



Q

## COMPLEX INTEGRATION

### Cauchy's integration theorem:

If a function  $f(z)$  is analytic and its derivative  $f'(z)$  is continuous with all points inside and on a simple closed curve  $C$ , then

$$\int_C f(z) dz = 0.$$

### Cauchy's integral formula:

If  $f(z)$  is analytic inside and on a simple closed curve  $C$  and 'a' be any point inside  $C$ , then

$$\int_C \frac{f(z)}{z-a} dz = 2\pi i f(a).$$

Where integration being taken in the anti-clockwise direction around  $C$ .



① Evaluate  $\int_c \frac{\cos \pi z}{z-1} dz$  where  $c$  is

$$|z| = 2$$

Soln:

$$f(z) = \cos \pi z$$

$$a = 1$$

$$c : |z| = 2$$

$a = 1$  lies inside  $c$ ,  $|1| < 2$

By Cauchy's integral formula,

$$\int_c \frac{f(z)}{z-a} dz = 2\pi i f(a)$$

$$\int_c \frac{\cos \pi z}{z-1} dz = 2\pi i f(1)$$

$$f(1) = \cos \pi$$

$$= -1$$

② Evaluate  $\int_c \frac{dz}{(z-3)^2}$  where  $c$  is the

$$\text{circle } |z| = 1$$

Soln:  $a = 3$  lies outside the circle  $|z| = 1$ .