Antenna Array

It is system of similar antennas oriented similarly to get greater directivity in a desired direction.

Linear Array

An antenna array is said to be linear if the individual antennas of the array are equally placed along a straight line.

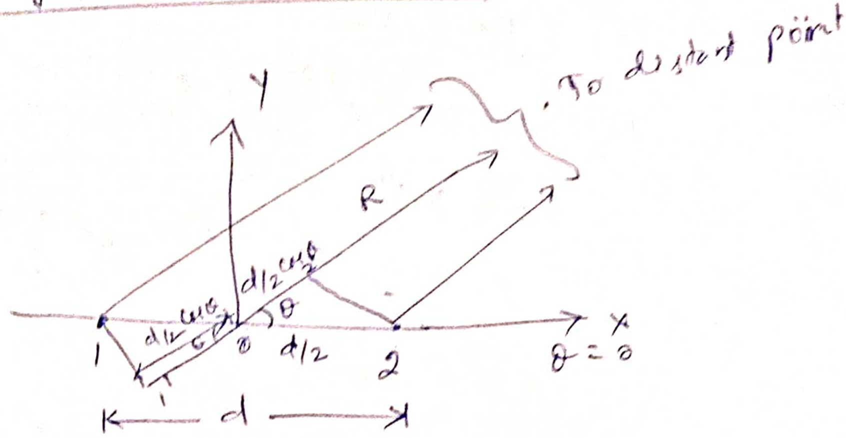
Individual antennas are called as elements.

Uniform linear array is one, in which the elements are fed with current of equal magnitude with uniform progressive phase shift along the line.

Point soure → zero volume radiator.

Two Element Arrays

Array of two point sources



Path difference = 1'2' metres.

$= d/2 \cos\theta + d/2 \cos\theta$

$= d \cos\theta$ metres

$= \frac{d}{\lambda} \cos\theta$ wavelengths.

Phase difference = $2\pi \times$ path difference

$= 2\pi \times \frac{d}{\lambda} \cos\theta$.

$\psi = \beta d \cos\theta$ radians.

E_1 = Far electric field at distant point P, due to source 1

E_2 = " " 2

E = Total electric field.

$\psi = \beta d \cos\theta$ radians
= phase angle difference between the fields of the two sources measured at an angle 'θ' along radius vector line.

Total far field at 'P'

$E = E_1 + E_2$

$= E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$

$E_1 e^{-j\psi/2} \rightarrow$ field component due to source 1

$E_1 e^{+j\psi/2} \rightarrow$ field component due to source 2

(iii)

i) Assuming amplitudes are same

$E_1 = E_2 = E_0$

$E = E_0 (e^{j\psi/2} + e^{-j\psi/2})$

$= 2E_0 \cos \psi/2$

$= 2 E_0 \cos\left(\frac{\beta d \cos\theta}{2}\right)$

If $\beta = 2\pi/\lambda$ & Let $d = \lambda/2$

$E = 2E_0 \cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \times \frac{\cos\theta}{2}\right)$

$= 2E_0 \cos(\pi/2 \cos\theta)$

$E_{norm} = \cos(\pi/2 \cos\theta)$

Maxima direction

E is max, when $\cos(\pi/2 \cos\theta)$ is max & its max^m value is ± 1.

~~$2E_0\cos$~~ $\cos(\pi/2 \cos\theta) = \pm 1$

$\pi/2 \cos\theta = \cos^{-1}(\pm 1)$

$\pi/2 \cos\theta = \pm n\pi$

where $n = 0, 1, 2, \ldots$

$n = 0$ for major lobe

$(\pi/2 \cos\theta)_{max} = 0$

$\pi/2 \cos\theta_{max} = 0$

$\cos\theta_{max} = 0$

$\theta_{max} = \cos^{-1}(0)$

$\boxed{\theta_{max} = 90^\circ \,\&\, 270^\circ}$

Minima direction

E is min when $\cos(\pi/2 \cos\theta)$ is min & its min^m value is 0

$\cos(\pi/2 \cos\theta) = 0$

(ii) Equal amplitude & opposite phase

$$E = -E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

$$E_1 = E_2 = E_0$$

$$E = -E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$= E_0 \left(e^{j\psi/2} - e^{-j\psi/2} \right)$$

$$= 2jE_0 \sin \psi/2$$

$$= 2jE_0 \sin\left(\frac{\beta d \cos\theta}{2} \right) \qquad \left[\because \psi = \beta d \cos\theta \right)$$

Normalized field $2jE_0 = 1$

$$E = \sin\left(\frac{\beta d}{2} \cos\theta \right)$$

$$= \sin\left(\frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{2}} \times \frac{1}{2} \cos\theta \right)$$

$$E = \sin\left(\pi/2 \cos\theta \right)$$

Maximum direction

E is max when, $\sin(\pi/2 \cos\theta)$ is max

& its max value is ± 1.

$$\sin(\pi/2 \cos\theta) = \pm 1$$

$$\pi/2 \cos\theta = \sin^{-1}(\pm 1)$$

$$\pi/2 \cos\theta = \pm (2n+1) \pi/2$$

Subs $n = 0$

$$\pi/2 \cos\theta = \pm \pi/2$$

$$\cos\theta = \pm 1$$

$$\theta_{max} = \cos^{-1}(\pm 1)$$

$$\boxed{\theta_{max} = 0^\circ \;\&\; 180^\circ}$$

$$\pi/2 \cos\theta = \cos^{-1}(0)$$
$$= \pm (2n+1)\pi/2 \qquad n = 0, 1, 2, \ldots$$

Sub n = 0

$$\pi/2 \cos\theta = \pm \pi/2$$
$$\cos\theta = \pm 1$$
$$\theta_{min} = \cos^{-1}(\pm 1)$$
$$0^\circ \text{ \& } 180^\circ$$

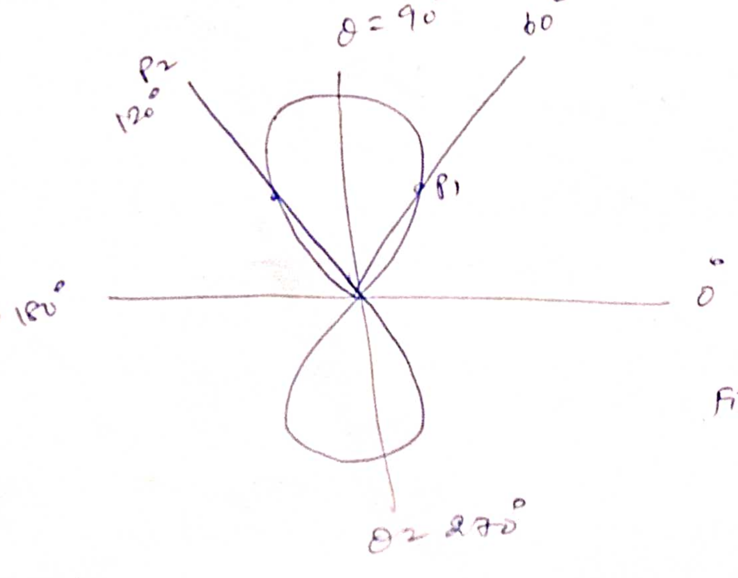
Half power point direction

$$\cos(\pi/2 \cos\theta) = \pm \frac{1}{\sqrt{2}}$$
$$\pi/2 \cos\theta_{HPPD} = \cos^{-1}(1/\sqrt{2})$$
$$= \pm (2n+1)\pi/4$$

n = 0,

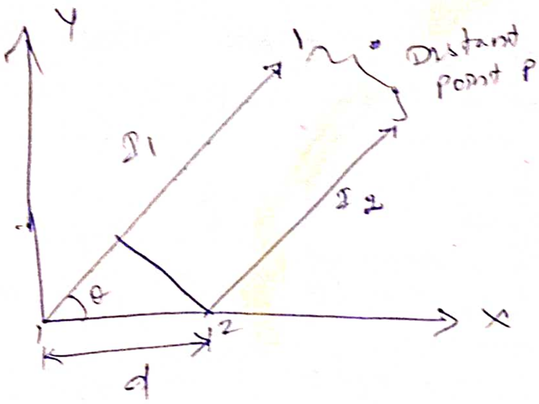
$$\pi/2 \cos\theta_{HPPD} = \pm \pi/4$$
$$\cos\theta_{HPPD} = \pm \frac{\pi}{4} \times \frac{2}{\pi}$$
$$\theta_{HPPD} = \cos^{-1}(\pm 1/2)$$
$$\boxed{\theta_{HPPD} = \pm 60^\circ \text{ \& } \pm 120^\circ}$$

Radiation pattern



Field pattern

(ii) Array of two point sources of unequal amplitude and any phase



$\psi = \beta d \cos\theta + \alpha$

$\alpha \rightarrow$ phase angle by which current I_2 of source 2 leads current I_1 of source 1.

E_1 & $E_2 \rightarrow$ amplitudes of fields of sources 1 & 2

$$E = E_1 e^{j0} + E_2 e^{j\psi}$$

($e^{j0} \rightarrow$ source 1 is taken as reference)

$$= E_1 \left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

$$E = E_1 (1 + k e^{j\psi})$$

$$k = \frac{E_2}{E_1}$$

if $E_1 > E_2$, $k < 1$

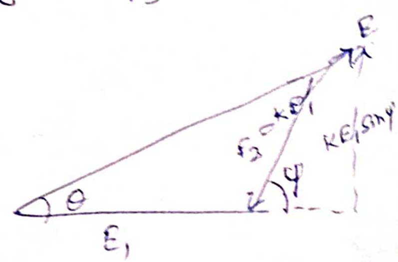
$$\text{i.e.} \quad E = E_1 (1 + k e^{j\psi})$$

$$= E_1 (1 + k \cos\psi + j k \sin\psi)$$

$$|E| = E_1 \sqrt{(1 + k\cos\psi)^2 + (k \sin\psi)^2}$$

Phase angle at P

$$\phi = \tan^{-1} \frac{k \sin\psi}{1 + k\cos\psi}$$



Minima direction

E is min, when

$\sin(\pi/2 \cos\theta) = 0$

$\pi/2 \cos\theta_{min} = \pm n\pi$ where $n = 0, 1, 2 \ldots$

$\cos\theta_{min} = 0$ (sub $n=0$)

$\theta_{min} = \cos^{-1}(0)$

$$\boxed{\theta_{min} = 90^\circ \ \& \ 270^\circ}$$

• Half power point directions

$\sin(\pi/2 \cos\theta) = \pm \frac{1}{\sqrt{2}}$

$\pi/2 \cos\theta_{HPPD} = \sin^{-1}\left(\pm \frac{1}{\sqrt{2}}\right)$

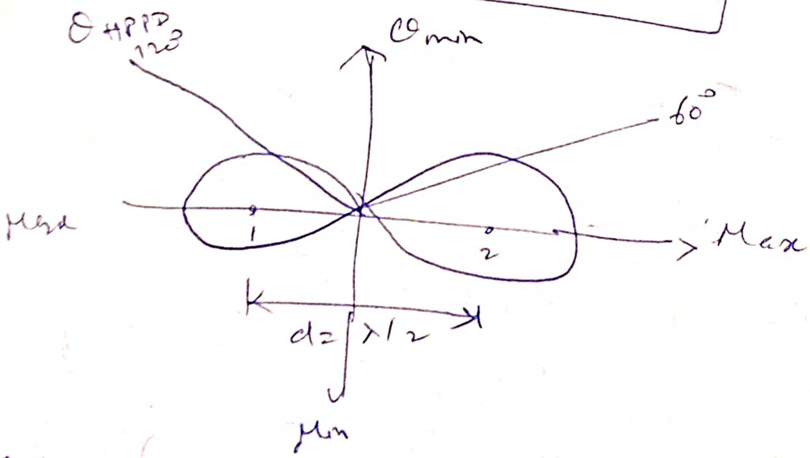
$= \pm (2n+1)\pi/4$

subs $n = 0$

$\pi/2 \cos\theta_{HPPD} = \pm \pi/4/2$

$\theta_{HPPD} = \cos^{-1}(\pm 1/2)$

$$\boxed{\theta_{HPPD} = \pm 60^\circ \ \& \pm 120^\circ}$$



Field pattern of two isotropic point sources of same amplitude ~~&~~ but opposite phase spaced $\lambda/2$ apart