MOTION OF PARTICLE UNDER GRAVITY:

Motion of a particle under gravity is the Special case of rectilinear motion under constant acceleration known as acceleration due to gravity denoted by g.

$$g = 9.81\ m/s^2$$

→ When a particle is dropped from a height, the earth attracts it, hence the velocity of particle will go on increase as it comes nearer to earth & hence it will be max when it strikes the ground, so g is +ve when moves downwards.

→ When moves upwards during projectile, the velocity will reduce(↓) & velocity is zero when it reaches the max. height. so g is -ve.

RECTILINEAR MOTION

Horizontal Motion	Vertical Motion	Vertical upward Motion
1) $v = u + at$	$v = u + gt$	$v = u - gt$
2) $s = ut + \frac{1}{2}at^2$	$h = ut + \frac{1}{2}gt^2$	$h = ut - \frac{1}{2}gt^2$
3) $v^2 = u^2 + 2as$	$v^2 = u^2 + 2gh$	$v^2 = u^2 - 2gh$
4) $S_n = u + \frac{a}{2}(2n-1)$	$h_n = u + \frac{g}{2}(2n-1)$	$h_n = u - \frac{g}{2}(2n-1)$

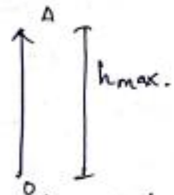
→ when a body starts moving vertically downwards its initial velocity $u = 0$.

→ when a body is projected vertically upwards, at the highest point, its final velocity $v = 0$.

* UPWARD MOTION:

→ Maximum height attained by upward particle.

$$\boxed{h_{max} = \frac{u^2}{2g}}$$



→ Time taken by the particle to reach max. height.

$$\boxed{t = u/g}$$

t = time taken by particle, projected upward.

Total time taken to return surface

$= 2 \times$ time up

$T = 2t$

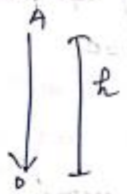
$$\boxed{T = \frac{2u}{g}}$$

$u = \sqrt{2g \times h_{max}}$ [If h_{max} is known]

$u = t \times g$ [If time is known]

* DOWNWARD MOTION:

Striking velocity of the particle moving downward from the portion of rest.



$v = \sqrt{2gh}$

7. A stone is thrown vertically upwards. It reaches the maximum height 12 m. Determine
(i) the velocity with which the stone was thrown
(ii) the time taken to reach the max. height
(iii) Total time taken by the stone, to return to the ground surface, after projected upwards

Solution

Given max. height attaining $h_{max} = 12\ m$

(i) Velocity with which the stone was thrown (u)

We know at max height, velocity $V = 0$

ie at $h_{max} = 12m$, $V = 0$

using this equation, $V^2 = u^2 - 2gh$

$$0 = u^2 - 2gh$$

$$u = \sqrt{2gh_{max}}$$

$$= \sqrt{2 \times 9.81 \times 12}$$

$$= 15.34\ m/s$$

(ii) Time taken to reach max. height.

We know at max. height velocity $V = 0$

using the eqn $v = u - gt$

$$0 = 15.34 - (9.81 \times t)$$

$$\therefore t = \frac{15.34}{9.81} = 1.56\ sec$$

(iii) Total time taken t

Total time taken by the stone T is equal to twice the time taken by the stone to reach the max. height

ie $T = 2t = 2 \times 1.5 sec = 3.12 sec$

2. A stone is projected upwards from the roof of a building with a velocity of 19.6 m/s & another stone is thrown downwards from the same point, three seconds later. If both the stones reach the ground at the same time, determine the height of the building take $g = 9.8 m/s^2$.

Solution

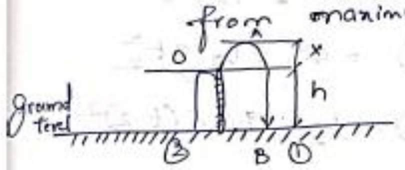
Let h be the height of the building. The stone (1) projected vertically upwards from the roof of building reach its max. height x & then starts moving downwards to strike the ground. The stone(2) thrown downwards three seconds later from the same point also reaches the ground. It is given that, both the stone reaches the ground at the same time. Now, let us consider the motion of the stones one by one.

motion of the first stone (thrown upwards)

Let, t = total time taken by the stone(1) to strike the ground.

t_1 = time taken to reach maximum height, x

t_2 = time taken to strike the ground, from maximum height



Clearly $t_1 + t_2 = t$

Consider the upward motion (from O to A)

Use the equation $V = u - gt$ $\left(u = 19.6 \text{ m/s}, \text{ time} = t_1\right)$

or $0 = 19.6 - 9.8 \times t_1$ (at max. hight $v = 0$)

or $t_1 = \dfrac{19.6}{9.8} = 2 \text{ sec}$

Using the equation $h = ut - \frac{1}{2} g t^2$

or $x = ut_1 - \frac{1}{2} g t_1^2$

$= (19.6 \times 2) - (\frac{1}{2} \times 9.8 \times 2^2)$

$= 19.6 \text{ m}$

Consider the downward motion (from A to B)

total distance = $(h+x)$; $u=0$; time $= t_2$

$= (h+19.6)$

But $t_2 = (t_1 - t_1) = (t-2)$ sec

Using the equation, $h = ut + \frac{1}{2}gt^2$ or

$$(h+x) = ut_2 + \frac{1}{2}gt_2^2$$

$$\text{or } (h+x) = ut_2 + \frac{1}{2}gt_2^2$$

$$\text{or } (h+x) = \frac{1}{2} \times 9.8 \times (t-2)^2$$

$$\text{or } (h+19.6) = 4.9(t-2)^2 \quad \text{......(i)}$$

motion of the second stone (thrown downwards)

$u = 0$; time taken $= (t-3)$ Since three seconds later)

Using the equation $h = ut + \frac{1}{2}gt^2$

$$\text{or } h = 0 + \left[\frac{1}{2} \times 9.8 \times (t-3)^2\right]$$

$$\text{or } h = 4.9(t-3)^2 \quad \text{......(ii)}$$

Subtract equation (ii) from equation (i)

$$\{(h+19.6) - h\} = \{4.9(t-2)^2 - 4.9(t-3)^2\}$$

$$\text{or } 19.6 = 4.9\{(t-2)^2 - (t-3)^2\}$$

$$= 4.9\{(t^2+4-4t) - (t^2+9-6t)\}$$

$= 4.9\{2t-5\}$

or $(2t-5) = \frac{19.6}{4.9} = 4$

$\therefore t = 9/2 = 4.5$ sec

Substitute $t = 4.5$ sec in equation (ii)

$h = 4.9(t-3)^2$

$= 4.9(4.5-3)^2$

$= 11.025$ m (Ans)

$\therefore$ height of the building is 11.025m