

Normal reaction $N_R = W \cos \alpha$

Frictional force $F = \mu N_R$
 $= \mu W \cos \alpha$

When $\mu W \cos \alpha > W \sin \alpha$ → Block is at rest
 $\mu W \cos \alpha < W \sin \alpha$, the impending downwards motion takes place.

When the angle of plane with horizontal α , is increased, $W \sin \alpha$ will be more than $\mu W \cos \alpha$ and sliding takes place.

⇒ "The angle of the inclined plane, at which the body tends to slide down, known as angle of repose, denoted by α_m ."

$\mu W \cos \alpha_m \leq W \sin \alpha_m$

$\mu W \cos \alpha_m = W \sin \alpha_m$

$\mu = \frac{W \sin \alpha_m}{W \cos \alpha_m}$

$= \tan \alpha_m \rightarrow \textcircled{1}$

$\mu = \tan \phi \rightarrow \textcircled{2}$

ϕ ⇒ angle of static friction



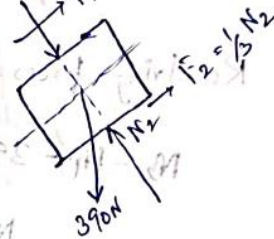
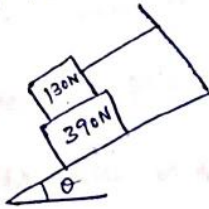


$$\tan \alpha_m = \tan \phi \quad \alpha_m = \phi$$

Angle of repose = angle of static friction.

Body on rough inclined plane

Q What should be the value of the angle θ so that motion of 390N block impends down the plane? The coefficient of friction $\mu = 1/3$.



FBD of upper block

FBD of lower block

FBD of upper block

T → tension of cable.

Resolving forces along plane

$$T - 130 \sin \theta - F_1 = 0$$

$$T - 130 \sin \theta - \mu N_1 = 0$$

$$T - 130 \sin \theta + \frac{1}{3} N_1 \rightarrow \text{①}$$

Resolving the force normal to plane

$$N_1 = 120 \cos \theta \rightarrow \text{②}$$

put ② in ①

$$T = 130 \sin \theta + \frac{1}{3} (120 \cos \theta)$$

$$T = 130 \sin \theta + 43.33 \cos \theta \rightarrow \text{③}$$



Consider FBD of lower block

Resolving forces along plane

$$F_1 + F_2 - 390 \sin \theta = 0$$

$$F_1 + F_2 = 390 \sin \theta$$

$$\mu N_1 + \mu N_2 = 390 \sin \theta$$

$$390 \sin \theta = \frac{1}{3} (130 \cos \theta + N_2) \rightarrow (4)$$

Resolving forces normal to plane

$$N_2 - N_1 - 390 \cos \theta = 0$$

$$N_2 = 390 \cos \theta + N_1$$

$$N_2 = 390 \cos \theta + 130 \cos \theta = 520 \cos \theta$$

Sub N_2 in eqn (4)

$$390 \sin \theta = \frac{1}{3} (130 \cos \theta + 520 \cos \theta)$$

$$390 \sin \theta = \frac{1}{3} (650 \cos \theta)$$

$$390 \sin \theta = 216.67 \cos \theta$$

$$\tan \theta = \frac{216.67}{390}$$

$$\theta = \tan^{-1} \left(\frac{216.67}{390} \right) = 29^\circ$$

