Initial and Final value Theorem :

Initial Value theorem :

If $L[f(t)] = F(s)$ then

$$\underset{t\to 0}{Lt}\ f(t) = \underset{s\to\infty}{Lt}\ s.F(s)$$

Final value theorem :

If $L[f(t)] = F(s)$ then

$$\underset{t\to \infty}{Lt}\ f(t) = \underset{s\to 0}{Lt}\ s.F(s)$$

7 mark

1) Verify Initial & Final Value theorem for the function $f(t) = 1 + e^{-t}(\sin t + \cos t)$

IVT

$$\underset{t\to 0}{Lt}\ f(t) = \underset{s\to\infty}{Lt}\ s.F(s)$$

L.H.S

$$\underset{t\to 0}{Lt}\ f(t) = \underset{t\to 0}{Lt}\left[1 + e^{-t}\sin t + e^{-t}\cos t\right]$$

$$= 1 + e^{0}\sin 0 + e^{0}\cos 0$$

$$= 1 + 1 = 2$$

R.H.S

$$\underset{s\to\infty}{Lt}\ s.F(s)$$

$$F(s) = L[f(t)]$$

$$= L\left[1 + e^{-t}\sin t + e^{-t}\cos t\right]$$

$$= L(1) + L(e^{-t}\sin t) + L(e^{-t}\cos t)$$

$$= \frac{1}{s} + \left[L(\sin t)\right]_{s\to s+1} + \left[L(\cos t)\right]_{s\to s+1}$$

$$= \frac{1}{s} + \left(\frac{1}{s^2+1}\right)_{s \to s+1} + \left(\frac{s}{s^2+1}\right)_{s \to s+1}$$

$$= \frac{1}{s} + \frac{1}{(s+1)^2+1} + \frac{s}{(s+1)^2+1}$$

$$F(s) = \frac{1}{s} + \frac{1}{s^2+2s+2} + \frac{s}{s^2+2s+2}$$

$$\therefore \lim_{s \to \infty} s.F(s) = \lim_{s\to\infty} s\left[\frac{1}{s} + \frac{1}{s^2+2s+2} + \frac{s}{s^2+2s+2}\right]$$

$$= \lim_{s\to\infty}\left[1 + \frac{s}{s^2+2s+2} + \frac{s^2}{s^2+2s+2}\right]$$

$$= \lim_{s\to\infty}\left[1 + \frac{\cancel{s}}{\cancel{s}(s+2+\frac{2}{s})} + \frac{\cancel{s^2}}{\cancel{s^2}(1+\frac{2}{s}+\frac{2}{s^2})}\right]$$

$$= \lim_{s\to\infty}\left[1 + \frac{1}{s+2+\frac{2}{s}} + \frac{1}{1+\frac{2}{s}+\frac{2}{s^2}}\right]$$

$$= \cancel{0}\ 1 + \frac{1}{\infty} + \frac{1}{1+0+0}$$

$$= 1 + 0 + 1 = 2$$

L.H.S = R.H.S

Hence initial value theorem is verified.

FVT

$$L[f(t)] = F(s)$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s.F(s)$$

L.H.S

$$\lim_{t\to\infty} f(t) = \lim_{t\to\infty}\left[1 + e^{-t}\sin t + e^{-t}\cos t\right]$$

$$= 1 + e^{-\infty}\sin\infty + e^{-\infty}\cos\infty$$