SOLENOIDAL VECTOR

A vector $\vec{F}$ is said to be solenoidal if $\text{div}.\vec{F} = 0$ or $\nabla.\vec{F} = 0$

Problems:

① . Show that the vector $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$ is solenoidal.

Sol

Given $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$

Solenoidal vector $\nabla.\vec{F} = 0$

$$\nabla\vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)(3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k})$$

$$= \frac{\partial}{\partial x}(3y^4z^2) + \frac{\partial}{\partial y}(4x^3z^2) + \frac{\partial}{\partial z}(-3x^2y^2)$$

$$= 0$$

IRROTATIONAL VECTOR

A vector F is called irrotational if $\text{curl}.\vec{F} = 0$ (or) $\nabla \times \vec{F} = 0$

Problems:

Show that the vector $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational

Sol

Given $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$

T.P $\nabla \times \vec{F} = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 6xy + z^3 & 3x^2 - z & 3xz^2 - y \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(3xz^2 - y) - \frac{\partial}{\partial z}(3x^2 - z)\right] - \vec{j}\left[\frac{\partial}{\partial x}(3xz^2 - y) - \frac{\partial}{\partial z}(6xy + x^3)\right] + \vec{k}\left[\frac{\partial}{\partial x}(3x^2 - z) - \frac{\partial}{\partial y}(6xy + z^3)\right]$$

$$= \vec{i}(-1+1) - \vec{j}(3z^2 - 3z^2) + \vec{k}(6x - 6x)$$

$$= 0$$

) Find the constants a, b, c. ~~so~~ if that the vector is irrotational $\vec{F} = (x+2y+az)\vec{i} + (bx+3y-z)\vec{j} + (4x+cy+2z)\vec{k}$

Sol

Given $\vec{F} = (x+2y+az)\vec{i} + (bx+3y-z)\vec{j} + (4x+cy+2z)\vec{k}$

We know $\nabla \times \vec{F} = 0$

But W.K.T $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+2y+az & bx+3y-z & 4x+cy+2z \end{vmatrix}$$

$$= \vec{i}\left[\partial/\partial y(4x+cy+2z) - \partial/\partial z(bx+3y-z)\right]$$

$$- \vec{j}\left[\frac{\partial}{\partial x}(\right.$$

$$+ \vec{k}$$

$$= \vec{i}(c+1) - \vec{j}(4-a) + \vec{k}(b-2)$$

$$(c+1) = 0 \qquad (4-a) = 0 \qquad b-2 = 0$$

$$a = 4, \ b = 2, \ c = -1 \qquad \because \nabla \times F = 0\vec{i} + 0\vec{j} + 0\vec{k}$$

ANGLE BETWEEN TWO SURFACE

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

CamScanner

Problems:

① Find the angle between the surface $x = y^2 - 1$ and $x^2 y = 2$ at the point $(1, 1, 1)$

Sol

Given $x = y^2 - 1$; $x^2 y = 2$

$\phi_1 = x - y^2 + 1$, $\phi_2 = x^2 y - 2$

$$\nabla \phi_1 = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

$= \vec{i}$

$= \vec{i} - 2y\vec{j} + 0\vec{k}$

$\nabla \phi_1 (1,1,1) = \vec{i} - 2\vec{j}$

$$\nabla \phi_2 = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

$$= \vec{i} \frac{\partial}{\partial x}(x^2 y - 2) + \vec{j} \frac{\partial}{\partial y}(x^2 y - 2) + \vec{k} \frac{\partial}{\partial x}(x^2 y - 2)$$

$= \vec{i}(2xy) + \vec{j}(x^2) + \vec{k}(0)$

$\nabla \phi_2 (1,1,1) = \vec{i}(2.1.1) + \vec{j}(1) + \vec{k}(0)$

$= 2\vec{i} + \vec{j}$

$|\nabla \phi_2| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$$\cos \theta = \frac{\nabla \phi_1 \; \nabla \phi_2}{|\nabla \phi_1| \, |\nabla \phi_2|}$$

$$= \frac{(\vec{i} - 2\vec{j}).(2\vec{i} + \vec{j})}{\sqrt{5}.\sqrt{5}}$$

$$= \frac{2 - 2}{5} = 0$$

$\cos \theta = 0$

$\theta = \cos^{-1}(0)$

$= \pi/2$