COMPLEX INTEGRATION

Cauchy's integration theorem:

If a function $f(z)$ is analytic and its derivative $f'(z)$ is continuous with all points inside and on a simple closed curve C, then

$$\int_C f(z)\,dz = 0.$$

Cauchy's integral formula:

If $f(z)$ is analytic inside and on a simple closed curve C and 'a' be any point inside C, then

$$\int_C \frac{f(z)}{z-a}\,dz = 2\pi i\, f(a).$$

Where integration being taken in the anti-clockwise direction around C.

(1) Evaluate $\int_C \frac{\cos \pi z}{z-1} dz$, where C is

$|z| = 2$

Soln:

$f(z) = \cos \pi z$

$a = 1$.

$C : |z| = 2$.

$a = 1$ lies inside C, $|1| < 2$

By Cauchy's integral formula,

$$\int_C \frac{f(z)}{z-a} dz = 2\pi i f(a)$$

$$\int_C \frac{\cos \pi z}{z-1} dz = 2\pi i f(1) \quad [\because f(z) = \cos \pi z$$

$$= -2\pi i \qquad f(1) = \cos \pi = -1]$$

(2) Evaluate $\int_C \frac{dz}{(z-3)^2}$ where C is the circle $|z| = 1$

Soln: $a = 3$ lies outside the circle $|z| = 1$.