$$= \int_0^2 (x)_{2-y}^{\sqrt{4-y^2}} dy$$

$$= \int_0^2 \left(\sqrt{4-y^2} - (2-y)\right) dy$$

$$= \int_0^2 \sqrt{4-y^2}\, dy - \int_0^2 (2-y)\, dy \qquad \left(\sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right)$$

$$= \left[\frac{y}{2}\sqrt{4-y^2} + \frac{4}{2}\sin^{-1}\left(\frac{y}{2}\right) - 2y + \frac{y^2}{2}\right]_0^2$$

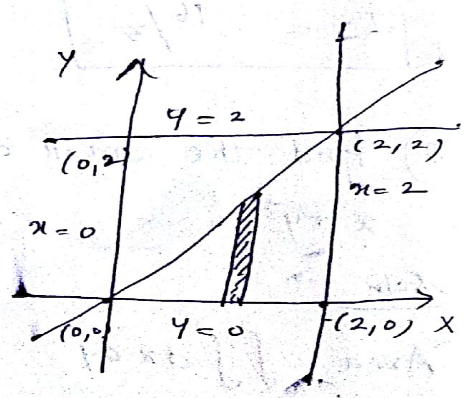
$$\boxed{I = \pi - 2}$$

Change of order of integration

(1) Change the order of integration in $\int_0^2 \int_y^2 \frac{x\, dy\, dx}{x^2+y^2}$ and hence evaluate it.

Soln:

$$I = \int_0^2 \int_y^2 \frac{x\, dx}{x^2+y^2} dy$$



Limits are $x = y$ & $x = 2$

$y = 0$ & $y = 2$.

Draw the strip parallel to y-axis.

Now the limits are x varies from 0 to 2

y varies from 0 to x.

$$I = \int_0^2 \int_0^x \frac{x \, dy \, dx}{x^2+y^2}$$

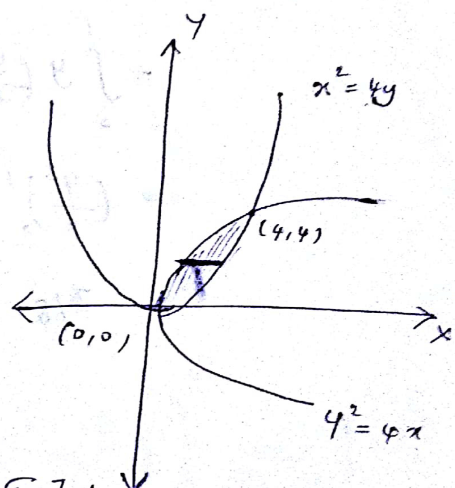
$$= \int_0^2 x \, dx \int_0^x \frac{dy}{x^2+y^2}$$

$$= \int_0^2 x \, dx \left[\frac{1}{x} \tan^{-1}\left(\frac{y}{x}\right) \right]_0^x$$

$$= \int_0^2 \left[\tan^{-1}\left(\frac{x}{x}\right) - \tan^{-1}(0) \right] dx$$

$$= \frac{\pi}{4} \times 2 = \pi/2 \Rightarrow \boxed{I = \pi/2}$$

(2) Evaluate by changing the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy \, dx$.



Soln:

Given : $y = \frac{x^2}{4}$ to $y = 2\sqrt{x}$

$\Rightarrow x^2 = 4y$ to $y^2 = 4x$

Now the limits are

$x = y^2/4$ to $x = \sqrt{4y}$.

$$\therefore I = \int_0^4 \int_{y^2/4}^{\sqrt{4y}} dx \, dy = \int_0^4 \left[-\frac{y^2}{4} + \sqrt{4y} \right] dy$$

$$\boxed{I = \frac{16}{3}}$$