Permutation & Combination:

Permutation:

A permutation is an arrangement of 'n' objects in which can be taken some (or) all at a time.

$$nP_r = \frac{n!}{(n-r)!}$$
(or) $P(n,r)$

Note:

$0! = 1$, $nP_0 = 1$, $nP_n = n!$

Problems

1. How many different bit strings are there of length 7?

No. of bit strings of length $7 = 7!$
$= 5040$

2. In how many ways can 6 persons occupy 3 vacant seats?

No. of persons $n = 6$
vacant seats $r = 3$

Total no. of ways $= nP_r = 6P_3 = 6\times5\times4 = 120$ ways

(or)

$$nP_r = \frac{n!}{(n-r)!} = \frac{6!}{(6-3)!} = \frac{6\times5\times4\times3\times2\times1}{3\times2\times1}$$
$= 120$ ways

3. How many permutations are there in the word for the following.

(i) MISSISSIPPI (ii) Radar (iii) Mathematical (iv) unusual

(i) No. of letters, $n = 11$

Repeated letters (r): $I\to4$, $S\to4$, $P\to2$

Required No. of permutations $= \frac{n!}{r!\,r!\,r!} = \frac{11!}{4!\,4!\,2!}$

$= 34,650$

(ii) No. of letters : $n = 5$

Repeated letters : $a \to 2$, $r \to 2$

(r)

Required No. of permutations $= \dfrac{5!}{2!\,2!} = 30$ ways

(iii) mathematical

No. of letters : $n = 12$

Repeated letters : $m \to 2$, $t \to 2$, $a \to 3$

Required No. of permutations $= \dfrac{12!}{3!\,2!\,2!}$

$= 19958400$

(iv) unusual

No. of letters : $n = 7$

Repeated letters : $u \to 3$, $n \to 1$, $s \to 1$, $a \to 1$, $l \to 1$

Required No. of permutations $= \dfrac{7!}{3!} = 840$ ways

4). Suppose there are 6 boys and 4 girls

(i) In how many ways can they sit in a row ?

(ii) In how many ways can they sit in a row if the boys and girls each sit together ?

(iii) In how many ways they can sit in a row if the girls can sit together ?

(iv) How many seating arrangements are there with no two girls sitting together ?

6 boys can sit in a row in $6!$ ways.

4 girls can sit in a row in $4!$ ways

(i) No. of ways can they sit in a row is $6! + 4!$

$= 10!$

$= 3,628,800$

(ii) No. of ways can they sit in a row if the boys and girls each sit together is

$$4!\,6!\,2! = 34,560$$

(iii) No. of ways they can sit in a row if the girls can sit together is $7!\,4! = 120960$

(iv) No. of seating arrangements are there with no two girls sitting together is $7P_4 \times 6!$

$$= \frac{7!}{3!} \times 6!$$
$$= 604800$$

Combinations:

A Combination is the selection of some (or) all of a number of different objects.

$$nC_r = \frac{n!}{r!\,(n-r)!}$$

Note:

$$nC_0 = nC_n = 1$$
$$nC_r = nC_{n-r}$$

Problems:

1. How many ways are there to select 5 players from 10 member tennis team to make a trip for match.

$n = 10$ members

$r = 5$

Now $10C_5 = \dfrac{10!}{5!\,(10-5)!} = \dfrac{10!}{5!\,5!} = 252$ ways

2]. A team of 11 players is to be chosen from 15 members. In how many ways this can be done if
(i) one particular player is always included ?
(ii) Two such players have always to be included ?

(i) one player is always included.
∴ Out of 14 members, we've to select 10 players in $14C_{10}$ ways $= \frac{14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4} = 1001$ ways

(ii) Two players are always included.
∴ Out of 13 members, we've to select 9 players = 715 ways

3]. A committee consisting of 6 men and 7 women in how many ways can be select a committee of
(i) 3 men and 4 women
(ii) 4 members which has atleast one women
(iii) 4 persons that has almost one man
(iv) 4 persons of both sexes
4 persons in which Mr. & Mrs. X is excluded.

(i) 3 mens can selected in $6C_3$ and
4 womens can selected in $7C_4$
∴ 3 men & 4 women are selected in $6C_3 \times 7C_4$
= 700 ways

(ii) committee of 4 members which has atleast 1 women.
1 woman & 3 men : $7C_1 \times 6C_3$
2 women & 2 men : $7C_2 \times 6C_2$
3 women & 1 man : $7C_3 \times 6C_1$
4 women & 0 man : $7C_4 \times 6C_0$
Selection can be done in
$7C_1 \times 6C_3 + 7C_2 \times 6C_2 + 7C_3 \times 6C_1 + 7C_4 \times 6C_0$
= 700 ways

(iii) Committee of 4 persons that has atmost one man.

1 man & 3 women : $6C_1 \times 7C_3$

0 man & 4 women : $6C_0 \times 7C_4$

Selection can be done in

$= 6C_1 \times 7C_3 + 6C_0 \times 7C_4$

$= 245$ ways

(iv) Committee of both sexes

1 man & 3 women : $6C_1 \times 7C_3$

2 men & 2 women : $6C_2 \times 7C_2$

3 men & 1 woman : $6C_3 \times 7C_1$

Selection can be done in

$= 6C_1 \times 7C_3 + 6C_2 \times 7C_2 + 6C_3 \times 7C_1$

$= 665$ ways

Hw Q. A box contains 6 white balls & 5 red balls. Find the number of ways 4 balls can be drawn from the box if

(i) They can be any colour

(ii) Two must be white and two red.

(iii) They must all be the same colour.

(i) They can be any colour : $11C_4 = 330$ ways

(ii) 2 must be white and 2 red : $6C_2 \cdot 5C_2$

$= 150$ ways

(iii) They must all be the same colour : $6C_4 + 5C_4$

$= 20$ ways