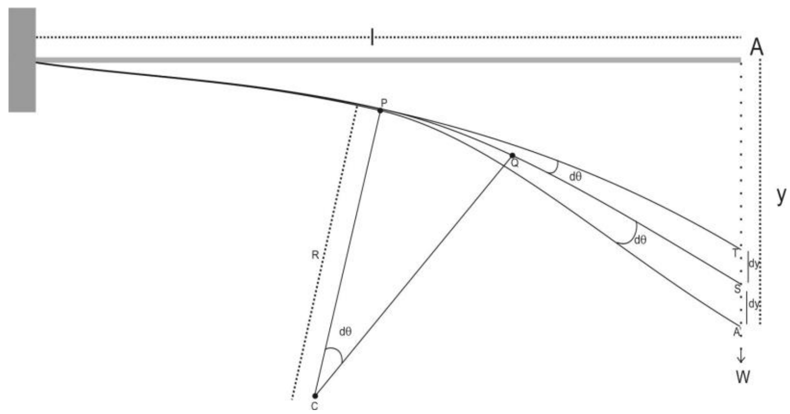
**CANTILEVER:**

A Cantilever is a beam fixed horizontally at one end and loaded at other end.

**Depression of a cantilever loaded at its ends:**
**Theory:**

The cantilever OA is fixed at O, its length is l and 'W' be the weight loaded at other end. Due to load it moves to OA'.




Let us consider an element PQ of the beam of length dx, at a distance x from fixed end. 'C' be the centre of curvature and R be the radius of curvature.

Due to the load (W) applied at free end, an external couple is created between A and Q, arm of couple is (l - x).

External bending movement = $W \times (l - x)$ --------------------(1)

Internal bending movement $= \frac{YI}{R}$ --------------------(2)

Under equilibrium condition,

External bending movement = Internal bending movement

$$\therefore R = \frac{YI}{W(l-x)} \text{ - - - - - - - - (3)}$$

From the figure arc length

$$PQ = Rd\theta = dx$$

$$d\theta = \frac{dx}{R} \quad \text{--------------------(4)}$$

On substituting R Value

$$d\theta = \frac{dx}{YI} W(l-x) \quad \text{--------------(5)}$$

From ΔQAS

$$\sin\theta = \frac{dy}{l-x}$$

$$dy = d\theta(l - x) \quad ------------(6)$$

On sub (5) in (6) we get,

$$dy = \frac{W(l - x)^2}{YI} dx$$

∴Total depression is by integrating the above within the limit 0 to l.

$$\therefore y = \frac{W}{YI} \int_0^l (l - x)^2 dx$$

On solving we get,

$$y = \frac{W}{YI} \cdot \frac{l^3}{3}$$

This equation gives the depression of the cantilever.

<u>Special Cases:</u>

(i) Rectangular cross section. For,

$$I = \frac{bd^3}{12}$$

Depression produced $y = \frac{4Wl^3}{Ybd^3}$

(ii) Circular cross section, For $I = \frac{\pi r^4}{4}$

Depression Produced $y = \frac{4Wl^3}{3\pi r^4 Y}$

'r' is the radius of the circular cross section.

**Young's Modulus(Y):**

It is defined as the ratio between the longitudinal stress to the longitudinal strain, within the elastic limits.

$$\text{Young's Modulus (Y)} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

**Uniform Bending**

Consider a beam supported by two knife edges A and B. Length between A and B is 'l'. Let equal weights (W), be added to either end of the beam C and D.

Let the distance CA and BD = a. Due to load applied the elevation 'x' produced from F to E. Let W be the reaction produced at the points A and B which acts vertically upwards.

From the fig

The external bending moment about P, written as

$$Mp = Wa$$

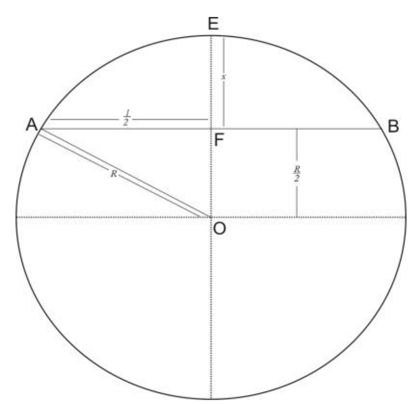
We know the internal bending moment = $\frac{YI_g}{R}$

On comparing (1) and (2)

$$Wa = \frac{YI_g}{R}$$

Here it is found that the elevation 'x' forms an arc of the circle of radius R

From ΔAFO




$$OA^2 = AF^2 + FO^2$$

Since OF= FE, therefore we can write $OA^2 = AF^2 + FE^2$

(or) $AF^2 = OA^2 - FE^2$

$$AF^2 = FE\left[\frac{OA^2}{FE} - FE\right]$$

Here, AF = $\frac{l}{2}$ , FE = $x = \frac{R}{2}$; OA=R

$$\left(\frac{l}{2}\right)^2 = x\left[\frac{R^2}{R/2} - x\right]$$

$$\left(\frac{l^2}{4}\right) = 2xR - x^2$$

If elevation x is small

$$\left(\frac{l^2}{4}\right) = 2xR$$

$$x = \frac{l^2}{8R} \quad \text{(or)} \quad R = \frac{l^2}{8x}$$

$$(or) \quad Wa = \frac{YI_g}{l^2/8x} \qquad \text{(or)} \quad Wa = \frac{8YI_g x}{l^2}$$

On Rearranging

The elevation of point 'E' above 'A' is $x = \frac{Wal^2}{8YI_g}$