$= 2 m/s^2$

CURVILINEAR MOTION:

When the path described by a moving particle is a curve line, then the motion of the particle is known as curvilinear motion of translation.

* If the continuous path described by a moving particle confined to a plane then it is known as plane motion

Cartesian System:

$$V_x = \frac{dx}{dt}$$

$$V_y = \frac{dy}{dt}$$

$$V = \sqrt{V_x^2 + V_y^2}$$

Angle of Inclination, $\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

Acceleration of Particle:

$$a_x = \frac{d^2x}{dt^2}$$

$$a_y = \frac{d^2y}{dt^2}$$

Resultant acceleration

$$a = \sqrt{a_x^2 + a_y^2}$$

angle of inclination

$$\tan\phi = \frac{a_y}{a_x}$$

1. The motion of a particle along a curved path is given by equation

$$x = t^2 + 8t + 4 \quad \& \quad y = t^3 + 3t^2 + 8t + 4$$

Determine,

(i) Initial Velocity of particle
(ii) Velocity of the particle at $t = 2$ Sec
(iii) Acceleration of particle at $t = 0$
(iv) Acceleration of particle at $t = 2$ Sec.

Velocity components of particle

Horizontal Component of Velocity

$$V_x = \frac{dx}{dt} = 2t + 8 \quad \text{--- (1)}$$

Velocity Component of Velocity

$$V_y = \frac{dy}{dt} = 3t^2 + 6t + 8 \quad \text{--- (2)}$$

Acceleration Components of Particle

Horizontal Component of acceleration

$$a_x = \frac{d^2x}{dt^2} = 2 \quad \text{--- (3)}$$

Vertical Component of acceleration

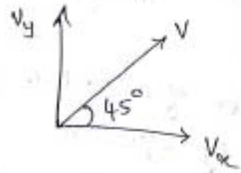
$$a_y = \frac{d^2y}{dt^2} = 6t + 6 \quad \text{--- (4)}$$

i) Initial Velocity of Particle

put $t = 0$ in eqn ① & ②

$V_x = 8 m/s$

$V_y = 8 m/s$



$V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{8^2 + 8^2}$

$= 11.31 m/s$

$\theta = \tan^{-1}\left(V_y/V_x\right)$

$= \tan^{-1}\left(8/8\right)$

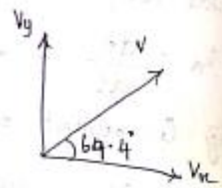
$= 45°$

ii) Velocity at 2 Sec.

Sub $t = 2$ Sec in ① & ②

$V_x = 12 m/s$

$V_y = 32 m/s$



Velocity at 2 Sec,

$V_2 = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{12^2 + 32^2}$

$= 34.17 m/s$

$\theta = \tan^{-1}\left(V_y/V_x\right)$

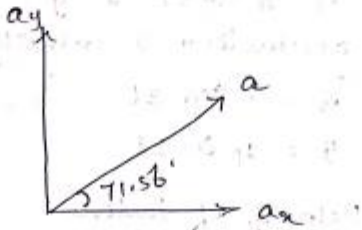
$= 69.4°$

(iii) Acceleration at $t = 0$

Sub $t = 0$ in (3) & (4)

$a_x = 2\,m/s^2$

$a_y = 6\,m/s^2$



$a = \sqrt{2^2 + 6^2}$

$= 6.324\,m/s^2$

$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

$= \tan^{-1}\left(\frac{6}{2}\right)$

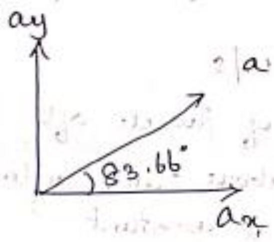
$= 71.56°$

(iv) acceleration at $t = 2$ Sec.

Sub $t = 2$ in (3) & (4)

$a_x = 2\,m/s^2$

$a_y = 18\,m/s^2$



$a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{2^2 + 18^2}$

$= 18.11\,m/s^2$

$\phi = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

$= \tan^{-1}\left(\frac{18}{2}\right)$

$= 83.66°$

2. The motion of a body moved on a curved path is given by the equation $x = 4 \sin 3t$ & $y = 4 \cos 3t$. Find the acceleration & velocity after 2 seconds.

Soln

$x = 4 \sin 3t$

$y = 4 \cos 3t$

Velocity of Particle

$V_x = \frac{dx}{dt}$

$= 12 \cos 3t$ $[\because \sin^2 a + \cos^2 a = 1]$

$V_y = \frac{dy}{dt}$

$= -12 \sin 3t$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{24 \cos^2 3t + 24 \sin^2 3t}$

$= 12 m/s$

Velocity of Particle after 2 Sec.

From above result, velocity at any time interval is constant

(ie) $V = 12 m/s$

Acceleration of Particle

$a_x = \frac{d}{dt}(V_x)$

$= -36 \sin 3t$

$a_y = \frac{d}{dt}(v_y)$

$= -36 \cos 3t$

$a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{(-36 \sin 3t)^2 + (-36 \cos 3t)^2}$

$= 36 m/s^2$

Acceleration of particle after 2 Sec.

From the above result, acceleration of particle at any time is constant

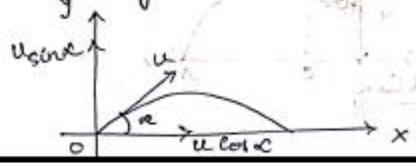
(ie) $a = 36 m/s^2$

PROJECTILE MOTION.

Projectile:
A Particle projected in Space at an angle to the horizontal plane.

Angle of Projection:
The angle to the horizontal at which the projectile is projected is called angle of projection denoted by 'α'.

Velocity of Projectile:
Velocity with which the projectile is thrown into Space. Denoted by 'u'.



Velocity 'u' can be resolved into 2 component along OX & OY axes.

Components of velocity along OX axis

$= u \cos \alpha$ (move projectile horizontally)

Components of velocity along OY axis

$= u \sin \alpha$ (move projectile vertically)

(4) Trajectory

path described by the projectile

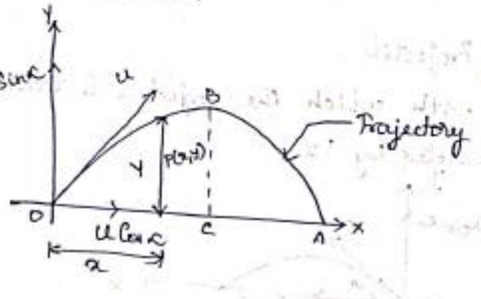
(5) Time of Flight

Total time taken by the projectile from the instant of projection upto the projectile into the plane again.

(6) Range

It is the distance along the plane between the point of projection & the point at which the projectile hits the plane at the end of its journey.

PATH OF A PROJECTILE :



x = velocity × Time Taken

$x = u \cos\alpha\, t$

$t = \frac{x}{u \cos\alpha}$

IIIly the vertical distance travelled by the projectile in any time t,

$y = u \sin\alpha\, t - \frac{1}{2} g t^2$

The above equation is arrived from $h = ut - \frac{1}{2} g t^2$

Sub, $u = u \sin\alpha$; $h = y$; $t = \frac{x}{u \cos\alpha}$

$$y = u \sin\alpha \left(\frac{x}{u \cos\alpha}\right) - \frac{1}{2} g \left(\frac{x}{u \cos\alpha}\right)^2$$

$$= \tan\alpha\, x - \frac{1}{2} \frac{g x^2}{u^2 \cos^2\alpha}$$

Std/. Results :-

Time of flight:

Time to reach the highest point or time to hit the ground from highest point

Time to reach the max height,

$t = \frac{u \sin\alpha}{g}$

∵ Time taken to reach up = Time taken to reach down

Time of flight, $T = 2t = \frac{2u \sin\alpha}{g}$