$$= \frac{1}{s}\frac{(1+e^{-2bs}-2e^{-bs})}{(1-e^{-2bs})}$$

$$= \frac{1}{s}\frac{(1-e^{-2bs})^2}{(1-(e^{-2bs})^2)}$$

$$= \frac{1}{s}\frac{(1-e^{-2bs})^2}{(1+e^{-2bs})(1-e^{-2bs})}$$

$$= \frac{1}{s}\tanh\frac{bs}{2}$$

Unit.5

~~II-Part~~

Inverse laplace Transform

If $L[f(t)] = F(S)$ Then $f(t)$ is called the Inverse laplace Transform of $F(S)$ & is denoted by

$$f(t) = L^{-1}[f(S)]$$

where, L^{-1} is the inverse Laplacian operator.

Laplace Transform	Inverse Laplace Transform
1. $L(1) = \frac{1}{S}$	$1 = L^{-1}\left(\frac{1}{S}\right)$
2. $L(R) = \frac{k}{S}$	$k = L^{-1}\left(\frac{k}{S}\right)$
3. $L(0) = 0$	$L^{-1}(0) = 0$
$L(t^n) = \frac{n!}{S^{n+1}}$	$t^n = L^{-1}\left(\frac{n!}{S^{n+1}}\right) \Rightarrow \frac{t^n}{n!} = L^{-1}($; $\boxed{\frac{t^{n-1}}{(n-1)!} = L^{-1}(}$

5. $L(e^{at}) = \frac{1}{s-a}$ $\qquad L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

6. $L(e^{-at}) = \frac{1}{s+a}$ $\qquad L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$

7. $L(\sin at) = \frac{a}{s^2+a^2}$ $\qquad L^{-1}\left(\frac{a}{s^2+a^2}\right) = \sin at$

8. $L(\cos at) = \frac{s}{s^2+a^2}$ $\qquad L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$

9. $L(\sin hat) = \frac{a}{s^2-a^2}$ $\qquad L^{-1}\left(\frac{a}{s^2-a^2}\right) = \sin hat$

10. $L(\cos hat) = \frac{s}{s^2-a^2}$ $\qquad L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cos hat.$

(1) $L^{-1}\left[\frac{1}{s} + \frac{1}{s^3} + \frac{1}{s+4} + \frac{s}{s^2+9} + \frac{1}{s^2+16}\right]$

$$= L^{-1}\left(\frac{1}{s}\right) + L^{-1}\left(\frac{1}{s^3}\right) + L^{-1}\left(\frac{1}{s+4}\right) + L^{-1}\left(\frac{s}{s^2+9}\right) + L^{-1}\left(\frac{1}{s^2+16}\right)$$

$$= 1 + \frac{t^{3-1}}{(3-1)!} + e^{-4t} + L^{-1}\left(\frac{s}{s^2+3^2}\right) + L^{-1}\left(\frac{1}{s^2+4^2}\right)$$

$$= 1 + \frac{t^2}{2} + e^{-4t} + \cos 3t + \frac{1}{4} L^{-1}\left(\frac{4}{s^2+4^2}\right)$$

$$= 1 + \frac{t^2}{2} + e^{-4t} + \cos 3t + \frac{1}{4} \sin 4t.$$

(2) $L^{-1}\left(\frac{7}{s^5}\right)$

$$\frac{t^{n-1}}{(n-1)!} = L^{-1}\left(\frac{1}{s^n}\right)$$

$$= 7\left(\frac{1}{s^5}\right)$$

$1 \times 2 \times 3 \times 4$

$$= 7\frac{t^4}{24}$$

3) $L^{-1}\left(\frac{1}{(s+2)^3}\right)$

$= e^{-2t} L^{-1}\left(\frac{1}{s^3}\right)$

$= e^{-2t} \frac{t^2}{2}$ //.

4) $L^{-1}\left(\frac{2}{s^2+9}\right)$

$= L^{-1}\left(\frac{2}{s^2+3^2}\right)$

$= 2\, L^{-1}\left(\frac{1}{s^2+3^2}\right)$

$= \frac{2}{3} L^{-1}\left(\frac{3}{s^2+3^2}\right)$

$= \frac{2}{3} \sin 3t$

5) $L^{-1}\left(\frac{5}{(s+2)^2+9}\right)$

$= L^{-1}\left[\frac{5}{(s+2)^2+3^2}\right]$

$= \frac{5}{3} L^{-1}\left[\frac{3}{(s+2)^2+3^2}\right]$

$= \frac{5}{3} e^{-2t} L^{-1}\left[\frac{3}{s^2+3^2}\right]$

$= \frac{5}{3} e^{-2t} \sin 3t$

6) $L^{-1}\left[\frac{s}{(s+2)^2+1}\right]$

$= L^{-1}\left[\frac{s+2-2}{(s+2)^2+1}\right]$

$= L^{-1}\left[\frac{s+2}{(s+2)^2+1}\right] - L^{-1}\left[\frac{2}{(s+2)^2+1}\right]$

$= e^{-2t} L^{-1}\left[\frac{s}{s^2+1}\right] - 2e^{-2t} L^{-1}\left[\frac{1}{s^2+1}\right]$

$= e^{-2t}\cos t - 2e^{-2t}\sin t$

7) $L^{-1}\left[\frac{s+1}{(s+3)^2+4}\right]$

$= L^{-1}\left[\frac{s+1+2-2}{(s+3)^2+4}\right]$

$= L^{-1}\left[\frac{s+3}{(s+3)^2+4}\right] - L^{-1}\left[\frac{2}{(s+3)^2+4}\right]$

$= e^{-3t} L^{-1}\left[\frac{s}{s^2+2^2}\right] - e^{-3t} L^{-1}\left[\frac{2}{s^2+2^2}\right]$

$= e^{-3t}\cos 2t - e^{-3t}\sin 2t$