TWO MARKS Q & A

1. A random variable 'X' has the following probability function.

Values of X	0	1	2	3	4	5	6	7	8
Probability $p(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Determine the value of 'a'. [A.U. May '04, Jan. '07]

Ans : We know that if P (x) is the probability mass function, then

$$\sum_{i=1}^{\infty} p(x_i) = 1 \qquad \text{[Here '}i\text{' varies from 0 to 8]}$$

$$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$81a = 1$$

$$\Rightarrow \quad \boxed{a = \frac{1}{81}}$$

2. A random variable 'X' has the following probability function

X :	0	1	2	3	4
$p(x)$:	K	3K	5K	7K	9K

Find the value of K. [A.U. May '06]

Ans : We know that $\sum p(x_i) = 1$

$$p(0) + p(1) + p(2) + p(3) + p(4) = 1$$

$$K + 3K + 5K + 7K + 9K = 1$$

$$25K = 1$$

$$\boxed{K = \frac{1}{25}}$$

3. Show that the function $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ is a p.d.f.

[A.U. June '06, May '09]

Ans :

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{2} \frac{x^2}{3}\,dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{9}[8+1]$$

$$= \frac{9}{9} = 1$$

4. If 'X' is a continuous random variable whose probability density function is given by $f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$. What is the value of 'c'?

Ans : We know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^2 c\,(4x - 2x^2)\,dx = 1 \qquad [\because 0 < x < 2]$$

$$2c \int_0^2 (2x - x^2)\,dx = 1$$

$$2c\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1 \Rightarrow 2c\left[4 - \frac{8}{3}\right] = 1$$

$$2c\left[\frac{12 - 8}{3}\right] = 1 \Rightarrow 2c \times \frac{4}{3} = 1$$

$$\boxed{c = \frac{3}{8}}$$

5. **Given that the p.d.f of a R.V 'X' is $f(x) = Kx$, $0 < x < 1$, find K and $P(X > 0.5)$.**
[A.U. June '05, Dec. '08]

Ans : We know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 Kx\,dx = 1 \qquad [\because 0 < x < 1]$$

$$K\left[\frac{x^2}{2}\right]_0^1 = 1$$

$$K\left[\frac{1}{2} - 0\right] = 1 \Rightarrow \boxed{K = 2}$$

$\therefore$

$$P(X > 0.5) = \int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{1} 2x\,dx \qquad [\because K = 2]$$

$$= 2\left[\frac{x^2}{2}\right]_{\frac{1}{2}}^{1} = \left[1 - \frac{1}{4}\right] = \frac{3}{4}$$

6. **A continuous random variable 'X' has a p.d.f $f(x) = K, 0 \le x \le 1$. Find 'K'.** **[A.U. Dec '05, Apr. '06]**

Ans : Since $f(x)$ is a $p.d.f$, we have

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_0^1 K\, dx = 1 \qquad [\because f(x) = K] \qquad (\because 0 \le x \le 1)$$

$$K[x]_0^1 = 1 \Rightarrow K(1-0) = 1 \Rightarrow K = 1$$

7. **If a random variable 'X' has the p.d.f $f(x) = \begin{cases} \frac{1}{4}, & |X| < 2 \\ 0, & \text{otherwise} \end{cases}$. Find P (X < 1)**

Ans : Given $f(x) = \begin{cases} \frac{1}{4}, & |X| < 2 \\ 0, & \text{otherwise} \end{cases}$ ... (1)

$$P(X<1) = \int_{-\infty}^{1} f(x)\, dx$$

$$= \int_{-2}^{1} \frac{1}{4}\, dx \qquad [\because \text{From (1)}] \qquad \left[\begin{array}{l} \because \ |X| < 2 \\ \Rightarrow -2 < x < 2 \end{array}\right]$$

$$= \frac{1}{4}[x]_{-2}^{1} = \frac{1}{4}[1+2] = \frac{3}{4}$$

8. **A random variable 'X' has the p.d.f.**

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \qquad \textbf{find } P\left(X < \frac{1}{2}\right).$$

[A.U. Apr '04, June '07 PST]

Ans : Given $f(x) = 2x, \quad 0 < x < 1$ ... (1)

$$P\left(X < \frac{1}{2}\right) = \int_0^{\frac{1}{2}} f(x)\, dx$$

$$= \int_0^{\frac{1}{2}} 2x\, dx \qquad [\because \text{From (1)}] \qquad [0 < x < 1]$$

$$= 2\left(\frac{x^2}{2}\right)_0^{1/2} = \left[\frac{1}{4} - 0\right] = \frac{1}{4}$$

9. A continuous random variable 'X' has a p.d.f. $f(x) = 3x^2, 0 \le x \le 1$. Find 'b' such that $P(X > b) = 0.05$. [A.U. May '05, Dec '07]

Ans : Given $f(x) = 3x^2, \quad 0 \le x \le 1$ ...(1)

When $P(X > b) = 0.05$

$$\Rightarrow \int_b^1 f(x)\,dx = 0.05$$

$$\Rightarrow \int_b^1 3x^2\,dx = 0.05 \qquad [\because f(x) = 3x^2]$$

$$\Rightarrow 3\left(\frac{x^3}{3}\right)_b^1 = 0.05$$

$$\Rightarrow 1 - b^3 = \frac{1}{20} \Rightarrow b^3 = 1 - \frac{1}{20} = \frac{19}{20}$$

$$\therefore \boxed{b = \left(\frac{19}{20}\right)^{\frac{1}{3}} = 0.9830}$$

10. For the following density function $f(x) = a\,e^{-|x|}, -\infty < x < \infty$, find the value of 'a'. [A.U June '06 PST]

Ans : Given $f(x) = a\,e^{-|x|}, -\infty < x < \infty$ ...(1)

Given $f(x)$ is p.d.f. $\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} a\,e^{-|x|}\,dx = 1 \qquad \text{[Using (1)]}$$

$$a \cdot 2\int_0^{\infty} e^{-x}\,dx = 1 \qquad [\text{In } (0, \infty), e^{-|x|} = e^{-x}]$$

$$2a\int_0^{\infty} e^{-x}\,dx = 1 \Rightarrow 2a\left[-e^{-x}\right]_0^{\infty} = 1$$

$$2a\left[-e^{-\infty} + e^0\right] = 1$$

$$2a\,[0 + 1] = 1 \qquad [\because e^{-\infty} = 0, e^0 = 1]$$

i.e., $\boxed{a = \frac{1}{2}}$

11. In a continuous random variable 'X' having the p.d.f

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$ **. Find $P(0 < X \le 1)$.**

Ans :

$$P(0 < X \le 1) = \int_0^1 f(x)\,dx = \int_0^1 \frac{x^2}{3}\,dx$$

$$= \frac{1}{3}\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}\left[\frac{1}{3}-0\right] = \frac{1}{9}$$

12. A random variable 'X' has the density function

$$f(x) = K \cdot \frac{1}{1+x^2} \text{ in } -\infty < x < \infty$$
$$= 0, \quad \text{otherwise}$$

. Find 'K'.

[A.U. Dec '07, Apr. '08]

Ans : Since $f(x)$ is p.d.f., we have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$K \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 1 \Rightarrow K(\tan^{-1} x)_{-\infty}^{\infty} = 1$$

$$K[\tan^{-1}(\infty) - \tan^{-1}(-\infty)] = 1$$

$$K\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 1 \qquad \left[\because \tan^{-1}(\infty) = \frac{\pi}{2},\ \tan^{-1}(-\infty) = -\frac{\pi}{2}\right]$$

$$K \cdot \pi = 1$$

$$K = \frac{1}{\pi}$$

13. For the following c.d.f. $F(x) = \begin{cases} 0, & x<0 \\ x, & 0 \le x \le 1 \\ 1, & x>1 \end{cases}$

find (i) $P(X > 0{\cdot}2)$ (ii) $P(0{\cdot}2 < X \le 0{\cdot}5)$. [A.U. Nov '04]

Ans : (*i*)
$$P(X > 0{\cdot}2) = 1 - P(X \le 0{\cdot}2)$$
$$= 1 - F(0{\cdot}2) = 1 - 0{\cdot}2 = 0{\cdot}8$$

(*ii*)
$$P(0{\cdot}2 < X \le 0{\cdot}5) = F(0{\cdot}5) - F(0{\cdot}2)$$
$$= 0{\cdot}5 - 0{\cdot}2 = 0{\cdot}3$$

14. The density function of a random variable 'X' is given by $f(x) = Kx(2-x), 0 \le x \le 2$. Find K.

(A.U Apr. '03, Dec. '04, June '07, Nov. '08)

Ans : Given $f(x) = Kx(2-x), \quad 0 \le x \le 2$ ... (1)

Given $f(x)$ is a $p.d.f$ $\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^2 Kx(2-x)\,dx = 1 \qquad \text{[Using (1)]}$$

$$K\int_0^2 (2x-x^2)\,dx = 1$$

$$K\left[\frac{2x^2}{2}-\frac{x^3}{3}\right]_0^2 = 1$$

$$K\left[\left(4-\frac{8}{3}\right)-(0-0)\right] = 1 \Rightarrow K\left[4-\frac{8}{3}\right] = 1$$

$$K\left[\frac{12-8}{3}\right] = 1$$

$$K\left(\frac{4}{3}\right) = 1 \Rightarrow \boxed{K = \frac{3}{4}}$$

15. Find the m.g.f for the distribution where $f(x) = \begin{cases} \frac{2}{3} & \text{at } x=1 \\ \frac{1}{3} & \text{at } x=2 \\ 0 & \text{Otherwise} \end{cases}$

Ans : Given : $f(1) = \frac{2}{3}$; $f(2) = \frac{1}{3}$

$f(3) = f(4) = \ldots\ldots = 0$

MGF of a R.V. 'X' is given by

$$M_X(t) = E[e^{tx}]$$

$$= \sum_{x=0}^{\infty} e^{tx} f(x)$$

$$= e^0 f(0) + e^t f(1) + e^{2t} f(2) + \ldots\ldots\ldots$$

$$= 0 + e^t\left(\frac{2}{3}\right) + e^{2t}\left(\frac{1}{3}\right) + 0 \ldots\ldots$$

$$= \frac{2}{3}e^t + \frac{1}{3}e^{2t}$$

$\therefore$ MGF is $M_X(t) = \frac{e^t}{3}[2+e^t]$

16. If a random variable 'X' has the MGF, $M_X(t) = \frac{2}{2-t}$, **find the variance of 'X'.** **[A.U. Apr. '07, Nov '0?]**

Ans : Given $M_X(t) = \frac{2}{2-t} = 2(2-t)^{-1}$ ...(1)

$$M_X'(t) = -2(2-t)^{-2}(-1) = 2(2-t)^{-2}$$

$$M_X''(t) = -4(2-t)^{-3}(-1)$$

$$= 4(2-t)^{-3} \quad \ldots (2)$$

$$E[X] = M_X'(0) = 2(2-0)^{-2}$$

$$= 2 \times 2^{-2} \quad \text{[From (1)]}$$

$$= \frac{2}{4} = \frac{1}{2}$$

$$E[X^2] = M_X''(0) = 4(2-0)^{-3}$$

$$= 4 \times 2^{-3} \quad \text{[From (2)]}$$

$$= \frac{4}{2^3} = \frac{4}{8} = \frac{1}{2}$$

$$\therefore \quad Var(X) = E[X^2] - [E(X)]^2$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

17. **Find the MGF of the distribution given by**

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

[A.U. Dec. '07]

Ans :

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$= \int_0^{\infty} e^{tx} \cdot \lambda e^{-\lambda x}\, dx \quad [\because x > 0, f(x) = \lambda e^{-\lambda x}]$$

$$= \lambda \int_0^{\infty} e^{-(\lambda - t)x}\, dx = \lambda \left[\frac{e^{-(\lambda - t)x}}{-(\lambda - t)} \right]_0^{\infty}$$

$$= -\frac{\lambda}{\lambda - t}\left[-e^{-\infty} - e^{-0}\right]$$

$$= -\frac{\lambda}{\lambda - t}[0 - 1] \quad [\because e^{-\infty} = 0, e^0 = 1]$$

$\therefore$ The MGF is $M_X(t) = \dfrac{\lambda}{\lambda - t}$

18. State and prove additive property of binomial distribution.

Ans : **The sum of two binomial variates is not a binomial variate.**

Let X and Y be two independent binomial variates with parameters (n_1, p_1) and (n_2, p_2) respectively.

Then
$$M_X(t) = (q_1 + p_1 e^t)^{n_1}$$
$$M_Y(t) = (q_2 + p_2 e^t)^{n_2}$$
$$\therefore M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$$

[$\because$ X and Y are independent R.V.'s]

$$= (q_1 + p_1 e^t)^{n_1} (q_2 + p_2 e^t)^{n_2}$$

The R.H.S. cannot be expressed in the form $(q + pe^t)^n$. Hence by uniqueness theorem of MGF X + Y is not a binomial variate.

19. Check whether the following data follow a Binomial distribution or not. Mean = 3; Variance = 4. [A.U. Apr. '04]

Ans : Given

$$\text{Mean} = np = 3 \quad \ldots (1)$$
$$\text{Variance} = npq = 4 \quad \ldots (2)$$
$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{4}{3} = 1\frac{1}{3}$$
$$q = 1\frac{1}{3} \text{ which is} > 1.$$

Since $q > 1$ which is not possible $(0 < q < 1)$. The given data does not follow Binomial distribution.

20. If 'X' is a random variate following binomial distribution with mean 2.4 and variance 1.44, find P (X ≥ 5). [A.U. May '03]

Ans : For a binomial distribution,

$$\text{Mean} = np = 2.4 \quad \ldots (1)$$
$$\text{Variance} = npq = 1.44 \quad \ldots (2)$$
$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{1.44}{2.4} \Rightarrow q = 0.6; \quad \therefore p = 0.4$$

Substituting $p = 0.4$ in (1),
$$n(0.4) = 2.4$$
$$n = \frac{2.4}{0.4} = 6$$

$\therefore$ The distribution function is
$$P(X = x) = 6C_x (0.4)^x (0.6)^{6-x}$$

Now
$$P(X \geq 5) = 1 - P(X < 5)$$
$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)]$$

$= 1 - \{6C_0 (0.4)^0 (0.6)^6 + 6C_1 (0.4)^1 (0.6)^5 + 6C_2 (0.4)^2 (0.6)^4$
$+ 6C_3 (0.4)^3 (0.6)^3 + 6C_4 (0.4)^4 (0.6)^2\}$

[Using calculator]

$= 0.04096$

21. With the usual notation find 'p' for a binomial random variate 'X' if $n = 6$ and if $9\,P(X = 4) = P(X = 2)$. [A.U. June '94, Dec. '08]

Ans : We know that

$$P(X = x) = nC_x\, p^x q^{n-x}.$$

Given

$$9\,P(X = 4) = P(X = 2)$$
$$9 \times 6C_4\, p^4 q^2 = 6C_2\, p^2 q^4$$
$$9p^2 = q^2$$
$$= (1-p)^2$$
$$9p^2 = 1 + p^2 - 2p$$
$$8p^2 + 2p - 1 = 0$$
$$p = \frac{-2 \pm \sqrt{4+32}}{16} = \frac{-2 \pm 6}{16} = \frac{1}{4} \text{ or } -\frac{1}{2}$$
$$p = -\frac{1}{2} \quad \text{(not possible)}$$

$\therefore$

$$p = 0.25,$$
$$q = 0.75$$

22. If the MGF of a r.v. X is of the form $(0.4e^t + 0.6)^8$. What is the MGF of $3X + 2$. [A.U. Dec '07]

Ans :

Given $\quad M_X(t) = (0.4\, e^t + 0.6)^8 = E[e^{tX}] \quad \ldots (1)$

$\therefore$ MGF of $3X + 2$ is given by

$$M_{3X+2}(t) = E[e^{(3X+2)t}]$$
$$= e^{2t} E[e^{3Xt}]$$
$$= e^{2t} E[e^{X(3t)}]$$

$[\because E[e^{3Xt}] = M_X(e^{3t})]$

$$= e^{2t}(0.4e^{3t} + 0.6)^8$$

23. If X is a Poisson variate $P(X = 2) = 9\,P(X = 4) + 90\,P(X = 6)$, find (i) mean of X, (ii) variance of X.

Ans :

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots\ldots$$

Given

$$P(X = 2) = 9\,P(X = 4) + 90\,P(X = 6)$$
$$\frac{e^{-\lambda}\lambda^2}{2!} = 9\,\frac{e^{-\lambda}\lambda^4}{4!} + 90\,\frac{e^{-\lambda}\lambda^6}{6!}$$
$$= e^{-\lambda}\lambda^2\left(\frac{9\lambda^2}{4!} + \frac{90\lambda^4}{6!}\right)$$

$$\frac{1}{2} = \frac{9\lambda^2}{4!} + \frac{90\lambda^4}{6!}$$

$$\frac{1}{2} = \frac{3\lambda^2}{8} + \frac{\lambda^4}{8}$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

$$\lambda^2 = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm 5}{2} = 1 \text{ or } -4$$

$$\lambda^2 = 1 \text{ or } \lambda^2 = -4$$

$$\lambda = \pm 1 \text{ or } \lambda = \pm 2i$$

$\therefore$ Mean $= \lambda = 1$ [λ – cannot be imaginary]

Variance $= \lambda = 1$

$\therefore$ Standard deviation $= 1$

24. **If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective ($e^{-3} = 0.0498$).** **[A.U. Nov. '02, Jan. '07]**

Ans : Let X be the R.V denoting the number of defective electric bulbs.

Given P (a bulb is defective) $= \frac{3}{100}$

$$p = 0.03$$

$$n = 100$$

$$\therefore \quad \lambda = np = 100 \times 0.03 = 3$$

P ('x' bulbs are defective) $= P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\therefore$ P(exactly 5 bulbs are defective)

$$= P(X = 5) = \frac{e^{-3}3^5}{5!} = \frac{0.0498 \times 243}{120} = 0.1008$$

25. **If the MGF of X is $(5 - 4e^t)^{-1}$, find the distribution of X and P (X = 5).** **[A.U., Dec. '04]**

Ans : Let the geometric distribution be

$$P(X = x) = q^x p, \quad x = 0, 1, 2, \ldots\ldots$$

The MGF of geometric distribution is given by

$$M_X(t) = \frac{p}{1 - qe^t} \quad \ldots (1)$$

Here $M_X(t) = (5 - 4e^t)^{-1} = 5^{-1}\left[1 - \frac{4}{5}e^t\right]^{-1}$ ... (2)

Comparing (1) and (2), we get $q = \frac{4}{5} \Rightarrow p = \frac{1}{5}$ [$\because p + q = 1$]

$$P(X=x) = pq^x, \quad x = 0, 1, 2, 3, \ldots\ldots$$

$$\therefore \quad = \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^x$$

$$P(X=5) = \frac{1}{5}\left(\frac{4}{5}\right)^5 = \frac{4^5}{5^6}$$

26. If the probability is 0.05 that a certain kind of measuring device will show excessive drift, what is the probability that the sixth of these measuring devices tested will be the first to show excessive drift ?

[A.U. Dec '07]

Ans : Let 'X' be the R.V denoting the no. of measuring devices to show excessive drift.

Here $p = 0.05 \Rightarrow q = 1 - 0.05 = 0.95$

$x = 6$

We know that $P(X=x) = q^{x-1}p = (0.05)(0.95)^5$

$= 0{\cdot}0387$

27. Find the moment generating function of uniform distribution.

[A.U. Ap. '05, Dec '06]

Ans :

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$

$$= \int_a^b e^{tx} f(x)\,dx = \int_a^b e^{tx} \cdot \frac{1}{b-a} \cdot dx$$

$$\left[\because f(x) = \frac{1}{b-a},\ a < x < b\right]$$

$$= \frac{1}{b-a}\left[\frac{e^{tx}}{t}\right]_a^b = \frac{1}{(b-a)t}\left[e^{bt} - e^{at}\right]$$

$\therefore$ The MGF of uniform distribution is

$$M_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$$

28. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. What is the probability that a repair takes atleast 10 hrs given that its duration exceeds 9 hours.

[A.U. Dec '04]

Ans : Let X be the R.V which represents the time to repair the machine. The density function of X is given by

$$f(x) = \lambda e^{-\lambda x}$$

$$= \frac{1}{2} e^{-\frac{1}{2}x}, \quad x > 0$$

$$\therefore \quad P(X>10/X>9) = P(X>9+1/X>9)$$

$$= P(X>1)$$ [by memory less property]

$$P(X>t) = e^{-\lambda t}$$

$$= e^{-\frac{1}{2}t} \quad \left[\because P(X>t) = e^{-\lambda t}\right]$$

$$P(X>1) = e^{-\frac{1}{2}} \quad [\because t=1]$$

$$= 0{\cdot}6065$$

29. The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$, what is the probability that the required time (i) exceeds 2 hours (ii) exceeds 5 hours.

[A.U. May '07]

Ans : Let X be the R.V which represents the time to repair the machine. Then the density function of X is given by

$$f(x) = \lambda e^{-\lambda x} = \frac{1}{2}\, e^{\frac{-1}{2}x}, x > 0.$$

(i) $$P(X>k) = e^{-\lambda k}$$

$$P(X>2) = e^{\frac{-1}{2}\times 2} \quad \left[\text{Here } \lambda = \frac{1}{2}, k=2\right]$$

$$= e^{-1}$$

(ii) $$P(X>5) = e^{\frac{-1}{2}\times 5} \quad \left[\text{Here } \lambda = \frac{1}{2}, k=5\right]$$

$$= e^{-\frac{5}{2}}$$

30. A normal distribution has mean $\mu = 20$ and S.D. $\sigma = 10$. Find $P(15 \le X \le 40)$.

[A.U. Nov '07, May '03]

Ans : Given $\mu = 20$

$\sigma = 10$

The normal variate $z = \dfrac{X-\mu}{\sigma} = \dfrac{X-20}{10}$

when

$$X = 15, \; z = \frac{X-20}{10} = \frac{15-20}{10} = -0.5$$

$$X = 40, \; z = \frac{40-20}{10} = 2$$

$$\therefore \quad P(15 \le X \le 40) = P(-0.5 \le z \le 2)$$

$$= P(-0.5 \le z \le 0) + P(0 \le z \le 2)$$
$$= P(0 \le z \le 0.5) + P(0 \le z \le 2)$$
$$= 0.1915 + 0.4772$$
$$= 0.6687$$

31. If X is a R.V. normally distributed with mean zero and variance σ^2, find $E(|X|)$. [A.U. Dec '07]

Ans : We know that $E(|X|)$ = Mean deviation about origin
= Mean deviation about mean (0)

Mean deviation about mean $= \frac{4}{5}\sigma$ $\left[\because \text{Mean deviation about mean of the normal distribution} = \frac{4}{5}\sigma\right]$

$$\therefore \quad E(|X|) = \frac{4}{5}\sigma$$

32. If the pdf of X is $f_X(x) = e^{-x}, x > 0$ find the pdf of $Y = 2X + 1$. [A.U., April, '03]

Ans :

Given p.d.f of 'X'

$$f_X(x) = e^{-x}, \quad x > 0 \qquad \dots (1)$$

Given $\quad Y = 2X + 1 \qquad \dots (2)$

$$y = 2x + 1,$$

$$\Rightarrow \quad x = \frac{y-1}{2} = f(y) \qquad \dots (3)$$

$$(3) \Rightarrow \quad \frac{dx}{dy} = \frac{1}{2}$$

$$\therefore \quad \left|\frac{dx}{dy}\right| = \frac{1}{2} \qquad \dots (4)$$

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$

$$= e^{-x}\frac{1}{2}, \; x > 0 \qquad \text{[Using (1) and (4)]}$$

$$= e^{-\left(\frac{y-1}{2}\right)} \left[\because x = \frac{y-1}{2}\right] \qquad \dots (5)$$

Since $\quad x > 0 \Rightarrow \frac{y-1}{2} > 0 \qquad \left[\because x = \frac{y-1}{2}\right]$

$$\Rightarrow y - 1 > 0$$
$$\Rightarrow y > 1 \qquad \dots (6)$$

Using (5) and (6) we get

$$f_Y(y) = e^{-\left(\frac{y-1}{2}\right)}, \quad y > 1$$

$$= e^{\frac{1-y}{2}}, \quad y > 1$$

33. If X is a uniformly distributed RV in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, find the pdf of Y = tan X.

Ans : **[A.U., Dec. '03, May '03 PQT, June '06 PQT]**

Since X is uniformly distributed in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, the p.d.f of 'X' is

$$f_X(x) = \frac{1}{\frac{\pi}{2} + \frac{\pi}{2}}, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$= \frac{1}{\pi}, \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \quad \ldots (1)$$

Given $Y = \tan X \quad \ldots (2)$

$$y = \tan x \Rightarrow x = \tan^{-1}(y) \quad \ldots (3)$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{1}{1+y^2} \quad \text{[Using (3)]}$$

$$\therefore \quad \left|\frac{dx}{dy}\right| = \frac{1}{1+y^2} \quad \ldots (4)$$

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$

$$= \frac{1}{\pi} \cdot \frac{1}{1+y^2} \quad \text{[Using (1) and (4)]} \quad \ldots (5)$$

Since $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\tan x \in (-\infty, \infty)$

$$\Rightarrow \quad y \in (-\infty, \infty) \quad \ldots (6)$$

Using (5) and (6) we get

$$\therefore \quad f_Y(y) = \frac{1}{\pi(1+y^2)}, \quad -\infty < y < \infty$$