Soln :

x varies from 0 to a

y varies from 0 to $\dfrac{b}{a}\sqrt{a^2 - x^2}$
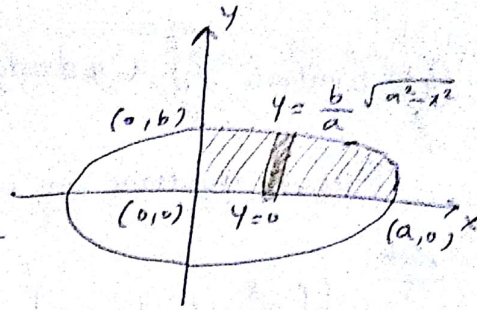
$$\iint_R x^2 y \, dy \, dx = \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^2 y \, dy \, dx$$

$$= \int_0^a x^2 \left(\frac{y^2}{2}\right)_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$$

$$= \frac{a^3 b^2}{15}$$

③ Evaluate $\iint_R xy \,(x+y)\, dy \, dx$ over the area between
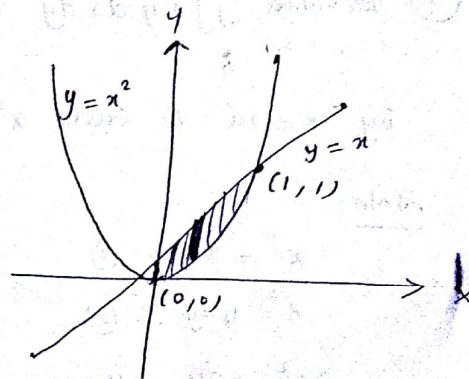
$y = x^2, \quad y = x$.

Soln :

$$\iint_R xy\,(x+y)\, dy \, dx$$

$$= \int_0^1 \int_{x^2}^{x} \left(x^2 y + x y^2\right) dy \, dx$$

$$= \int_0^1 \left[\frac{x^2 y^2}{2} + \frac{x y^3}{3}\right]_{x^2}^{x} dx$$

$$= \int_0^1 \left[\frac{x^4}{2} + \frac{x^4}{3} - \frac{x^6}{2} - \frac{x^7}{3}\right] dx$$

$$= \frac{3}{56}$$

$y = x \longrightarrow$ ①

$y = x^2 \longrightarrow$ ②

$x = x^2$

$x^2 - x = 0$

$x = 0, 1$

$y = 0, 1$

$\therefore (0,0) \; (1,1)$

(4) Evaluate $\iint\limits_{R} x^2 y \, dx \, dy$ over the region in the Positive quadrant in which $x+y \le 1$.
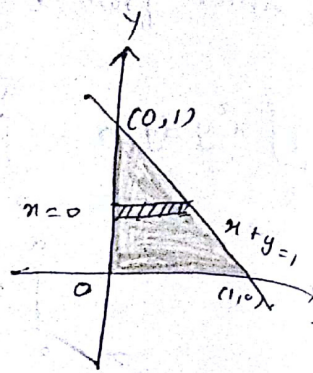
Soln:

$$\iint\limits_{R} x^2 y \, dx \, dy = \int_0^1 \int_0^{1-y} x^2 y \, dx \, dy$$

$$= \int_0^1 \left(\frac{x^3}{3} y \right)_0^{1-y} dy$$

$$= \int_0^1 \frac{y}{3} (1-y)^3 \, dy$$

$$= \frac{1}{60}.$$

(5) Evaluate $\iint\limits_{A} xy \, dx \, dy$ where A is a region bounded by $x = 2a$ & curve $x^2 = 4ay$

Soln:

$$x = 2a \rightarrow \text{①}$$
$$x^2 = 4ay \rightarrow \text{②}$$
$$4a^2 = 4ay \Rightarrow y = a.$$

x varies from $2\sqrt{ay}$ to $2a$.
y varies from 0 to a.

$$I = \int_0^a \int_{2\sqrt{ay}}^{2a} xy \, dx \, dy = \int_0^a \left(\frac{x^2}{2} \right)_{2\sqrt{ay}}^{2a} \cdot y \, dy$$

$$= \frac{a^4}{3}.$$