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SNS College of Technology, Coimbatore - 35.
(An Autonomous Institution)
Internal Assessment -II
Academic Year 2022-2023(Even)
Second Semester
Department of Mathematics
19MAB102- Integral Calculus & Laplace Transforms

B

Time: 1.30 Hours**Maximum Marks: 50**

PART – A (5 x 2 = 10 MARKS)
ANSWER ALL QUESTIONS

- | | | | BLO
OMS |
|----|---|-----|--------------------|
| 1. | State Stoke's theorem. | CO2 | (R) |
| 2. | Find $\nabla \cdot \vec{F}$ for the vector point function $\vec{F} = xz^3\vec{i} - 2x^2yz\vec{j} + 2yz^4\vec{k}$ at the point (1,-1,1). | CO2 | (U) |
| 3. | List the conditions necessary and sufficient condition for a function to be analytic. | CO3 | (R) |
| 4. | Determine whether the function $w = 2xy + i(x^2 + y^2)$ is analytic or not. | CO3 | (U) |
| 5. | Obtain the fixed points of the transformation $w = \frac{2z-5}{z+4}$ | CO3 | (U) |

PART –B (2 x 13 = 26 MARKS)
ANSWER ALL QUESTIONS

- | | | | |
|-------|--|-----|-------------------|
| 6. | a)
Verify Stoke's Theorem for the vector $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region $x = 0; y = 0; x = a; y = b$ | CO2 | (App)

(13) |
| (or) | | | |
| b)(i) | Show that the function $u = x^3 + x^2 - 3xy^2 + 2xy - y^2$ is harmonic and hence find the analytic function. | CO3 | (App)
(8) |
| (ii) | Find the bilinear transformation that maps $0, 1, \infty$ onto $i, 1, -i$ | CO3 | (An)
(5) |

7. a)(i) If $f(z) = u + iv$ is an regular function of z in the domain D , then show that
- $$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] |f(z)|^2 = 4 |f'(z)|^2$$
- (An)
CO3 (10)
- (ii) Interpret the critical points of the transformation $w = z + \frac{1}{z}$
- (An)
CO3 (3)
- (or)
- b)(i) Prove that $u = \frac{1}{2} \log(x^2 + y^2)$ is a harmonic function and establish its Harmonic Conjugate.
- (App)
CO3 (8)
- (ii) Identify the image of $|z - 2i| = 2$ under the translation $w = \frac{1}{z}$
- (App)
CO3 (5)
8. a) Verify Gauss divergence theorem for the vector $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ over the rectangular parallelepiped $0 \leq x \leq a; 0 \leq y \leq b; 0 \leq z \leq c$.
- (App)
CO2 (14)
- (or)
- b) Discuss in detail about the Applications of flow problems and categorize using analytic functions.
- (App)
CO3 (14)

R/U: Remember/ Understand **App:** Apply **An:** Analyze **E:** Evaluate **C:** Create

Prepared by

Verified by

Dean (S & H)