TEST OF SIGNIFICANCE OF LARGE SAMPLES:

TEST FOR SINGLE MEAN:

Null Hypothesis, $H_0 : \mu = \mu_0$

Test statistics, $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ (or) $z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

1) A sample of 900 members is found to have a mean of 3.4 cm and S.D. 2.61 cms. Is the sample from a large population of mean 3.25 cm and S.D. 2.61 cms. If the population is normal and its mean is unknown find the 95% confidential (fiducial) limits of true mean.

Soln: Given: $n = 900$, $\bar{x} = 3.4$, $\mu = 3.25$, $\sigma = 2.61$

step 1: Formulating H_0 & H_1:

$H_0 : \mu = 3.25$

$H_1 : \mu \neq 3.25$ (two tailed test)

step 2: Level of significance $\alpha = 5\% = 0.05$

step 3: Test statistic,
$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}} = 1.724$$

step 4: Critical value at 5% is $z_\alpha = 1.96$.

step 5: Conclusion: Since $|z| = 1.724 < 1.96 = z_\alpha$,

H_0 is accepted at 5% level of significance.

$\therefore$ The sample is taken from population where mean 3.25 cm.

Confidence limits:

$$\mu = \bar{x} \pm z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$= 3.4 \pm 1.96 \times \frac{2.61}{\sqrt{900}} = 3.4 \pm 0.17$$

$$= 3.23, \ 3.57$$

(ie) $3.23 < \mu < 3.57$.

2) A random sample of 200 employees at a large corporation showed their average to be 42.8 years with a S.D. of 6.89 years. Test the hypothesis $H_0: \mu = 40$, $H_1: \mu > 40$ at $\alpha = 0.01$ level of significance.

Soln:

Given: $n = 200$, $\bar{x} = 42.8$, $\mu = 40$, $\sigma = 6.89$

Step 1: Formulating H_0 and H_1:

$H_0: \mu = 40$

$H_1: \mu > 40$ (one tail test – right)

Step 2: Level of significance, $\alpha = 0.01$.

Step 3: Test statistic, $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$$= \frac{42.8 - 40}{\frac{6.89}{\sqrt{200}}}$$

$$= 5.747$$

Step 4: Critical value at 1% (one tailed - right) is $z_\alpha = 2.33$

Step 5: Conclusion: Since $|z| = 5.747 > 2.33 = z_\alpha$

$\therefore$ H_0 is rejected at 1% level of significance.

$\therefore$ the hypothesis, $H_1: \mu > 40$ is accepted.

3) The mean height of college students in a city are normally distributed with S.D. 6 cms. A sample of 100 students has mean height of 158 cms. Test the hypothesis that the mean height of college students in the city 160 cms. Also obtain 99% confidence limits for the true mean.

Soln: Given: $n = 100$, $\bar{x} = 158$, $\mu = 160$, $\sigma = 6$

Step 1: Formulating H_0 and H_1:

$H_0: \mu = 160$

$H_1: \mu \neq 160$ (two tailed test)

Step 2: Level of significance, $\alpha = 1\%$

Step 3: Test statistic, $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{158 - 160}{6/\sqrt{100}}$

$= 3.33$

Step 4: Critical value at 1% (two side test) is

$z_\alpha = 2.58$.

Step 5: Conclusion; Since $|z| = 3.33 > 2.58 = z_\alpha$

$\therefore$ H_0 is rejected at 1% level of significance.

$\therefore$ The mean height of the college students in the city is 160 cms is not true.

Confidence limit:

$\mu = \bar{x} \pm z_\alpha \dfrac{\sigma}{\sqrt{n}}$

$= 158 \pm 2.58 \times \dfrac{6}{\sqrt{100}}$

$= 158 \pm 1.548$

$= 156.452, 159.548$

(ie) $156.452 < \mu < 159.548$, here $\mu = 160$ does not lies in the interval.