



Triple integrals

① Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} x \, dz \, dy \, dx$

$$I = \int_0^1 \int_0^{1-x} (z)_0^{x+y} x \, dy \, dx$$

$$= \int_0^1 \int_0^{1-x} (x+y)x \, dy \, dx = \int_0^1 \int_0^{1-x} (x^2 + xy) \, dy \, dx$$

$$= \int_0^1 \left[x^2 y + \frac{xy^2}{2} \right]_0^{1-x} dx$$

$$= \int_0^1 \left[x^2(1-x) + \frac{x(1-x)^2}{2} \right] dx$$

$$I = \frac{1}{8}$$

② $\int_0^a \int_0^b \int_0^c e^{x+y+z} \, dx \, dy \, dz$

$$= \int_0^a \int_0^b (e^x)_0^c e^{y+z} \, dy \, dz$$

$$= (e^c - e^0) \int_0^a (e^y)_0^b \, dz$$

$$= (e^c - 1) (e^b - 1) (e^z)_0^a$$

$$= (e^c - 1) (e^b - 1) (e^a - 1)$$