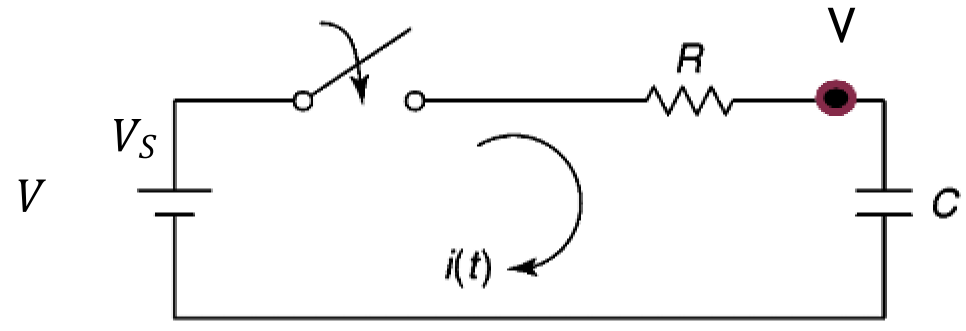

Applying Laplace => $V(S) - \frac{V_S}{S} = -RC[SV(S)]$

$$V(S)[1 + SCR] = \frac{V_S}{S}$$

$$V(S) = \frac{V_S}{S(1+SCR)} = \frac{V_S}{RC}\frac{1}{S(S+\frac{1}{RC})}$$

$$= [\frac{1}{S} - \frac{1}{S+\frac{1}{RC}}]$$

$$= V_S[\frac{1}{S} - \frac{1}{S+\frac{1}{RC}}]$$

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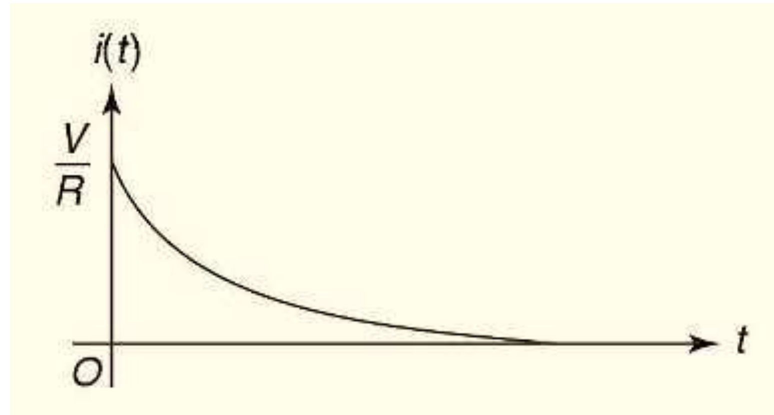
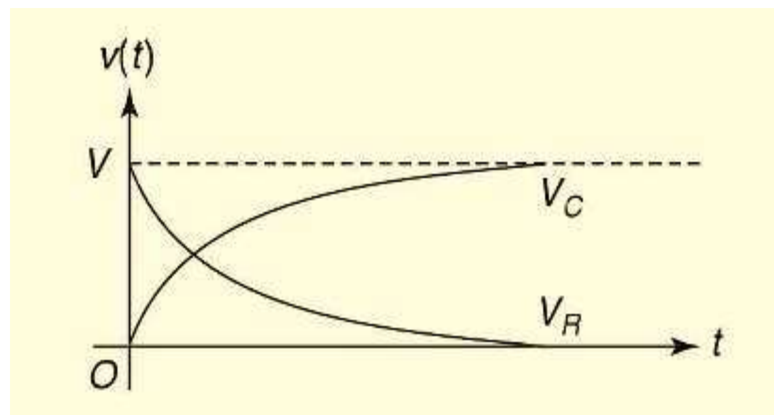




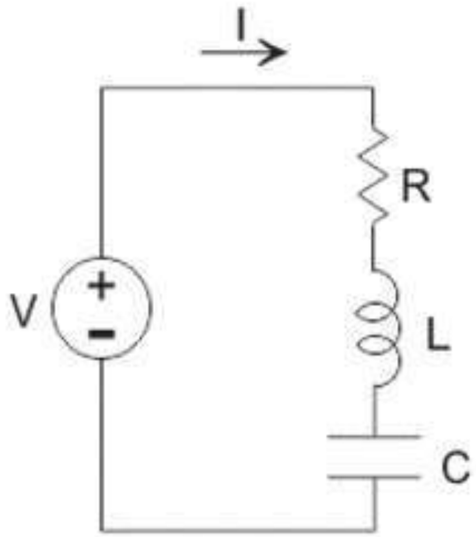
$$V(t) = V_s[1 - e^{-t/RC}]$$

$$i = \frac{V}{R}e^{-\frac{1}{RC}t} \quad \text{for } t > 0$$

Transient Analysis of series RLC Circuit (Differential Approach)

Equation of RLC Circuit

Consider a **RLC circuit** having resistor R, inductor L, and capacitor C connected in series and are driven by a voltage source V. Let Q be the charge on the capacitor and the current flowing in the circuit is I. Apply Kirchhoff's voltage law



$$L.I'(t) + Q.I(t) + \frac{1}{C}Q(t) = V(t)$$

In this equation; resistance, inductance, capacitance and voltage are known quantities but current and charge are unknown quantities. We know that an current is a rate of electric charge flowing, so it is given by

$$\frac{dQ}{dt}(t) = I(t) \; or \; I(t) = Q(t)$$

Differentiating again $I'(t) = Q''(t)$

$$L.Q''(t) + R.Q'(t) + \frac{1}{C} = V(t)$$

Differentiating the above equation with respect to 't' we get,

$$L.I''(t) + Q.I'(t) + \frac{1}{C}I(t) = V'(t)$$

Now at time t = 0, V(0) = 0 and at time t = t, $V(t) = E_0 \sin\omega t$

Differentiating with respect to 't' we get $V'(t) = \omega E_0 \cos\omega t$

Substitute the value of V'(t) in above equation

$$L.I''(t) + R.I'(t) + \frac{1}{C}I(t) = \omega E_o \cos\omega t$$

Let us say that the solution of this equation is $I_P(t) = A\sin(\omega t - \acute{\phi})$ and if $I_P(t)$ is a solution of above equation then it must satisfy this equation,

$$L.I_P(t) + R.I_P(t) + \frac{1}{C}I_P(t) = \omega E_o \cos\omega t$$

$$- L\omega 2A\sin(\omega t - \phi) + R\omega A\cos(\omega t - \phi) + \frac{1}{C}A\sin(\omega t - \phi) = \omega E_o \cos \omega t$$

$$- L\omega 2A\sin(\omega t - \phi) + R\omega A\cos(\omega t - \phi)\frac{1}{C}A\sin(\omega t - \phi) = \omega E_o \cos(\omega t - \phi + \phi)$$

Apply the formula of cos (A + B) and combine similar terms we get,

$$\left(\frac{1}{C} - L\omega^2\right)A\sin(\omega t - \varphi) + R\omega A\cos(\omega t - \phi)$$

$$= \omega E_o \cos\phi \cos(\omega t - \varphi) - \omega E_o \sin\varphi \cos(\omega t - \phi)$$

Match the coefficient of sin(ωt - φ) and cos(ωt - φ) on both sides we get,

$$\left(-\frac{1}{C} + 2L\omega\right)A = \omega E_o \sin\phi \; and \; R\omega A = \omega E_o \cos\phi$$

Now we have two equations and two unknowns i.e φ and A, and by dividing the above two equations we get,

$$\tan\phi = \frac{-\frac{1}{C} + 2L\omega}{R\omega}$$

Squaring and adding above equation, we get

$$A\sqrt{\left(-\frac{1}{C}+2L\omega\right)^2+(R\omega)^2}=\omega\,E_o$$

$$or\ A=\frac{\omega\,E_o}{\sqrt{\left(-\frac{1}{C}+2L\omega\right)^2+(R\omega)^2}}$$

Analysis of RLC Circuit Using Laplace Transformation

Step 1 : Draw a phasor diagram for given circuit.

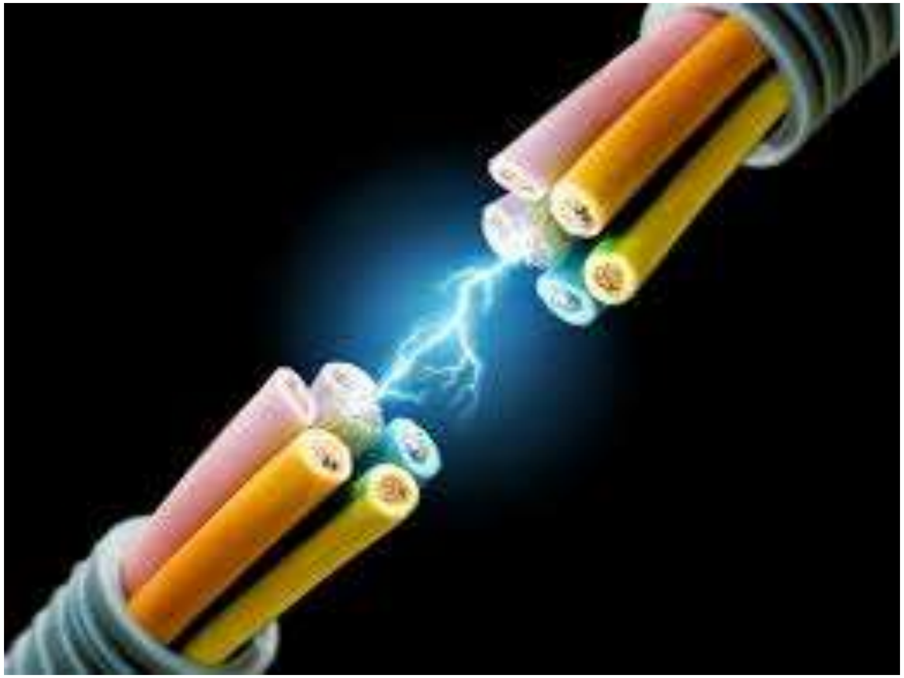
Step 2 : Use Kirchhoff's voltage law in RLC series circuit and current law in RLC parallel circuit to form differential equations in the time-domain.

Step 3 : Use Laplace transformation to convert these differential equations from time-domain into the s-domain.

Step 4 : For finding unknown variables, solve these equations.

Step 5 : Apply inverse Laplace transformation to convert back equations from s-domain into time domain.

RECAP....



...THANK YOU