change of order of integration

1. Change of order of integration for $\int_0^1 \int_0^x dx\, dy$

Sol

Given integral is not in the correct form

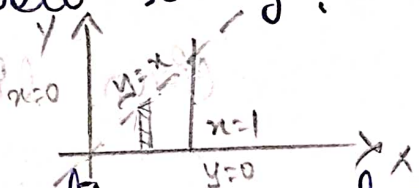
$$\int_0^1 \int_0^x dy\, dx$$

Given limits x limit $x=0$ to $x=1$

y limit $y=0$ to $y=x$

Inner limit is with respect to y.

∴ It is vertical strip

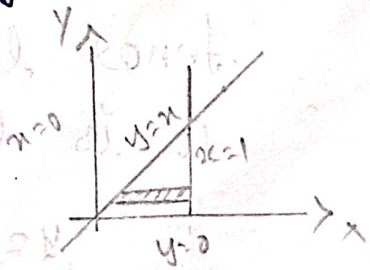


By changing order of integration we have to draw a horizontal strip

x limit $x=y$ to $x=1$

y limit $y=0$ to $y=1$

$$I = \int_0^1 \int_y^1 dx\, dy$$

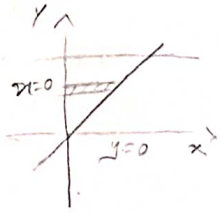


2. Change the order of integration for $\int_0^1 \int_0^y dx\, dy$

Sol

Given integral is in the correct form

$\int_0^1 \int_0^y dx\, dy$



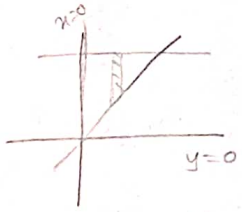
Given limits : x limit $x=0$ to $x=y$

y limit $y=0$ to $y=1$

Inner limit is with respect to x.

∴ It is horizontal strip.

By changing order of integration we have to draw vertical strip



x limit : $x=0$ to $x=1$

y limit : $y=x$ to $y=1$

$$I = \int_0^1 \int_x^1 dy\, dx$$

3. Evaluate by changing the order of integration

$\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\, dx$

Sol

Given integral is in the correct form

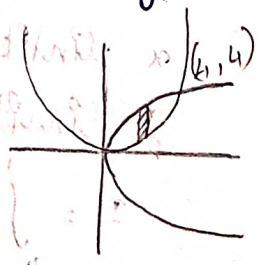
$$I = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\, dx$$

Given limits: x limit $x=0$ to $x=4$

y limit $y=x^2/4$ to $2\sqrt{x}$

Inner limit is with respect to y.

It is the vertical strip



$y = \frac{x^2}{4}$, $y = 2\sqrt{x}$

$4y = x^2 \rightarrow (1)$ $y^2 = 4x \rightarrow (2)$

$x^2 = 4y$ squaring on both side

$x^4 = 16 y^2$

$x^4 = 16(4x)$

$x^4 = 64x$

$x^3 = 64$

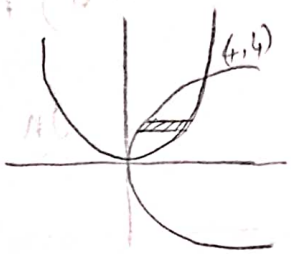
$\boxed{x = 4}$ sub in (2)

$y^2 = 4x$

$y^2 = 4(4) = 16$

$\boxed{y = 4}$

By changing order of integration we have to draw horizontal strip



x limit $x = \frac{y^2}{4}$ to $x = 2\sqrt{y}$

y limit $y = 0$ to $y = 4$

$$I = \int_0^4 \int_{y^2/4}^{2\sqrt{y}} dx\, dy$$

$$= \int_0^4 [x]_{y^2/4}^{2\sqrt{y}} dy$$

$$= \int_0^4 \left[2\sqrt{y} - \frac{y^2}{4}\right] dy$$

$$= \int_0^4 \left(2y^{1/2} - \frac{y^2}{4}\right) dy$$

$$I = \left[\frac{2y^{3/2}}{3/2} - \frac{y^3}{4\times 3}\right]_0^4$$

$$= \frac{2(4)^{3/2}}{3/2} - \frac{4^3}{3/2}$$

$$= \frac{2\times 4\sqrt{4}}{3/2} - \frac{4^2}{3}$$

$$= \frac{32-16}{3} = \frac{16}{3}$$