TEST OF SIGNIFICANCE OF SMALL SAMPLES:

STUDENT'S t-TEST:

TEST FOR SINGLE MEAN:

Null hypothesis: $H_0: \mu = \mu_0$.

Test Statistic, $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n-1}}$ if SD is given.

$t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$ if SD is not given.

[To find S:

$$S^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$]

Degrees of freedom: $\nu = n-1$

NOTE: Confidence Limit: $\bar{x} \pm t_\alpha \dfrac{S}{\sqrt{n-1}}$

1) A random sample of 10 boys had the following IQ's. 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ's of 100? Find a reasonable range to which most of the mean IQ's value of sample 10 boys.

Soln: given: $n = 10$, $\mu = 100$

$$\bar{x} = \frac{70+120+110+101+88+83+95+98+107+100}{10}$$

$$= 97.2.$$

To find s : $s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$

x :	70	120	110	101	88	83	95	98	107	100
$x-\bar{x}$:	-27.2	22.8	12.8	3.8	-9.2	-14.2	-2.2	0.8	9.8	2.8
$(x-\bar{x})^2$:	739.84	519.84	163.84	14.44	84.64	201.64	4.84	0.64	96.04	7.84

$\sum (x-\bar{x})^2 = 1833.6$

$\therefore s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{1833.6}{10-1}$

$= 203.73$

$\Rightarrow s = 14.27$.

Step 1 : Formulating H_0 and H_1 :

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$ (two tailed test)

Step 2 : LOS. at $\alpha = 5\% = 0.05$

Step 3 : Test Statistic, $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

$= \frac{97.2 - 100}{14.27/\sqrt{10}}$

$= -0.62$

$|t| = 0.62$.

step 4: t_{tab} for degree of freedom, $\nu = n-1$
$\nu = 10-1 = 9$

(ie) t_{tab} : 2.262 (t_α)

step 5: Conclusion: $t = 0.62 < 2.262 = t_\alpha$

$\therefore$ Ho is accepted at 5% Los.

(ie) the population mean Iq's is 100.

Confidence limit:

$$\mu = \bar{x} \pm t_\alpha \frac{s}{\sqrt{n-1}}$$

$$= 97.2 \pm 2.262 \times \frac{14.27}{\sqrt{10-1}}$$

$$= 97.2 \pm 10.759$$

$$= 107.95, 86.45$$

3) the weights of 10 peoples of a locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 kg it is resonable to believe that the average weights of people locality greater than 64 kg. test at 5% Los.

Soln: Given: $n = 10$, $\mu = 64$

$$\bar{x} = \frac{70+67+62+68+61+68+70+64+64+66}{10}$$

$$\bar{x} = 66$$

To find s:

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

x:	70	67	62	68	61	68	70	64	64	66
$x-\bar{x}$:	4	1	-4	2	-5	2	4	-2	-2	0
$(x-\bar{x})^2$:	16	1	16	4	25	4	16	4	4	0

$$\sum (x-\bar{x})^2 = 90$$

$$\therefore \quad s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{90}{10-1} = 10$$

$$s = 3.16$$

Step 1: Formulating H_0 and H_1:

$H_0: \mu = 64$

$H_1: \mu > 64$ (one tailed test - right)

Step 2: LOS at $\alpha = 5\%$.

Step 3: Test statistic, $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

$$= \frac{66-64}{3.16/\sqrt{10}}$$

$$= 2.02$$

Step 4: t_{tab} for degree of freedom, $\nu = n-1$

$= 10-1$

$= 9$

$\therefore t_{tab}: t_\alpha = 1.833$ (at two tailed at 10%.)

Step 5 : Conclusion: $t = 2.02 > \cancel{0.9165}\ 1.833 = t_\alpha$

$\therefore$ H_0 is rejected at 5% LOS

(i.e) the avg. weight of people locality is greater than 64.kg.