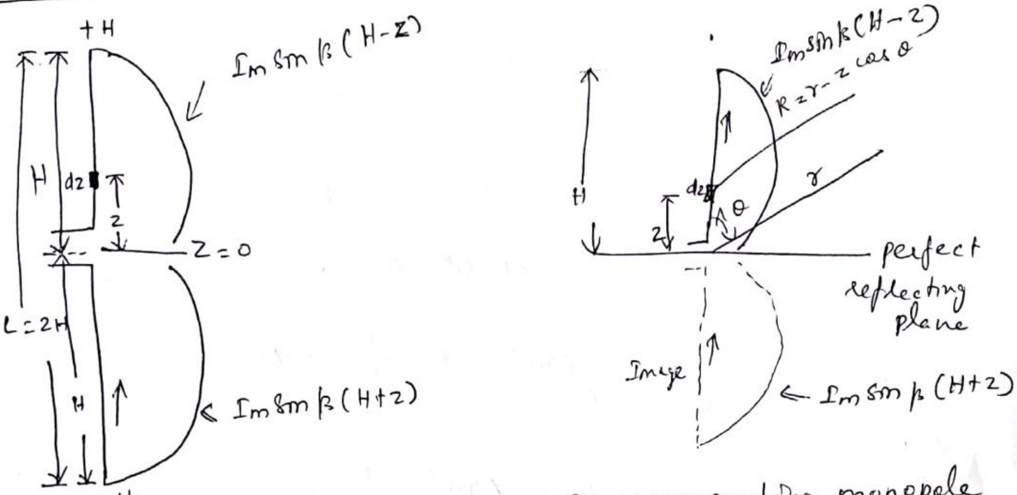
Radiation from half wave dipole and quarter-wave monopole



(a) centre-fed dipole with assumed sinusoidal current distribution

(b) corresponding monopole

Figure (a) shows a centre-fed dipole with a sinusoidal current distribution & (b) shows corresponding monopole.

"A dipole antenna" is a straight radiator, usually fed in the center and producing a maximum of radiation in the plane normal to the axis. The length specified is the overall length.

Radiation from a Quarter-wave monopole or halfwave dipole

It will be assumed that the current is sinusoidally distributed as shown in figure.

Then,

$$I = I_m \sin\beta(H-z), \quad z > 0$$

$$I = I_m \sin\beta(H+z), \quad z < 0$$

$I_m \rightarrow$ the value of current at the current loop (or) current maximum.

The expression for the vector potential at point P due to the current element $I dz$ is

$$dA_z = \frac{\mu I e^{-j\beta R} dz}{4\pi R} \rightarrow (1)$$

$R \rightarrow$ distance from the current element to point P.

The total vector potential at P due to all the current elements will be,

$$A_z = \frac{\mu}{4\pi} \int_{-H}^{0} \frac{I_m \sin\beta(H+z) e^{-j\beta R} dz}{R}$$

$$+ \frac{\mu}{4\pi} \int_{0}^{H} \frac{I_m \sin\beta(H-z) e^{-j\beta R}}{R} dz \rightarrow (2)$$

~~$R \simeq \theta$~~ $\boxed{R \simeq r} \rightarrow$ (in denominator)

$Nr \rightarrow R$ in the phase factor, so the difference between R and r is important.

$$R = r - z\cos\theta.$$

Then the expression for A_z becomes,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[\int_{-H}^{0} \sin\beta(H+z) e^{j\beta z\cos\theta} dz \right.$$

$$\left. + \int_{0}^{H} \sin\beta(H-z) e^{j\beta z\cos\theta} dz \right] \rightarrow (3)$$

[if $H = \lambda/4$.

$$\sin\beta(H+z) = \sin\beta(H-z) \qquad \beta H = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\sin(\pi/2 + \beta z) = \sin(\pi/2 - \beta z)$$

$$= \cos\beta z.]$$

Then eq (3) becomes,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[\int_{-H}^{0} \cos\beta z \, e^{j\beta z\cos\theta} dz + \int_{0}^{H} \cos\beta z \, \cancel{\sin\beta(H-z)} \, e^{j\beta z\cos\theta} dz \right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_0^H \cos\beta z\, e^{-j\beta z\cos\theta}\,dz + \int_0^H \cos\beta z\, e^{j\beta z\cos\theta}\,dz\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^H \cos\beta z\left(e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta}\right)dz$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^H \cos\beta z \times 2\cos\beta z\cos\theta\, dz$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^H \left[\cos(\beta z + \beta z\cos\theta) + \cos(\beta z - \beta z\cos\theta)\right]dz$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^H \left[\cos\beta z(1+\cos\theta) + \cos\beta z(1-\cos\theta)\right]dz$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin\beta z(1+\cos\theta)}{\beta(1+\cos\theta)} + \frac{\sin\beta z(1-\cos\theta)}{\beta(1-\cos\theta)}\right]_0^{Hz}$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{(1-\cos\theta)\cos(\pi/2\cos\theta) + (1+\cos\theta)\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)\,(1-\cancel{\cos\theta}+1+\cancel{\cos\theta})}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{\cancel{2}\,\cancel{4}\,\pi r\beta} \times \frac{\cancel{2}\cos(\pi/2\cos\theta)}{\sin^2\theta}$$

$$\boxed{A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]} \longrightarrow (4)$$

when current is entirely in z direction,

$$\mu H_\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r \times -A_z\sin\theta\right)$$

$$\cancel{\mu H_\phi = -\frac{\partial A_z}{\partial r}\sin\theta} \rightarrow (5)$$

$$H_\phi = \frac{1}{\mu r}\left[-\frac{\partial}{\partial r} \times \mu\frac{I_m e^{-j\beta r}}{2\pi\beta r}\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\times r\right]$$

$$= \frac{1}{\mu r}\left[\frac{\mu I_m e^{-j\beta r}}{2\pi r \beta}\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]\times(j\beta)/\cancel{r}$$

$$H_\phi = \frac{jI_m}{2\pi r}e^{-j\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right] \rightarrow (6)$$

The electric field strength at a distant point will be,

$$E_\theta = \eta H_\phi$$

$$= \cancel{120}^{60}\cancel{\pi}\times\frac{jI_m}{\cancel{2}\cancel{\pi}r}e^{-j\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$E_\theta = \frac{j60\,I_m}{r}e^{-j\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right] \rightarrow (7)$$

The magnitude of the electric field strength for the radiation field of half-wave dipole (or) quarter-wave monopole is

$$|E_\theta| = \frac{60\,I_m}{r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]\ V/m. \rightarrow (8)$$

Maxl. value Poynting vector

$$P_{max} = |E_\theta|\times|H_\phi|$$

$$= \frac{60\,I_m}{r}\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\times\frac{I_m}{2\pi r}\frac{\cos(\pi/2\cos\theta)}{\sin\theta}$$

$$P_{av} = \frac{P_{max}}{2}$$

$$= \frac{\cancel{60}^{15}}{\cancel{2}}\frac{I_m}{r}\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\times\frac{I_m}{\cancel{2}\pi r}\frac{\cos(\pi/2\cos\theta)}{\sin\theta}$$

$$P_{av} = \frac{15\,I_m^2}{\pi r^2}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right\}^2$$

Interms of rms current

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad I_m = \sqrt{2}I_{rms}$$

$$P_{av} = \frac{30 I_{rms}^2}{\pi r^2}\left[\frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}\right] \ W/m^2.$$

Power radiation by a Half wave dipole and its Radiation resistance

Poynting vector Method

Elemental area of the spherical shell

$$ds = 2\pi r^2 \sin\theta \, d\theta \rightarrow (1)$$

$$\text{Power radiated } W = \oint_S P_{av}\, ds = \int_0^{\pi} \frac{30 I_{rms}^2}{\pi r^2}\left\{\frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}\right\} \times 2\pi r^2 \sin\theta \, d\theta$$

$$= 60 I_{rms}^2 \int_0^{\pi} \left\{\frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta}\right\} d\theta$$

$$= 60 I_{rms}^2 \int_0^{\pi} 1/2 \left\{\frac{(1+\cos(\pi\cos\theta)}{\sin\theta}\right\} d\theta$$

$$(2\cos^2\theta = 1 + \cos 2\theta$$

$$= 60 I_{rms}^2 \times I$$

$I \rightarrow$ numerical integration method (or) graphical method (or) Simpson's (or) trapezoidal rules,

$$I = 1.219$$

$$W = 60 I_{rms}^2 \times 1.219$$

$$W = 73.140 I_{rms}^2$$

$$W = I_{rms}^2 R_r$$

$$\therefore \boxed{R_r = 73.14 = 73\,\Omega}$$

quarter-wave monopole $\quad R_r = \frac{73.14}{2} = 36.57\,\Omega.$