



SNS COLLEGE OF TECHNOLOGY
(AN AUTONOMOUS INSTITUTION)
COIMBATORE - 35
DEPARTMENT OF MATHEMATICS



Tautology:

A statement formula which is true always irrespective of the truth values of the individual variables is called a tautology.

Eg:

$P \vee \neg P$ is a tautology.

Contradiction:

A statement formula which is always false is called a contradiction (or) absurdity.

Eg:

$P \wedge \neg P$ is a contradiction.

Contingency:

A statement formula which is neither tautology nor contradiction is called contingency.

Eg:

$P \leftrightarrow Q$ is contingency.

1. Show that $[P \wedge (P \wedge Q)] \rightarrow Q$ is a tautology.

P	Q	$\neg P$	$P \wedge Q$	$\neg P \wedge (P \wedge Q)$	$[P \wedge (P \wedge Q)] \rightarrow Q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	F	F	T

Since all the entries in the resulting column is true, the given expression is a tautology.

2. Show that $(P \wedge Q) \wedge \neg (P \vee Q)$ is a contradiction.

P	Q	$P \wedge Q$	$P \vee Q$	$\neg (P \vee Q)$	$(P \wedge Q) \wedge \neg (P \vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since all the entries in the resulting column are F, the given expression is a contradiction.

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3. Identify the given expression

$$7(P \vee Q) \vee (7P \vee 7Q)$$

P	Q	7P	7Q	$P \vee Q$	$7(P \vee Q)$	$7P \vee 7Q$	$7(P \vee Q) \vee (7P \vee 7Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T

Since the entries in the resulting column are T as well as F, the given expression is a contingency.

Examine whether $[(P \vee Q) \rightarrow R] \leftrightarrow [A \rightarrow \sim(P \vee Q)]$ is a tautology?

Equivalence :

Two statement formulas p and q are equivalent iff $p \leftrightarrow q$ is a tautology. It is denoted by $p \Leftrightarrow q$.

Show that the propositions are logically equivalent for the following.

- (i) $P \rightarrow Q \Leftrightarrow 7P \vee Q$
(ii) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

(i) $P \rightarrow Q \Leftrightarrow 7P \vee Q$

P	Q	$P \rightarrow Q$	7P	$7P \vee Q$	$P \rightarrow Q \Leftrightarrow 7P \vee Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Since $(P \rightarrow Q) \leftrightarrow (7P \vee Q)$ is a tautology.

Hence $(P \rightarrow Q) \Leftrightarrow (7P \vee Q)$

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(ii) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

P	Q	R	$Q \vee R$	$P \rightarrow (Q \vee R)$	$P \rightarrow Q$	$P \rightarrow R$	$(P \rightarrow Q) \vee (P \rightarrow R)$	$P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Here $[P \rightarrow (Q \vee R)] \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$ is a
tautology.
Hence $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$.

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